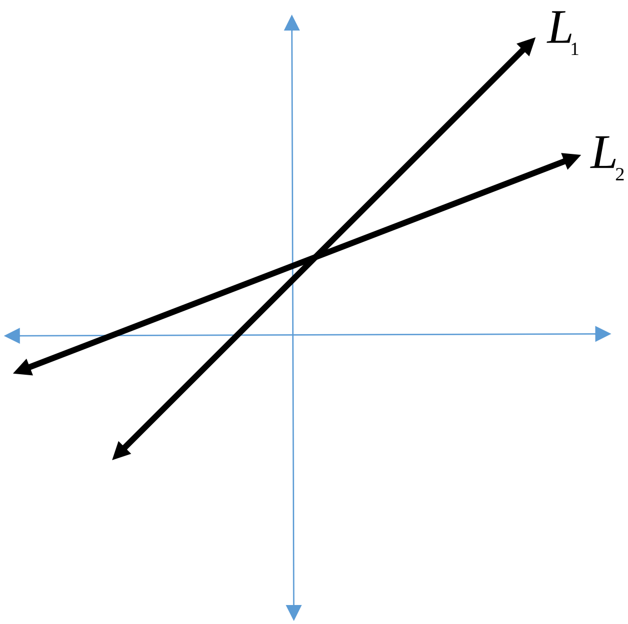
***Non-linear Systems of Equations:***

**Let's start with a review of linear systems of equations:**

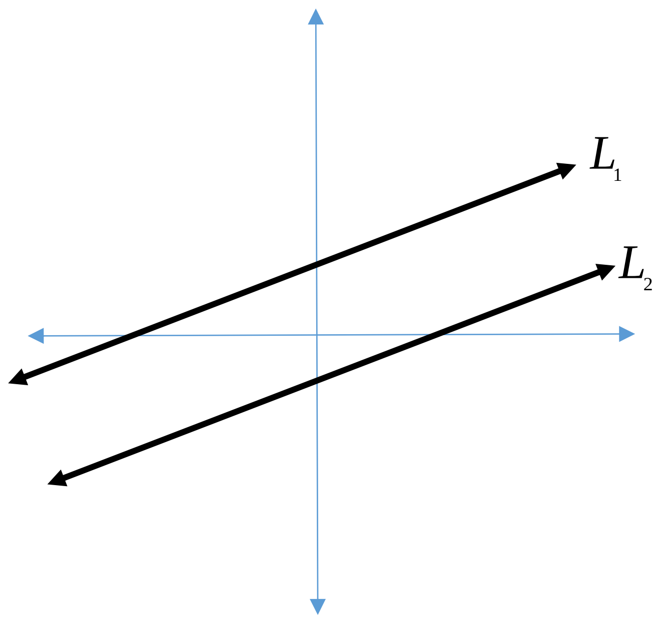
$$E_1$$

$$E_2$$

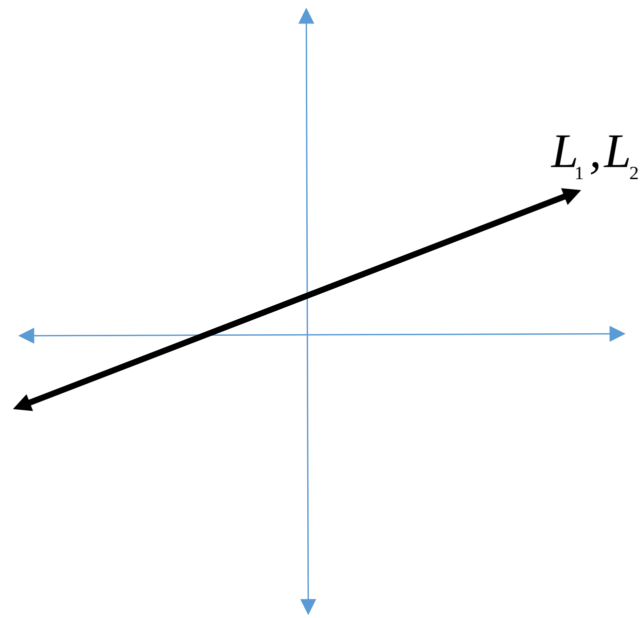
**Possibilities:**




**1 solution**




**No solution**

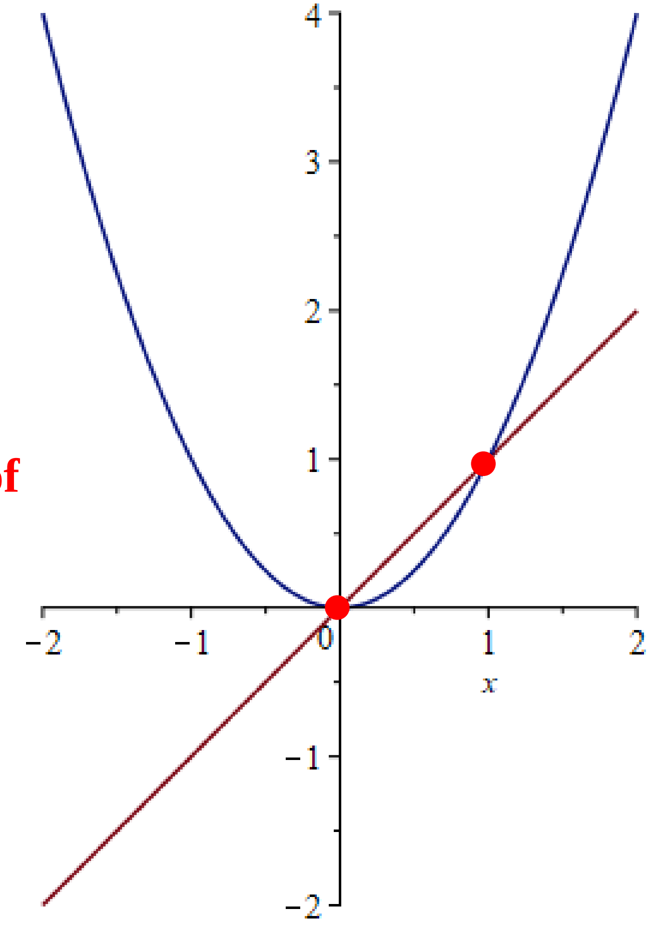



**Infinitely many solutions**

**If at least one of the equations in the system is non-linear, then it's a non-linear system of equations. Unlike linear systems, anything can happen.**

**Examples:**

**1.** $\begin{array}{l} y = x \\ y = x^2 \end{array}$



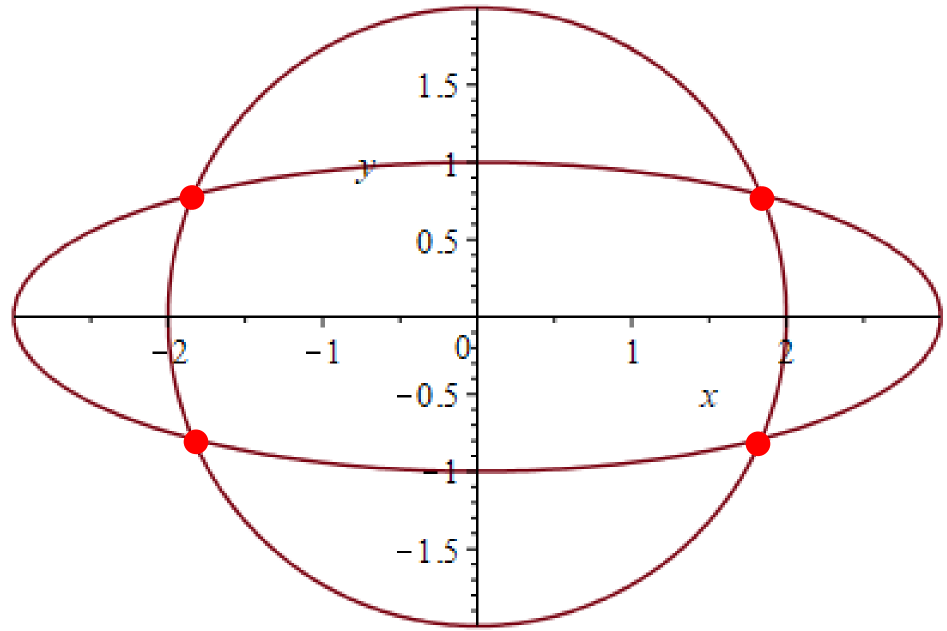

**How many solutions?**

**There are 2 solutions corresponding to the 2 points of intersection.**

**Eyeball the solutions.**

**As ordered pairs, the solutions are $(0,0)$ and $(1,1)$. They can also be written as $x = 0, y = 0$ and $x = 1, y = 1$.**

**2.** $$\begin{aligned} x^2 + y^2 &= 4 \\ \frac{x^2}{9} + y^2 &= 1 \end{aligned}$$




**How many solutions?**

**There are 4 solutions.**

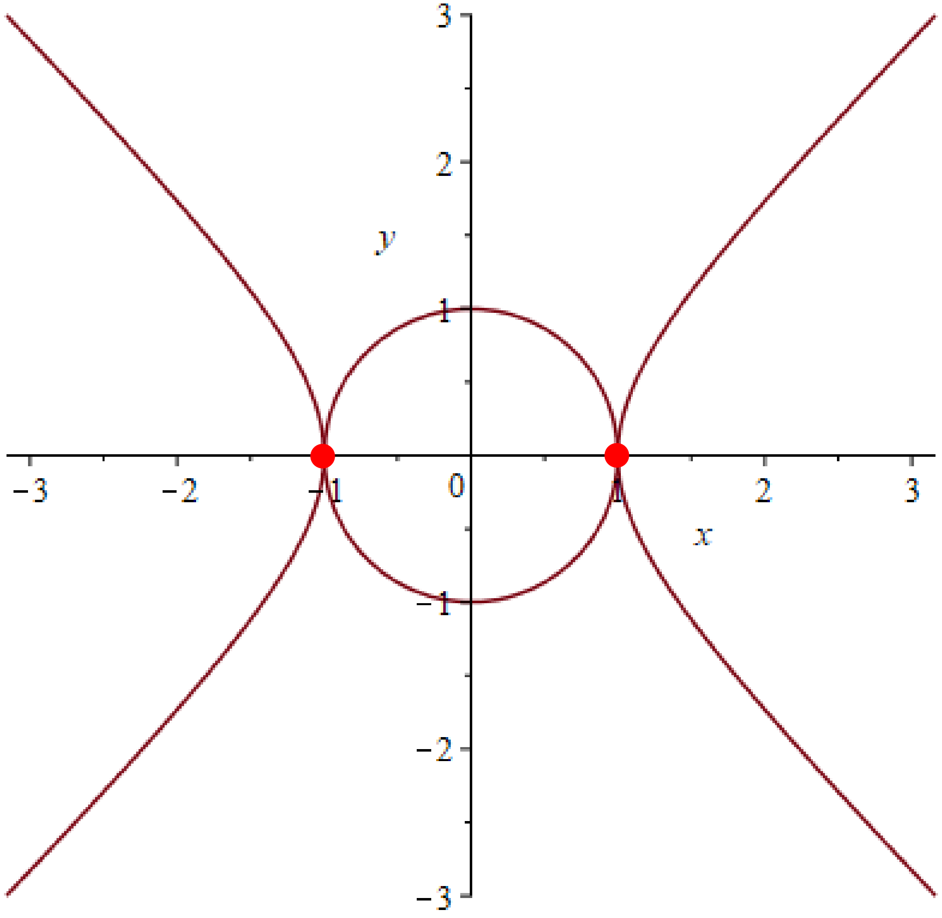
**Use elimination to find the solutions.**

**Subtract the second equation from the first equation to get** $\frac{8x^2}{9} = 3 \Rightarrow x^2 = \frac{27}{8} \Rightarrow x = \pm\sqrt{\frac{27}{8}}$**. To find the matching *y*-coordinates, substitute the *x*-values into the first equation to get** $\frac{27}{8} + y^2 = 4 \Rightarrow y^2 = \frac{5}{8} \Rightarrow y = \pm\sqrt{\frac{5}{8}}$**. So the four solutions are** $\left(-\sqrt{\frac{27}{8}}, -\sqrt{\frac{5}{8}}\right), \left(-\sqrt{\frac{27}{8}}, \sqrt{\frac{5}{8}}\right), \left(\sqrt{\frac{27}{8}}, -\sqrt{\frac{5}{8}}\right), \left(\sqrt{\frac{27}{8}}, \sqrt{\frac{5}{8}}\right)$**.**

**3.** $\begin{array}{l} x^2 - y^2 = 1 \\ x^2 + y^2 = 1 \end{array}$

**How many solutions?**

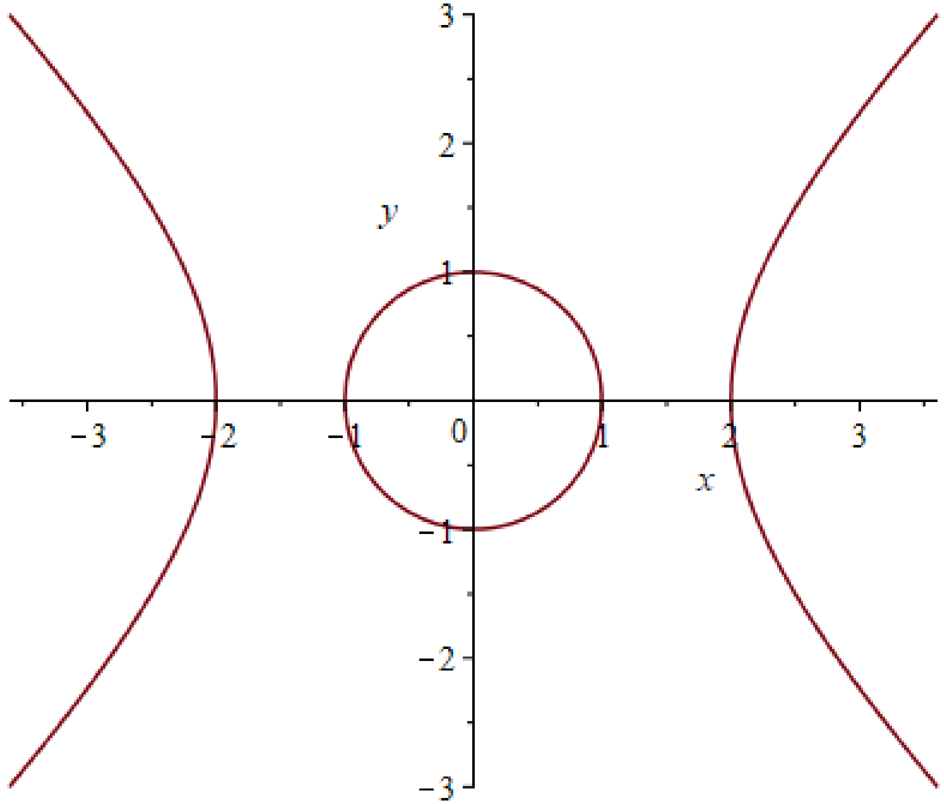
**There are 2 solutions.**




**Eyeball the solutions.**

**The two solutions are $(-1,0)$ and $(1,0)$.**

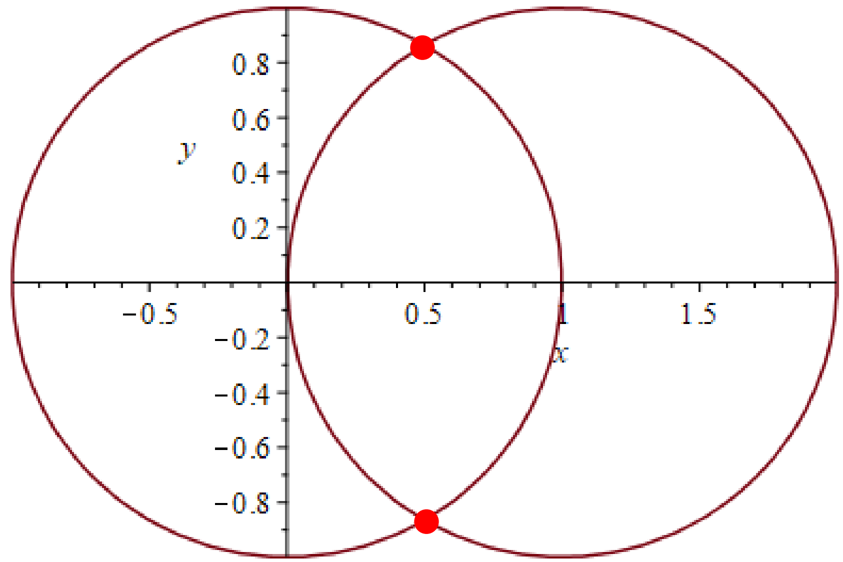
**4.** $x^2 - y^2 = 4$
$x^2 + y^2 = 1$




**How many solutions?**

**There are no solutions, since there are no points of intersection.**

**5.** $x^2 + y^2 = 1$
$(x-1)^2 + y^2 = 1$



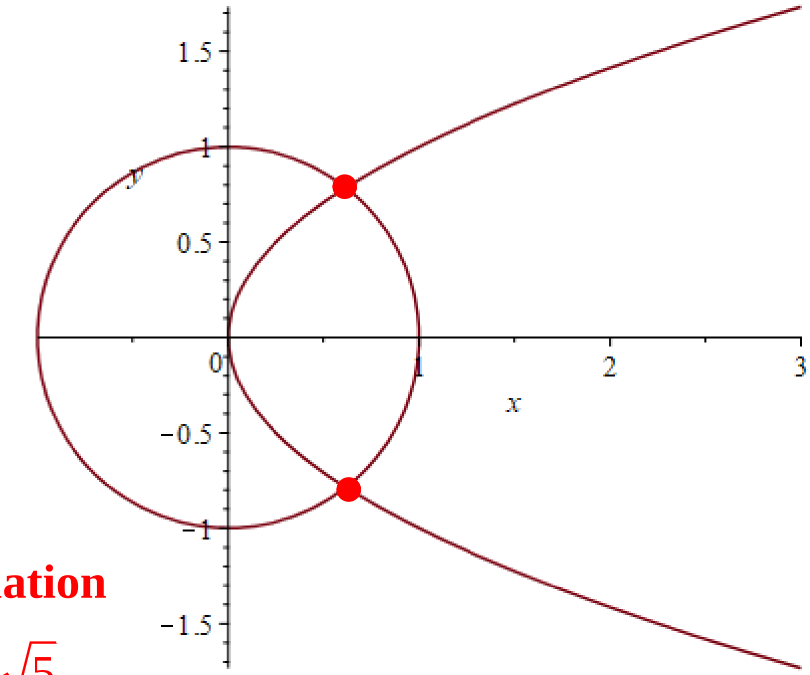

**How many solutions?**

**There are 2 solutions.**

**Use elimination to find the solutions.**

**Subtract the second equation from the first equation to get** $x^2 - (x-1)^2 = 0 \Rightarrow 2x - 1 = 0 \Rightarrow x = \frac{1}{2}$**. To find the matching** ***y*****-coordinates, substitute the** ***x*****-value into the first equation to get** $y^2 = \frac{3}{4} \Rightarrow y = \pm\frac{\sqrt{3}}{2}$**. So the two solutions are** $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ **and** $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$**.**

**6.** $y^2 = x$
$x^2 + y^2 = 1$




**How many solutions?**

**There are 2 solutions.**

**Use substitution to find the solutions.**

**Substitute the first equation into the second equation to get** $x^2 + x - 1 = 0 \Rightarrow x = \frac{-1 \pm \sqrt{1+4}}{2} \Rightarrow x = \frac{-1 \pm \sqrt{5}}{2}$**, but from the graph *x* must be positive, so** $x = \frac{-1+\sqrt{5}}{2}$**. To find the matching *y*-coordinates, substitute the *x*-value into the first equation to get** $y^2 = \frac{-1+\sqrt{5}}{2} \Rightarrow y = \pm\sqrt{\frac{-1+\sqrt{5}}{2}}$**. So the two solutions are** $\left(\frac{-1+\sqrt{5}}{2}, \sqrt{\frac{-1+\sqrt{5}}{2}}\right)$ **and** $\left(\frac{-1+\sqrt{5}}{2}, -\sqrt{\frac{-1+\sqrt{5}}{2}}\right)$**.**