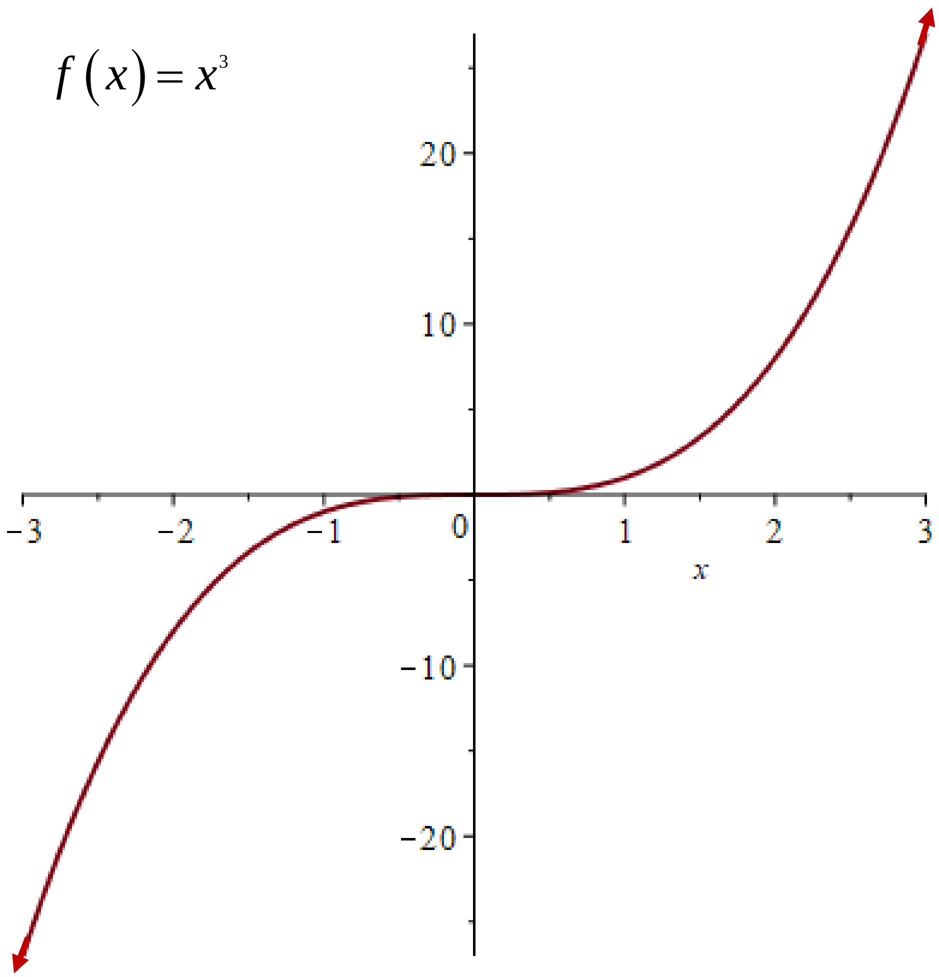
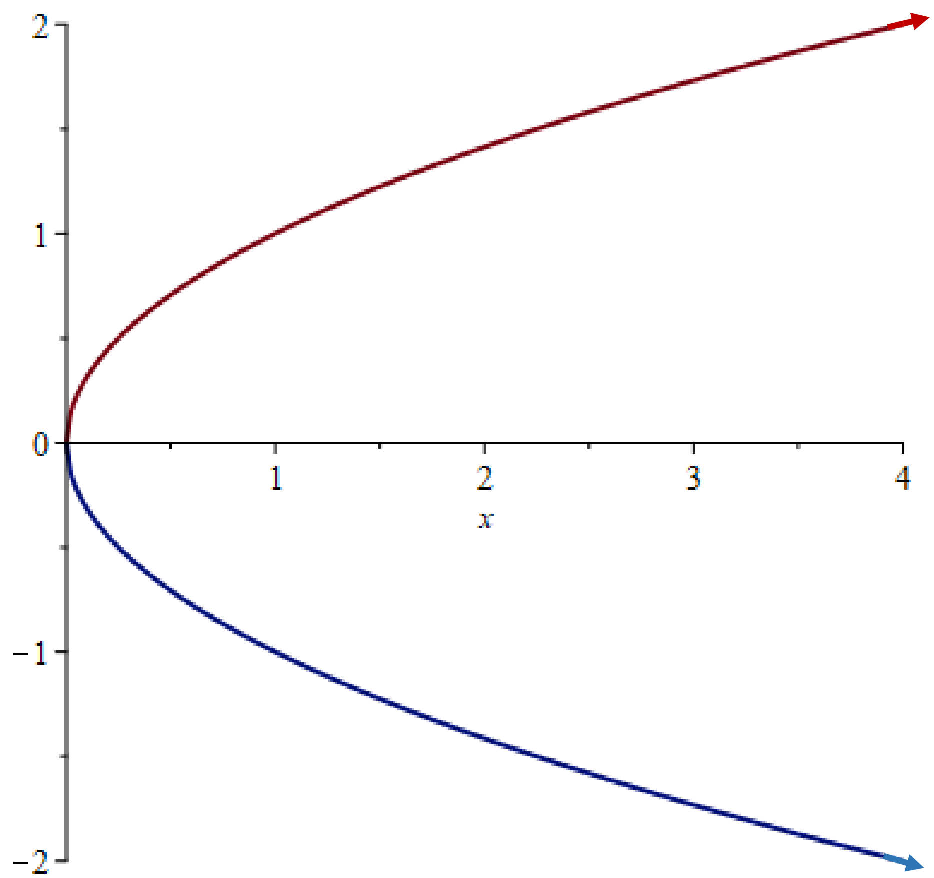
Review of Representing curves in the plane:

Graph of a function of *x*:



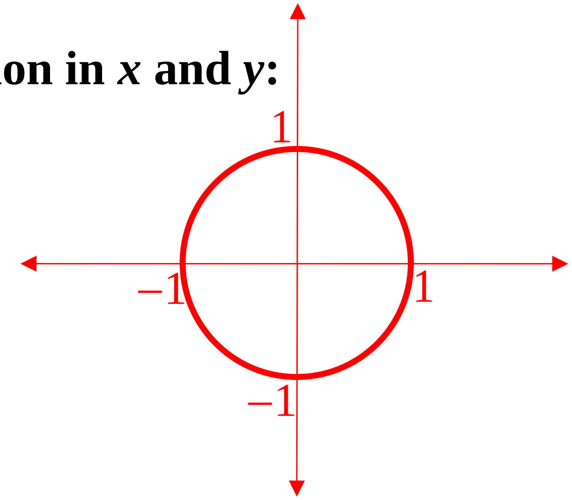
Graph of a function of *y*:

$$g(y) = y^2$$



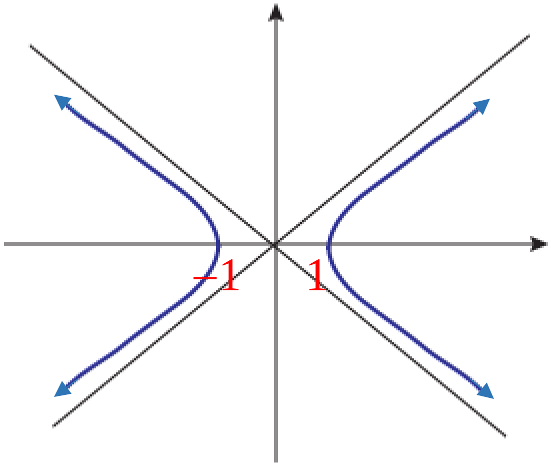
Graph of the solutions of an equation in *x* and *y*:

$$x^2 + y^2 = 1$$



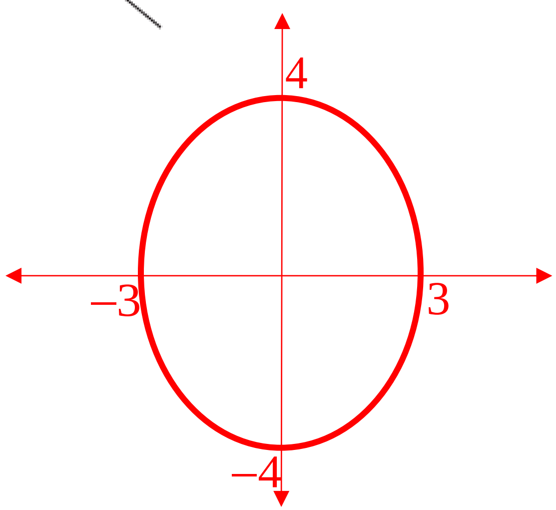
circle

$$x^2 - y^2 = 1$$



hyperbola

$$\frac{x^2}{9} + \frac{y^2}{16} = 1$$



ellipse

Parametric Equations:

$$\begin{cases} x = f(t) \\ \\ y = g(t) \end{cases} \quad ;\ t \text{ is restricted to some interval}$$

t **is called the parameter.**

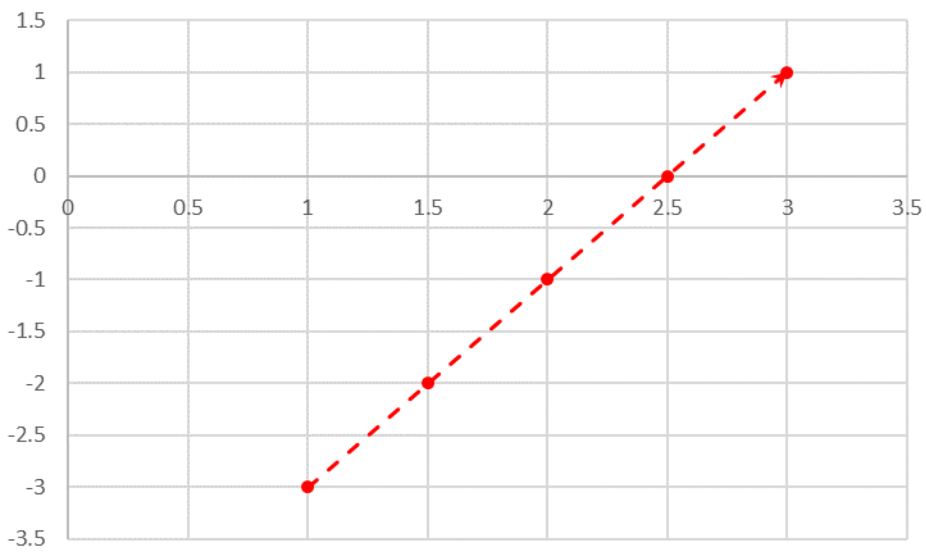
Here is another way to represent a curve. The *x* and *y* coordinates of the points on the curve are given by formulas involving a third variable called a parameter.

Examples:

1. $\begin{cases} x = t + 1 \\ y = 2t - 3 \end{cases} \quad ;0 \le t \le 2$

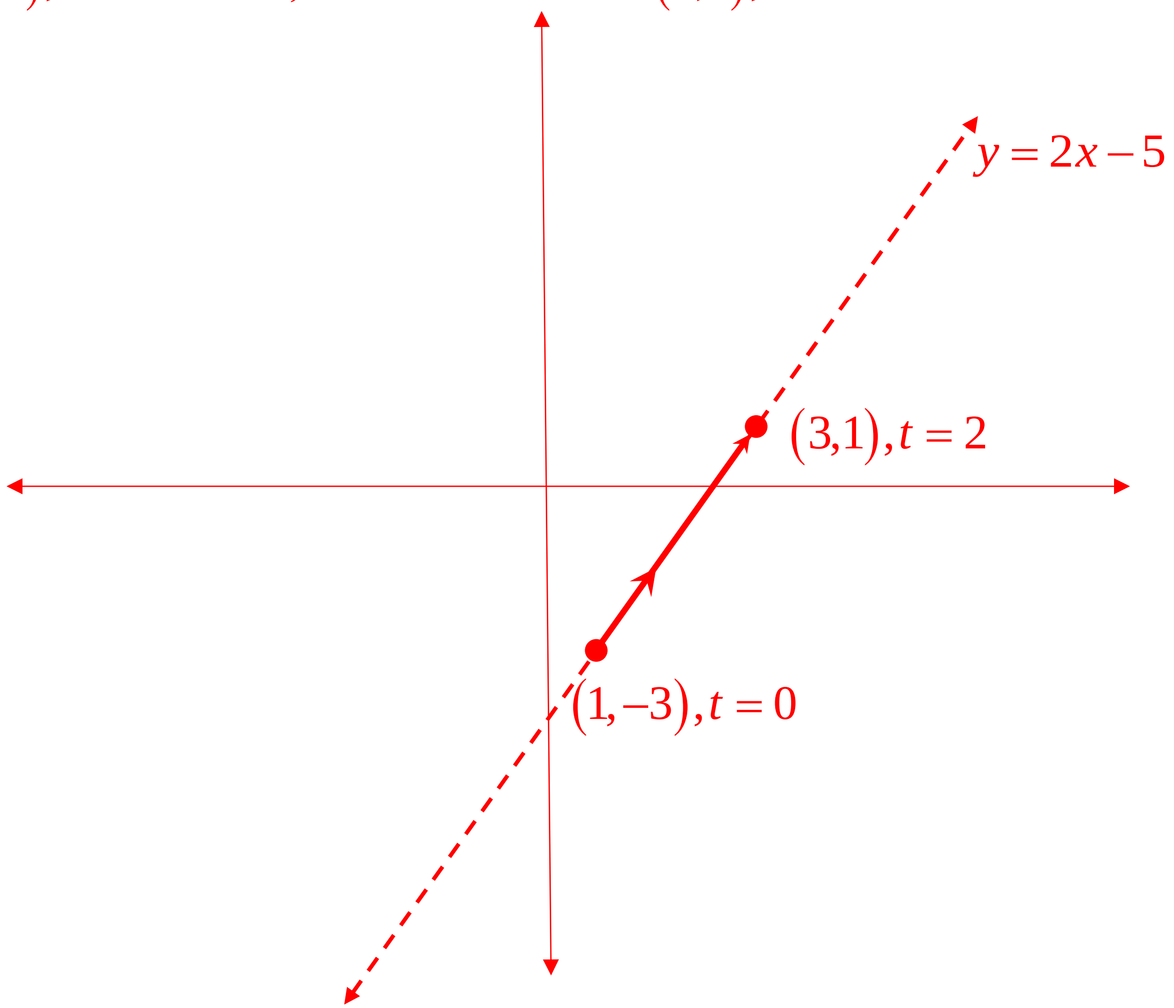
t	x	y	(x, y)
0	**1**	**-3**	$(1,-3)$
.5	**1.5**	**-2**	$(1.5,-2)$
1	**2**	**-1**	$(2,-1)$
1.5	**2.5**	**0**	$(2.5,0)$
2	**3**	**1**	$(3,1)$

We can make a table of points on the curve corresponding to selected values of the parameter to get an idea of the shape of the curve.



The parametric equations appear to be tracing out a line segment from the point $(1,-3)$ to the point $(3,1)$.

Another method for dealing with parametric equations is called eliminating the parameter. We'll solve for *t* in the first equation and substitute into the second equation to get $y = 2(x-1)-3 \Rightarrow y = 2x-5$**. So now we know for sure, that the parametric equations will trace a portion of the graph of** $y = 2x-5$**. We know that we'll start at** $(1,-3)$**, when** $t=0$**, and we'll end at** $(3,1)$**, when** $t=2$**.**

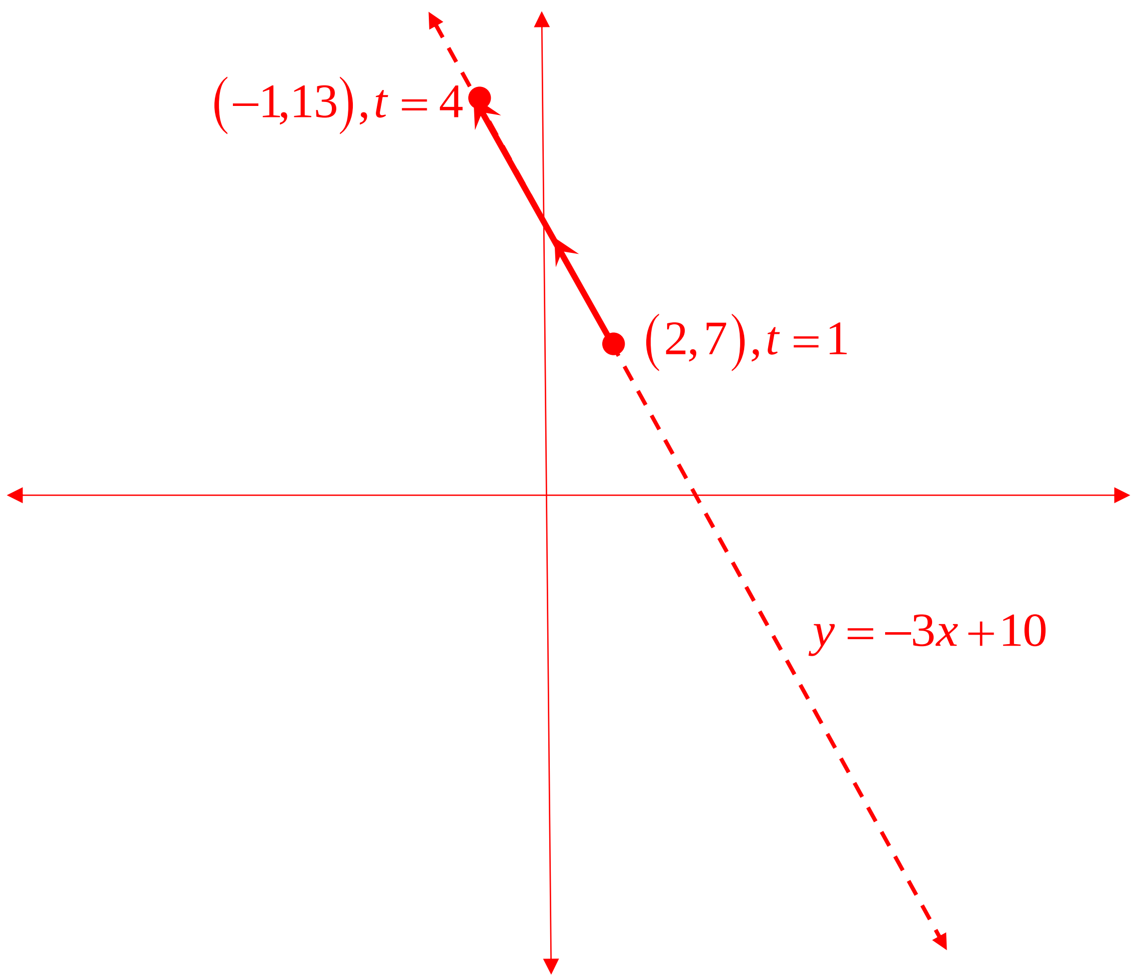


2. $\begin{cases} x = -t + 3 \\ \\ y = 3t + 1 \end{cases} ;1 \le t \le 4$

Let's eliminate the parameter:

$x = -t + 3 \Rightarrow t = 3 - x \Rightarrow y = 3(3 - x) + 1 \Rightarrow \boxed{y = -3x + 10}$**, a line.**

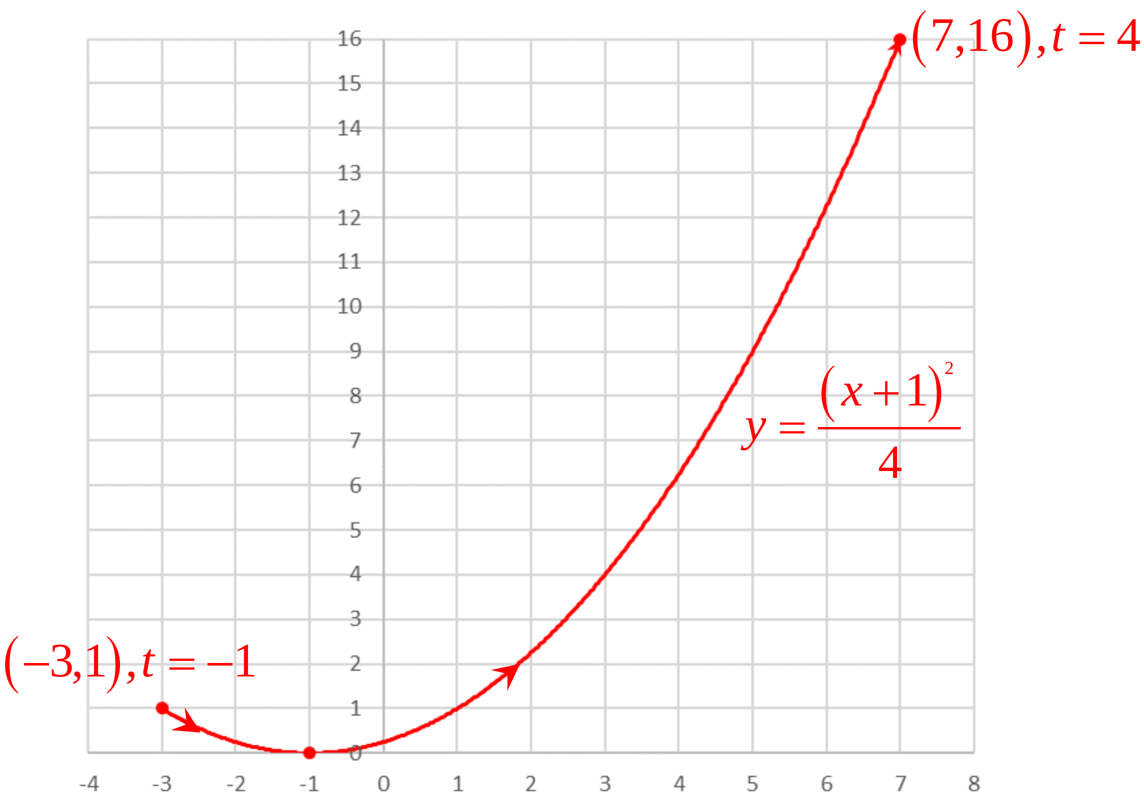
The parametric equations trace the segment from right to left.



3. $\begin{cases} x = 2t - 1 \\ y = t^2 \end{cases} ; -1 \le t \le 4$

Let's eliminate the parameter: $x = 2t - 1 \Rightarrow t = \dfrac{x+1}{2} \Rightarrow y = \left(\dfrac{x+1}{2}\right)^2 \Rightarrow \boxed{y = \dfrac{(x+1)^2}{4}}$**, a parabola.**

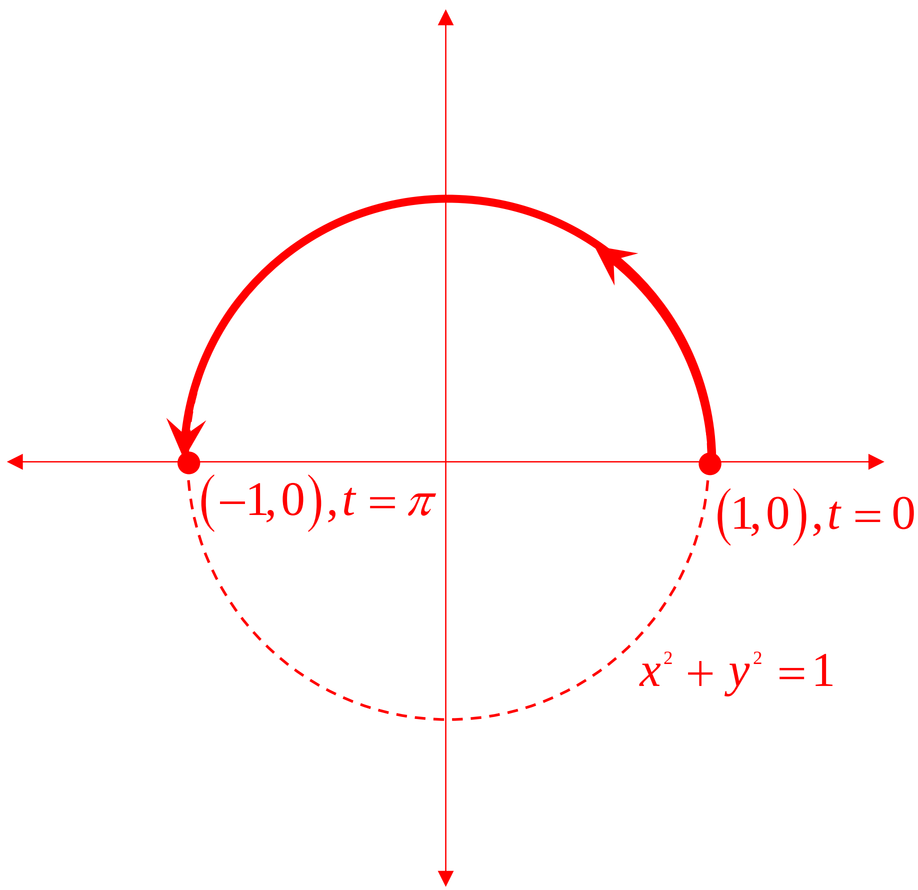
The parametric equations trace the parabolic segment from left to right.



4. $\begin{cases} x = \cos t \\ y = \sin t \end{cases} ; 0 \le t \le \pi$, $\sin^2\theta + \cos^2\theta = 1$

$x^2 = \cos^2 t, y^2 = \sin^2 t \Rightarrow x^2 + y^2 = \cos^2 t + \sin^2 t = 1 \Rightarrow \boxed{x^2 + y^2 = 1}$**, a circle.**

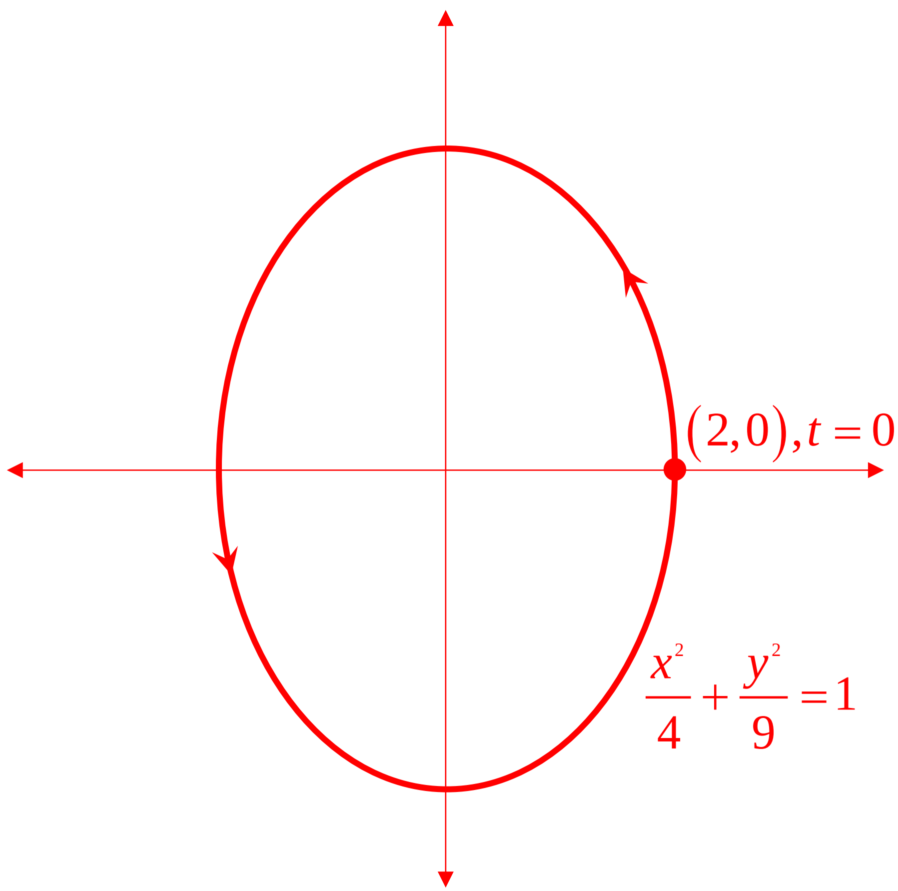
The parametric equations trace the upper half of the unit circle in the counterclockwise direction.



5. $\begin{cases} x = 2\cos t \\ y = 3\sin t \end{cases} ;0 \le t < 2\pi$ **,** $\sin^2\theta + \cos^2\theta = 1$

$\frac{x}{2} = \cos t, \frac{y}{3} = \sin t \Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = \cos^2 t + \sin^2 t = 1 \Rightarrow \boxed{\frac{x^2}{4} + \frac{y^2}{9} = 1}$**, an ellipse.**

The parametric equations trace the entire ellipse in the counterclockwise direction.

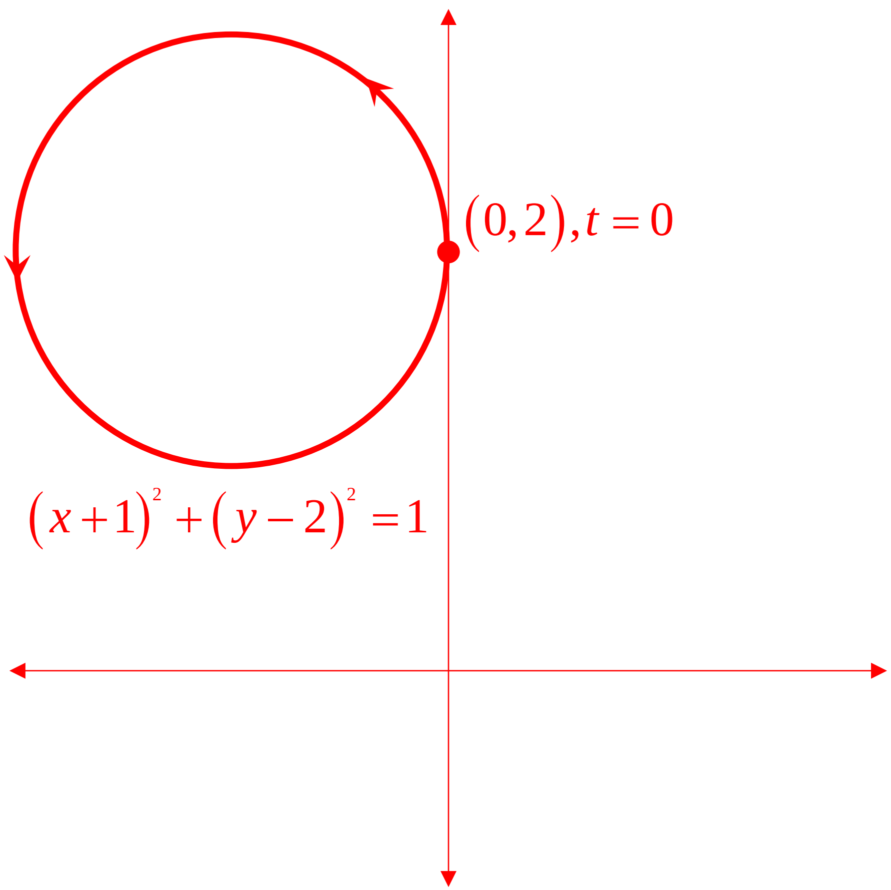


6. $\begin{cases} x = \cos t - 1 \\ y = \sin t + 2 \end{cases} ; 0 \le t < 2\pi$ **,** $\sin^2\theta + \cos^2\theta = 1$

$x+1=\cos t, y-2=\sin t \Rightarrow (x+1)^2+(y-2)^2=\cos^2 t+\sin^2 t=1 \Rightarrow \boxed{(x+1)^2+(y-2)^2=1}$**,**

a circle.

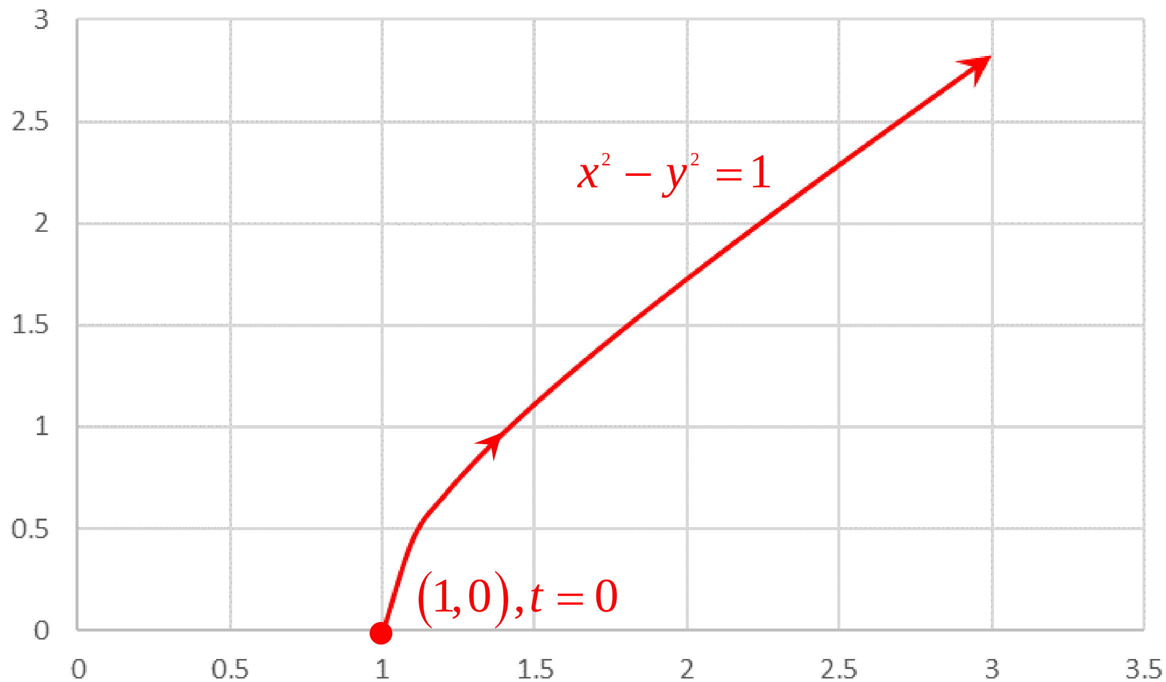
The parametric equations trace the entire circle in the counterclockwise direction.



7. $\begin{cases} x = \sec t \\ y = \tan t \end{cases} ;0 \le t < \dfrac{\pi}{2}$, $\quad \sec^2\theta - \tan^2\theta = 1$

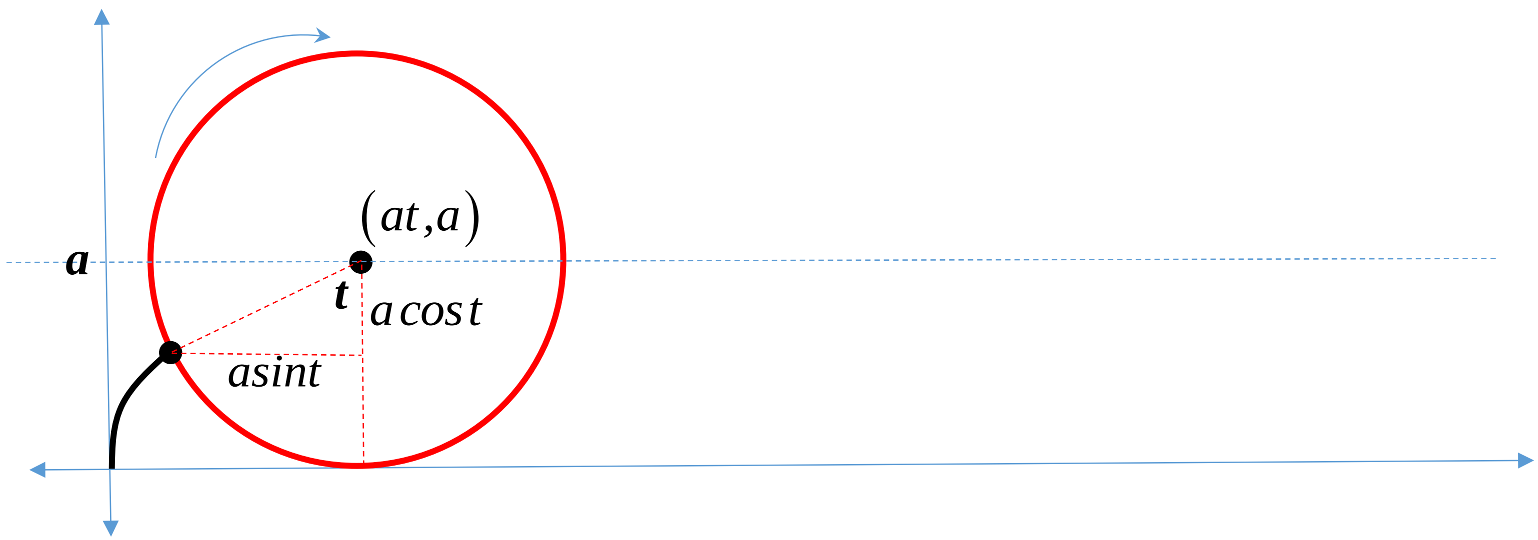
$x^2 = \sec^2 t, y^2 = \tan^2 t \Rightarrow x^2 - y^2 = \sec^2 t - \tan^2 t = 1 \Rightarrow \boxed{x^2 - y^2 = 1}$**, a hyperbola.**

The parametric equations trace the upper half of the right branch of the hyperbola from left to right.



<u>The Cycloid:</u>

The Cycloid is the curve traced out by a fixed point on a circle of radius *a* that is rolling along the *x*-axis, starting at the origin.



So the parametric equations are

$$\begin{cases} x = at - a\sin t \\ \\ y = a - a\cos t \end{cases} \quad ; t \geq 0.$$

It would be difficult/impossible to express a formula for the cycloid, other than a parametric representation.