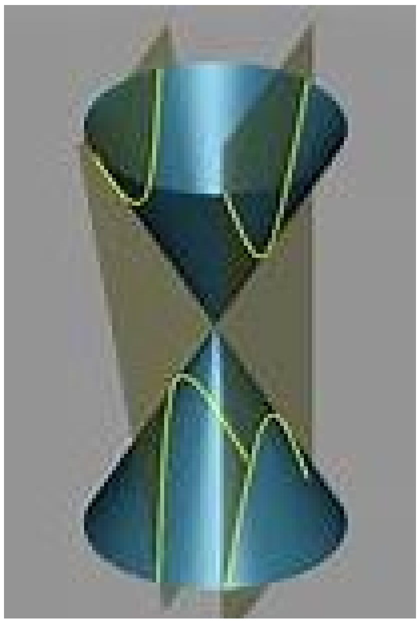
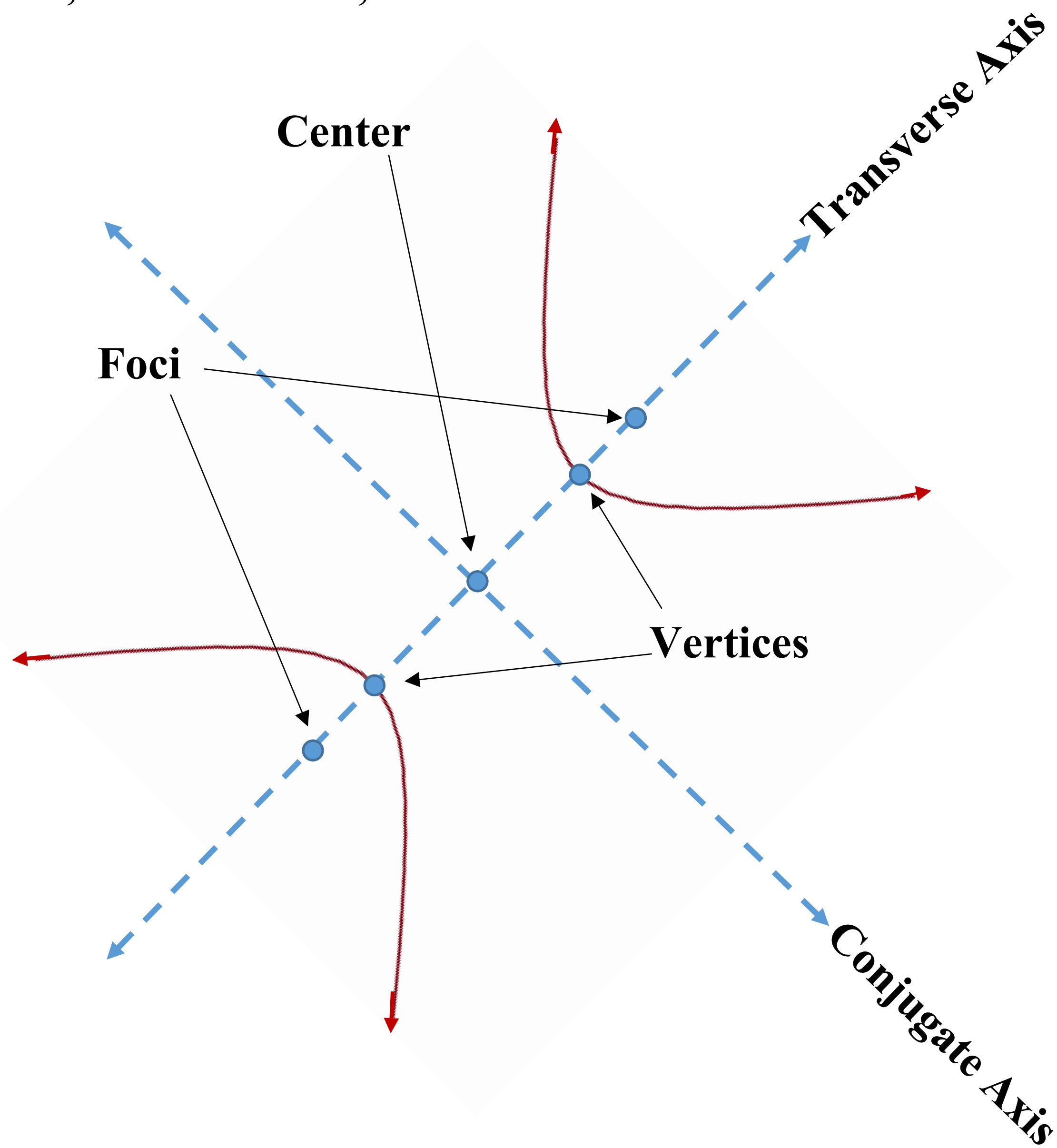
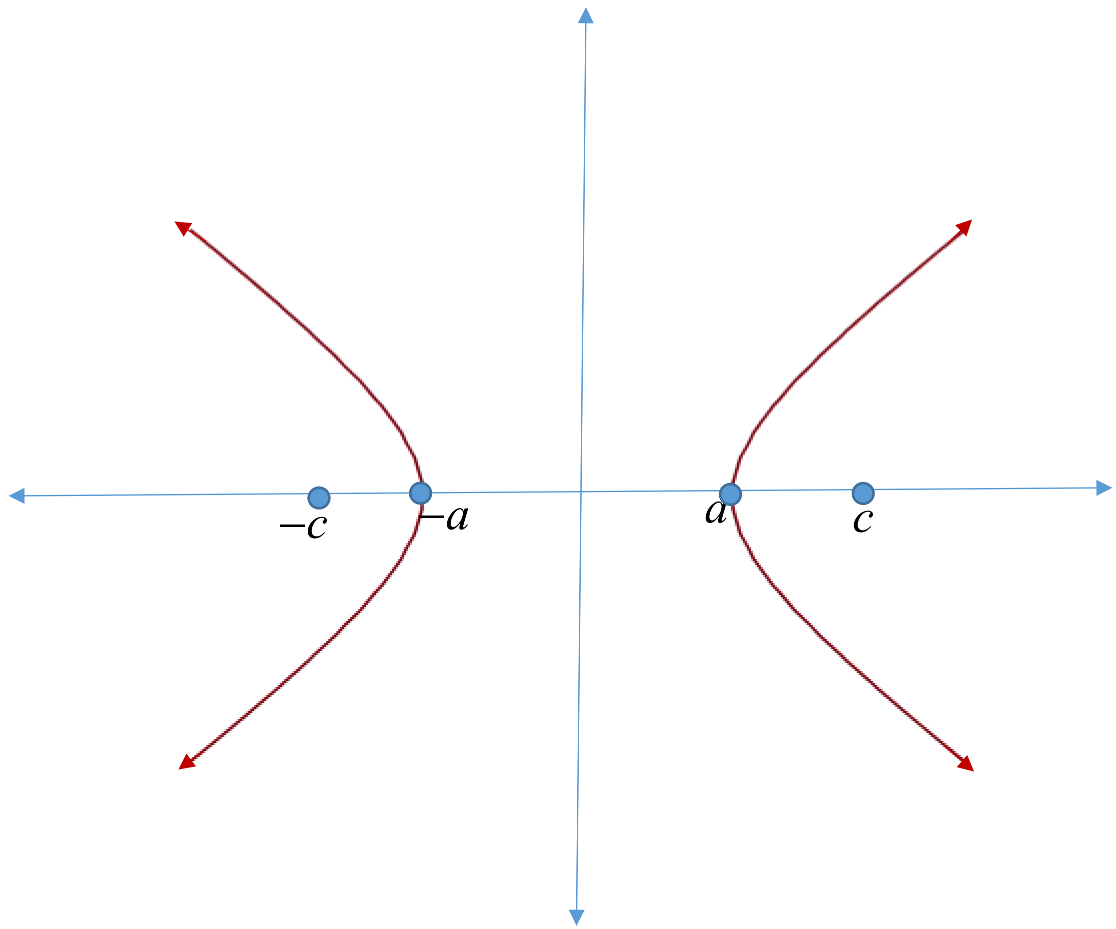
Hyperbola: The set of points in the plane the difference of whose distances to two fixed points, called the foci, is a constant.

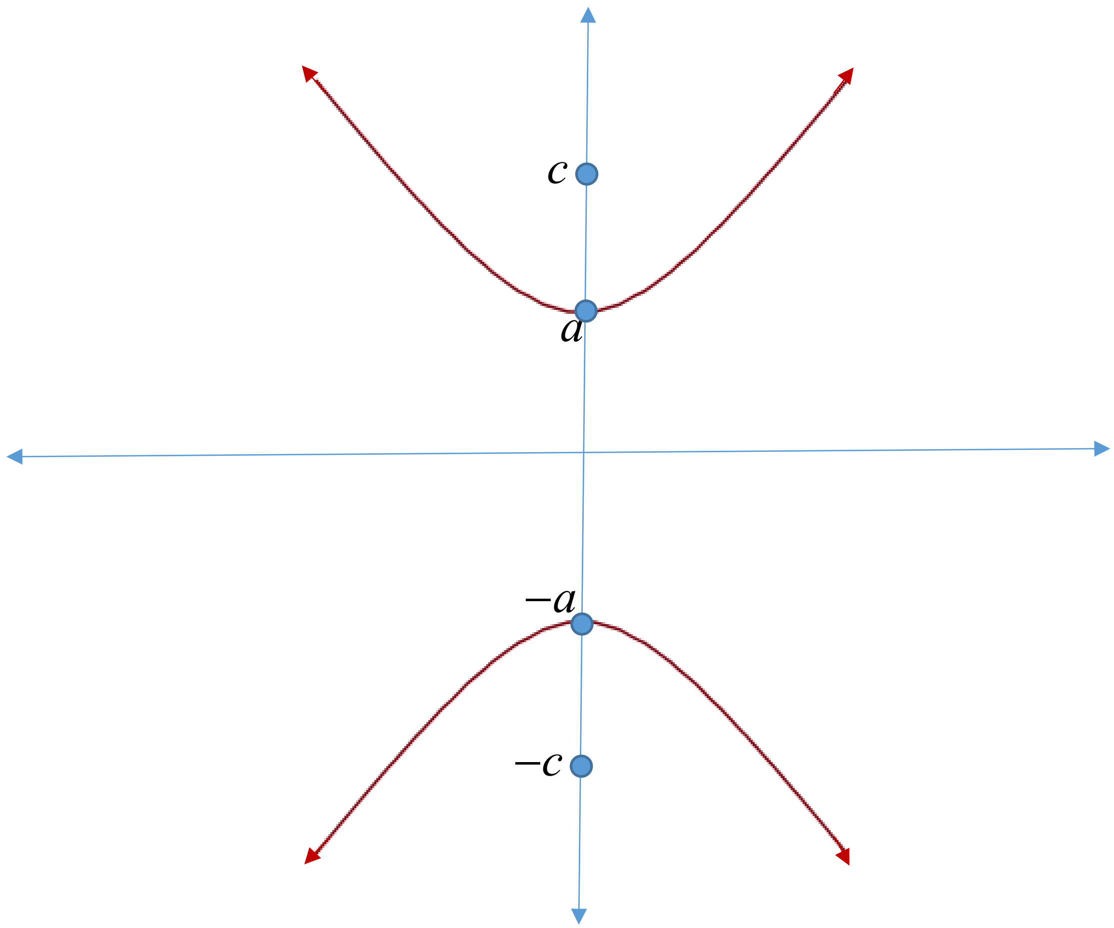




Hyperbolas centered at the origin with transverse and conjugate axes of the *x* and *y* axes.



$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1,\; c^2=a^2+b^2$$



$$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1,\ c^2 = a^2 + b^2$$

Where is *b* on the graph?

<u>Asymptotes:</u> Hyperbolas have a pair of asymptotes.

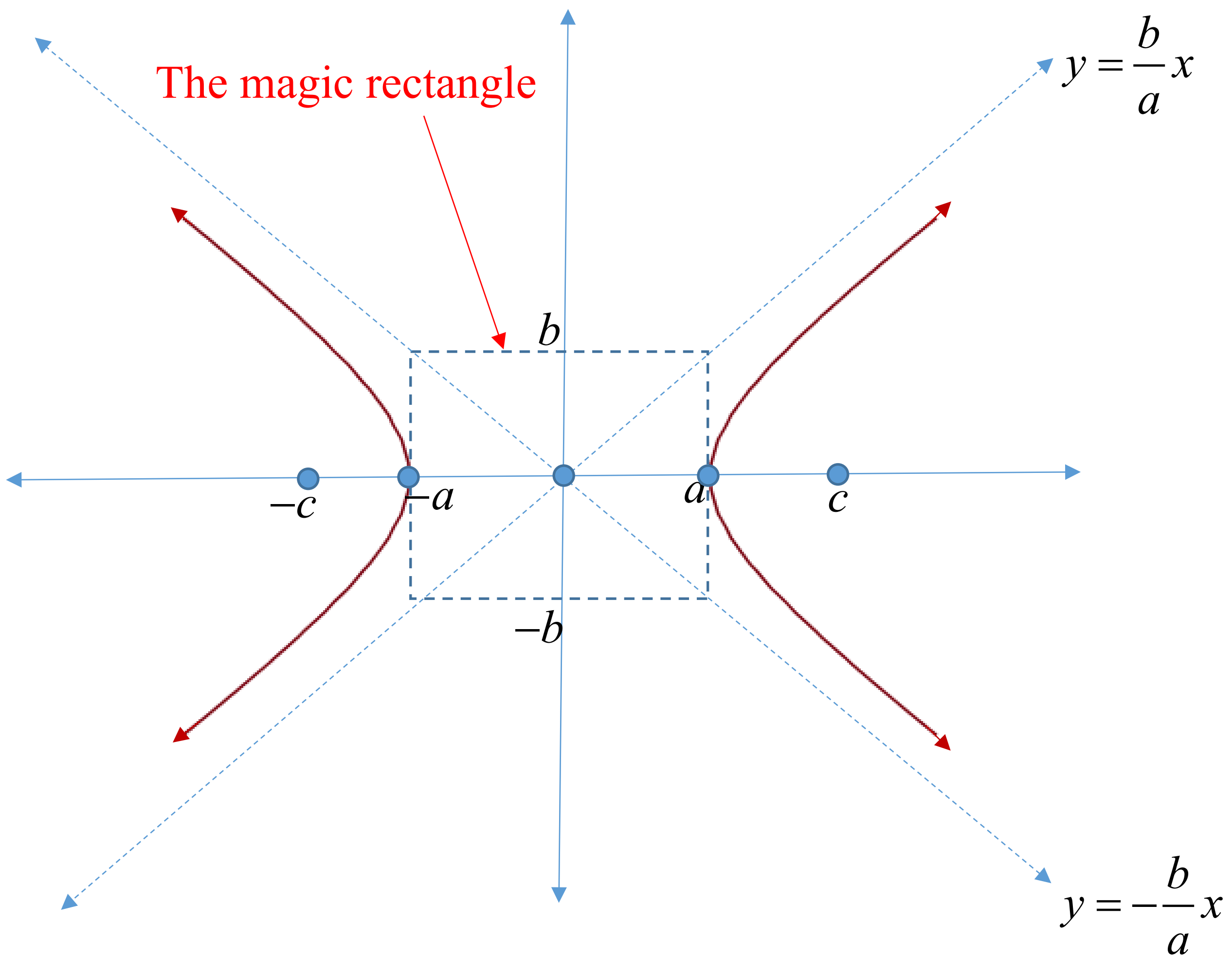
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1$$

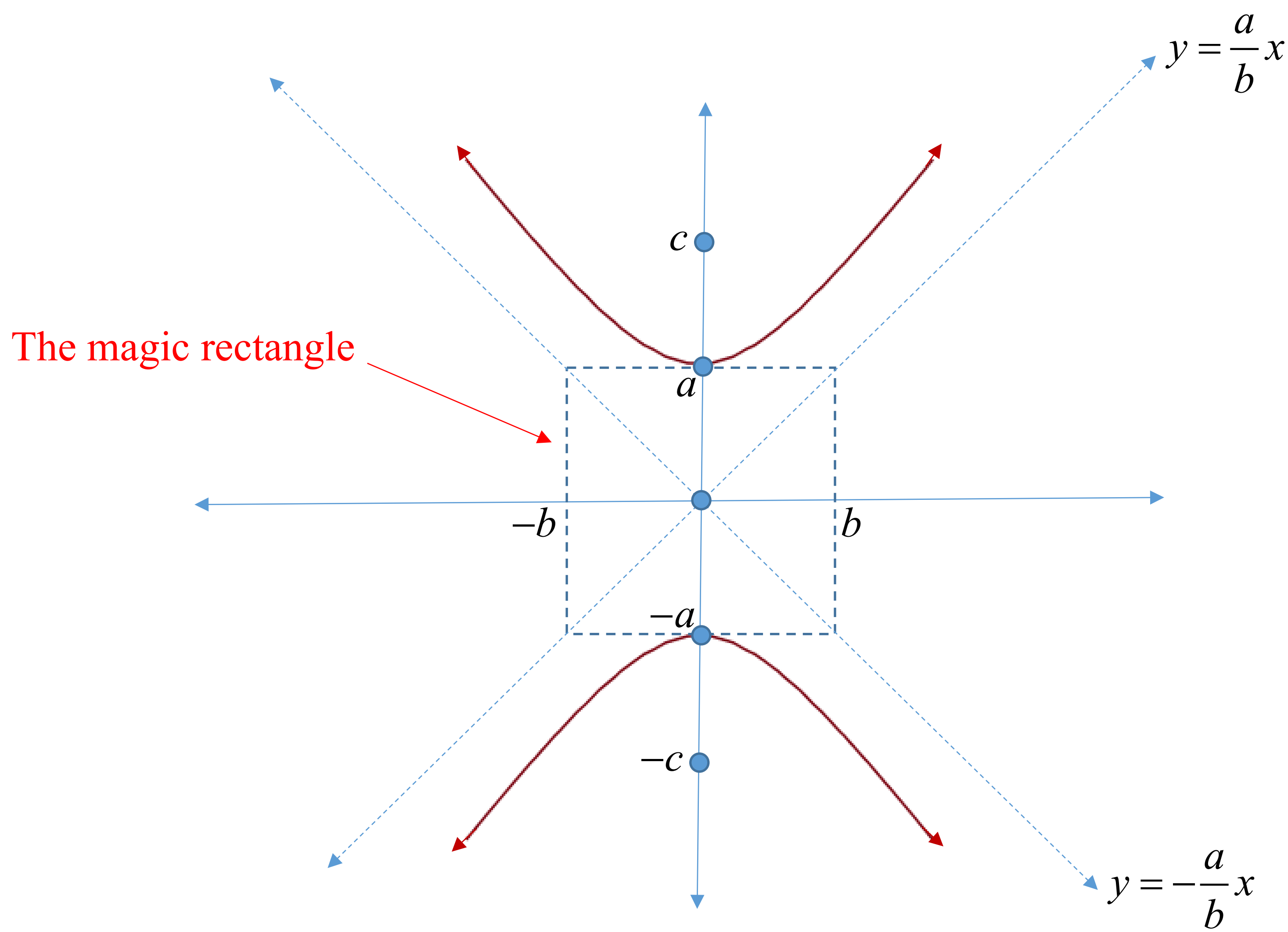
$$y^2 = \frac{b^2 x^2}{a^2} - b^2$$

$$y^2 = x^2\left[\frac{b^2}{a^2} - \frac{b^2}{x^2}\right]$$

So for x^2 large, $y^2 \approx \frac{b^2}{a^2}x^2$, so the asymptotes are $y = \pm\frac{b}{a}x$.



The extensions of the diagonals of the magic rectangle form the asymptotes.

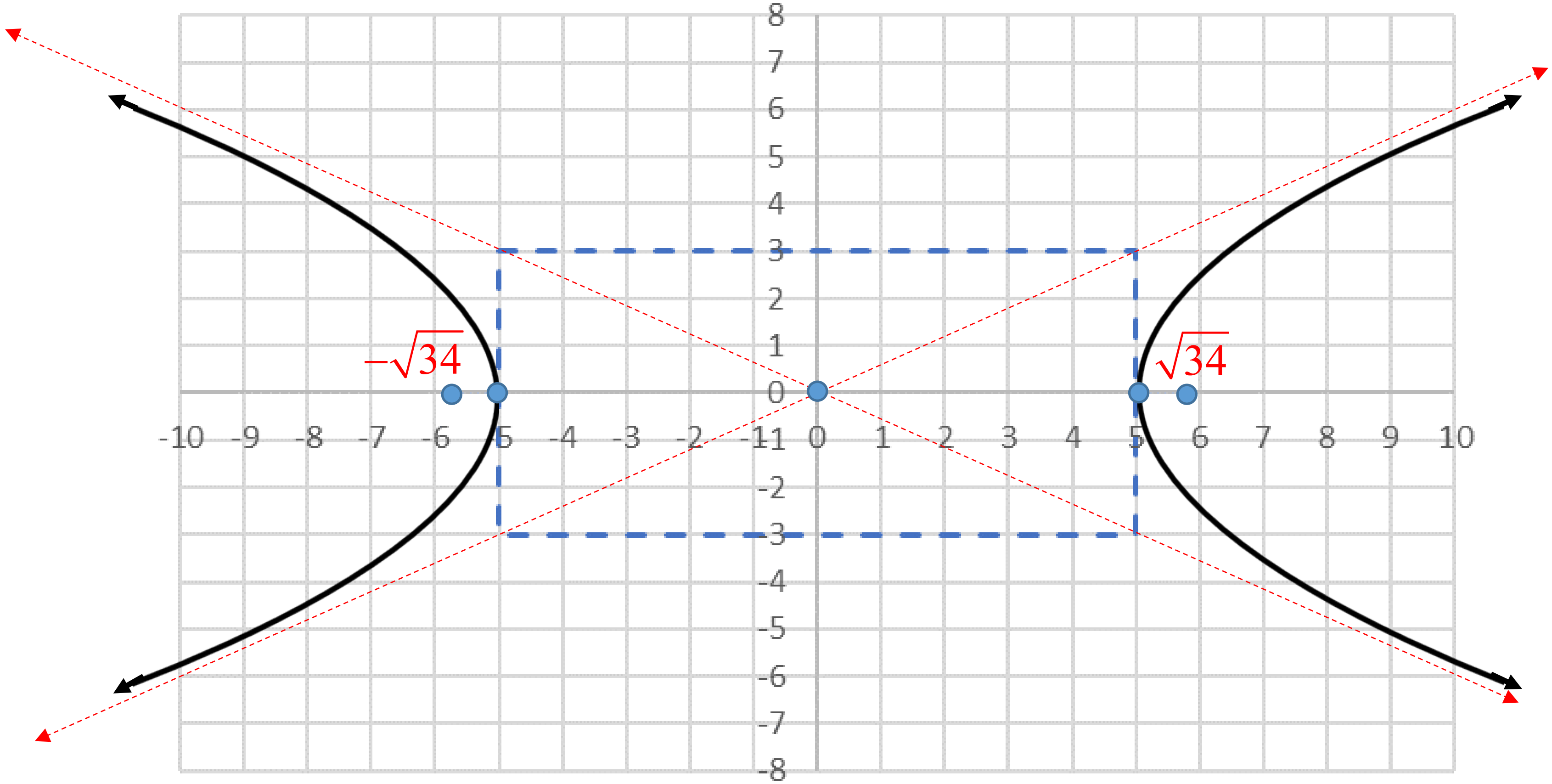


The extensions of the diagonals of the magic rectangle form the asymptotes.

Sketch the graphs of the following hyperbolas.

1. $\frac{x^2}{25}-\frac{y^2}{9}=1$

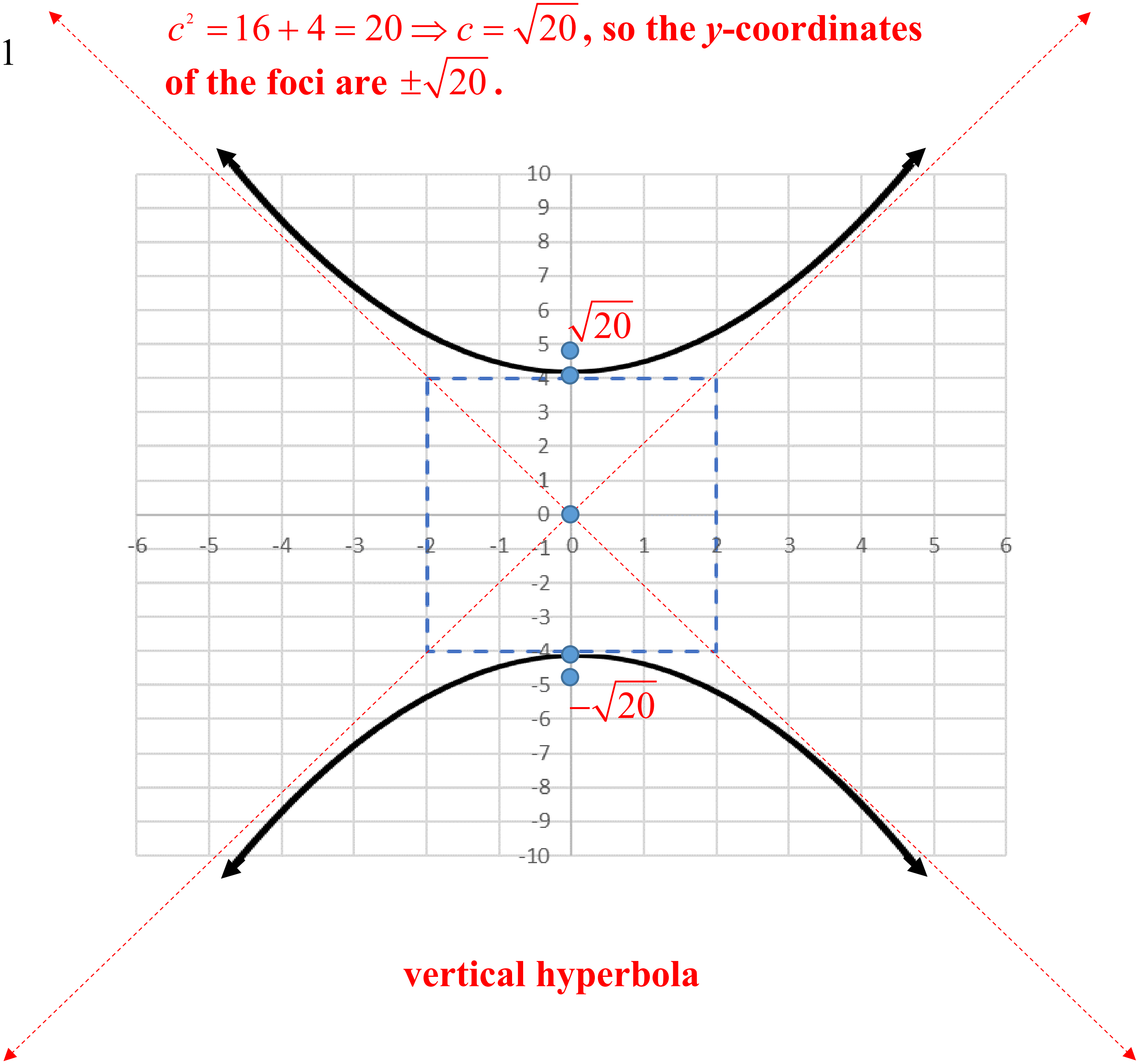
$c^2=25+9=34 \Rightarrow c=\sqrt{34}$**, so the *x*-coordinates of the foci are** $\pm\sqrt{34}$**.**



horizontal hyperbola

2. $\frac{y^2}{16} - \frac{x^2}{4} = 1$

$c^2 = 16 + 4 = 20 \Rightarrow c = \sqrt{20}$**, so the *y*-coordinates of the foci are** $\pm\sqrt{20}$**.**

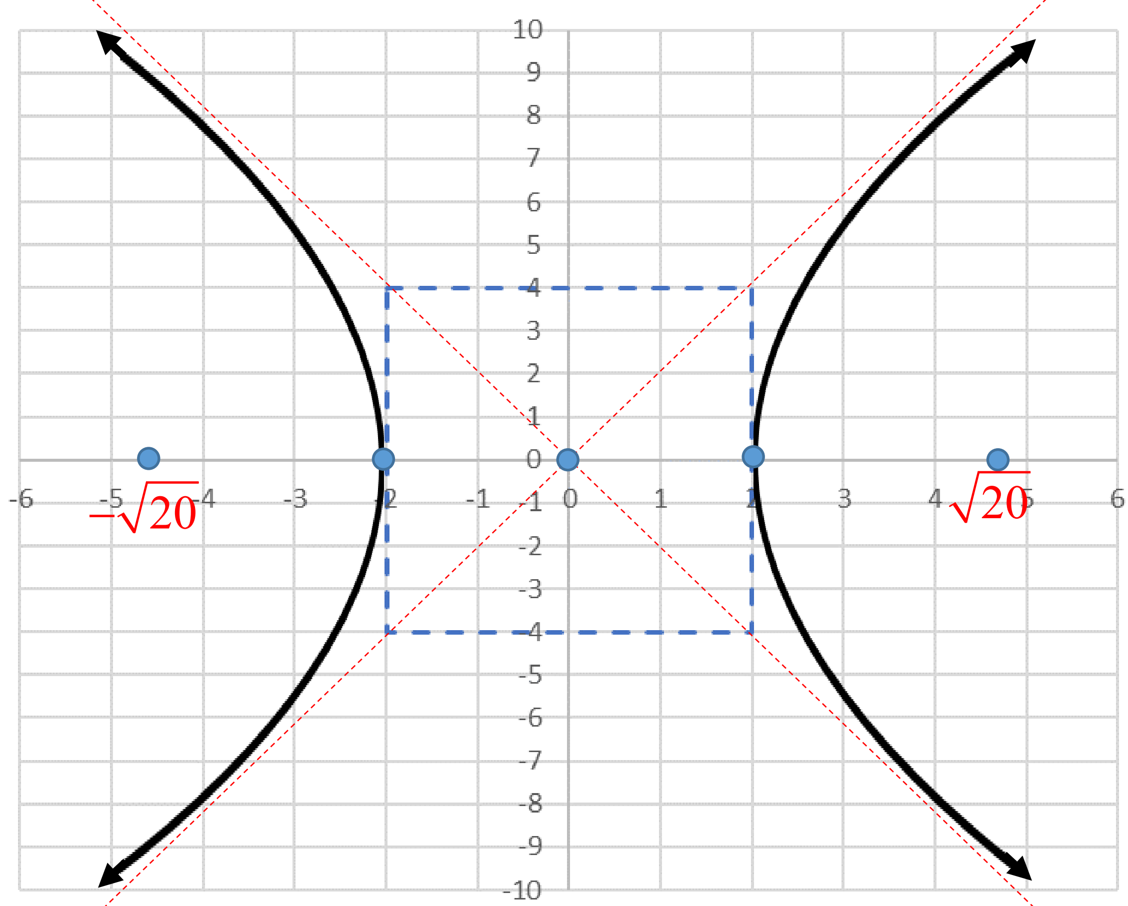


vertical hyperbola

3. $4x^2 - y^2 = 16$

$$\frac{4x^2 - y^2}{16} = \frac{16}{16} \Rightarrow \frac{x^2}{4} - \frac{y^2}{16} = 1$$

$c^2 = 16 + 4 = 20 \Rightarrow c = \sqrt{20}$, **so the *x*-coordinates of the foci are** $\pm\sqrt{20}$**.**



horizontal hyperbola

Not necessarily centered at the origin

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

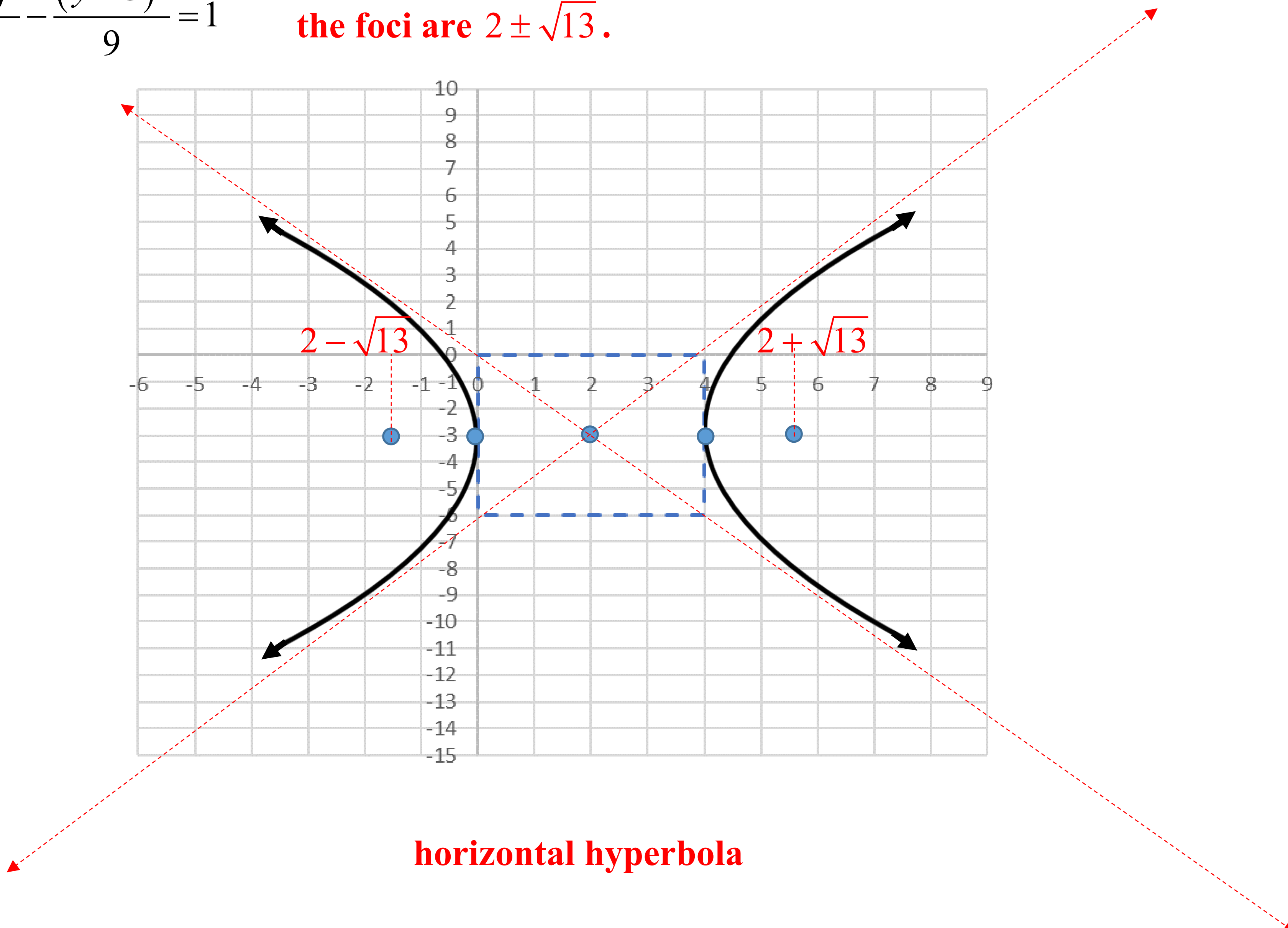
Or

$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

Sketch the following hyperbolas.

1. $\dfrac{(x-2)^2}{4}-\dfrac{(y+3)^2}{9}=1$

$c^2 = 4+9=13 \Rightarrow c=\sqrt{13}$**, so the *x*-coordinates of the foci are** $2\pm\sqrt{13}$**.**

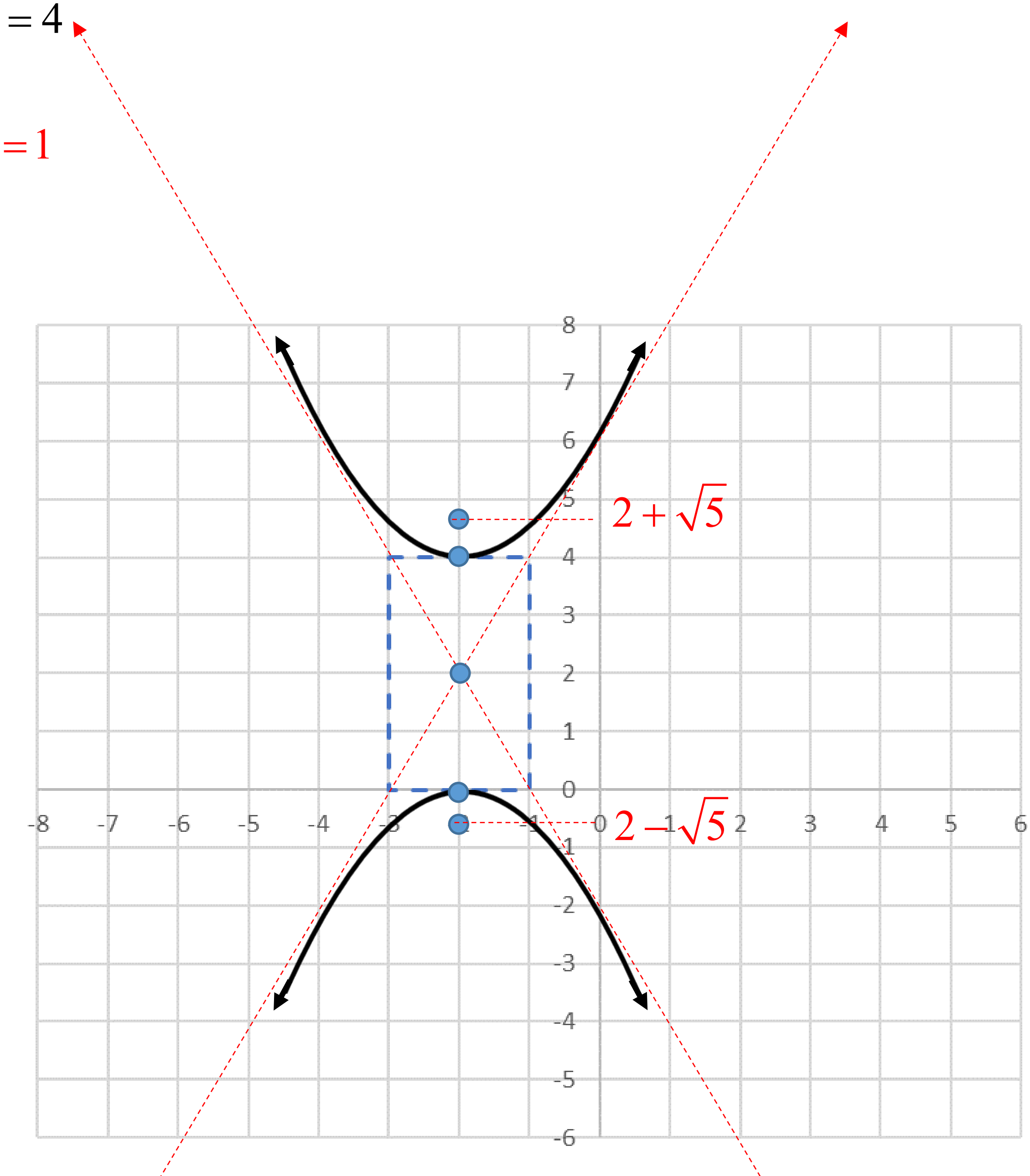


horizontal hyperbola

2. $(y-2)^2 - 4(x+2)^2 = 4$

$$\Rightarrow \frac{(y-2)^2}{4} - \frac{(x+2)^2}{1} = 1$$

$c^2 = 4 + 1 = 5 \Rightarrow c = \sqrt{5}$ **, so the *y*-coordinates of the foci are** $2 \pm \sqrt{5}$**.**

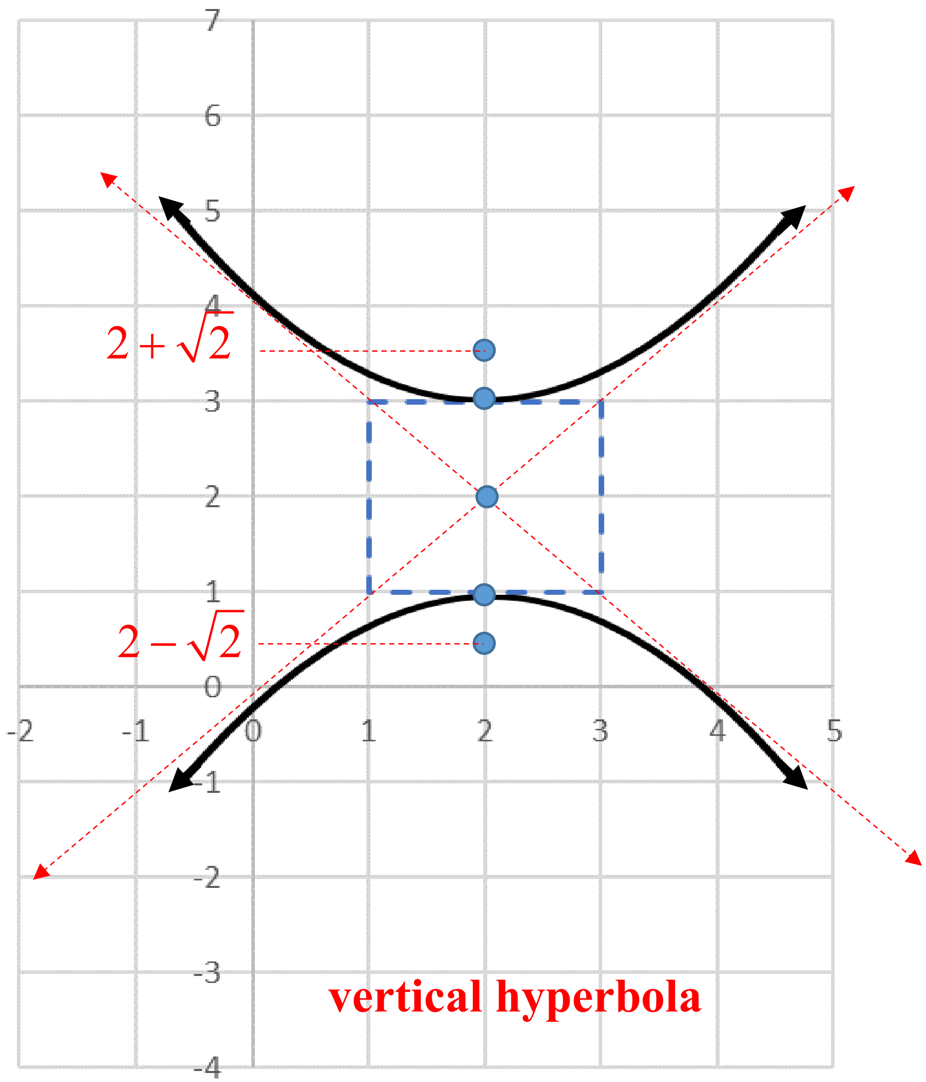


vertical hyperbola

3. $y^2 - x^2 - 4y + 4x - 1 = 0$

$$\left(y^2 - 4y\right) - \left(x^2 - 4x\right) = 1 \Rightarrow \left(y^2 - 4y + 4\right) - \left(x^2 - 4x + 4\right) = 1 \Rightarrow \frac{(y-2)^2}{1} - \frac{(x-2)^2}{1} = 1$$

$c^2 = 1 + 1 = 2 \Rightarrow c = \sqrt{2}$, **so the *y*-coordinates of the foci are** $2 \pm \sqrt{2}$**.**



Find an equation for the hyperbola being described.

1. Center at $(0,0)$, focus at $(3,0)$, and vertex at $(1,0)$.

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, **so from the vertex,** $a=1$, **from the focus,** $c=3$, **so** $b^2=9-1=8$, **and therefore an equation for the hyperbola is** $\boxed{\frac{x^2}{1}-\frac{y^2}{8}=1}$ **.**

2. Focus at $(0,6)$ and vertices at $(0,\pm 2)$.

$\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$, **so from the vertices,** $a=2$, **from the focus,** $c=6$, **so** $b^2=36-4=32$, **and therefore an equation for the hyperbola is** $\boxed{\frac{y^2}{4}-\frac{x^2}{32}=1}$**.**

3. Center at $(4,-1)$, focus at $(7,-1)$, and vertex at $(6,-1)$.

$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$, **from the center,** $\frac{(x-4)^2}{a^2}-\frac{(y+1)^2}{b^2}=1$, **from the vertex,** $a=2$, **from the focus,** $c=3$, **so** $b^2=9-4=5$, **and therefore an equation for the hyperbola is** $\boxed{\frac{(x-4)^2}{4}-\frac{(y+1)^2}{5}=1}$.

4. Focus at $(-4,0)$ and vertices at $(-4,4)$ and $(-4,2)$.

$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$, **from the vertices,** $\frac{(y-3)^2}{a^2}-\frac{(x+4)^2}{b^2}=1$, **from the vertices,** $a=1$, **from the focus,** $c=3$, **so** $b^2=9-1=8$, **and therefore an equation for the hyperbola is** $\boxed{\frac{(y-3)^2}{1}-\frac{(x+4)^2}{8}=1}$.