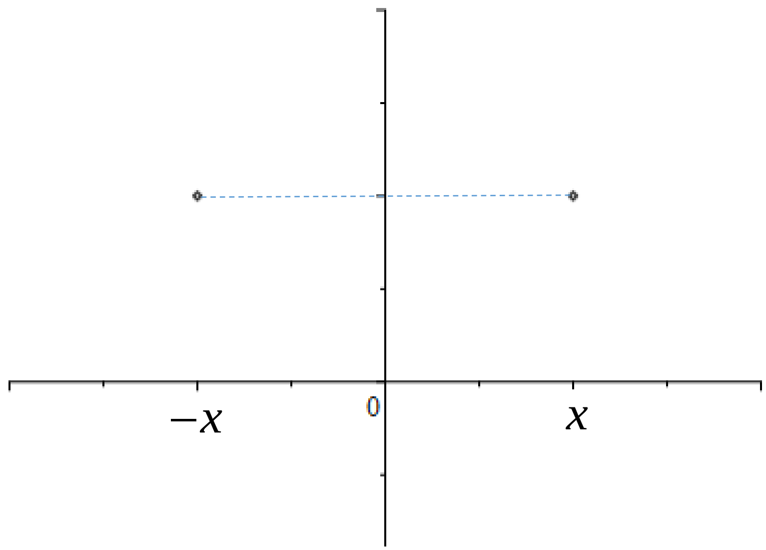
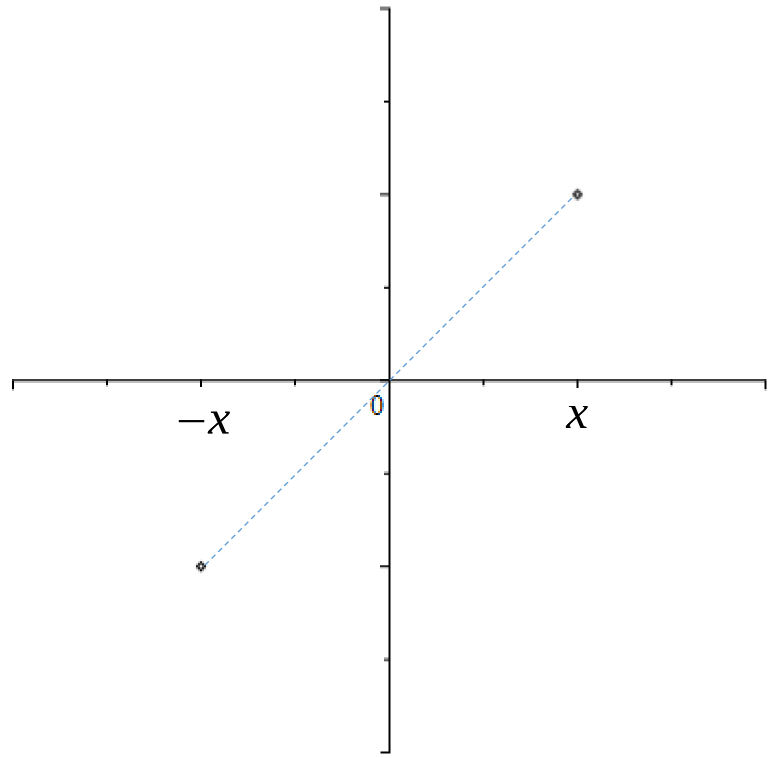
Review of Even and Odd Functions:

A function *f* is even if $f(-x) = f(x)$ for all *x* in the domain of *f*.



The graph has *y*-axis symmetry.

A function *f* is odd if $f(-x) = -f(x)$ **for all *x* in the domain of *f*.**

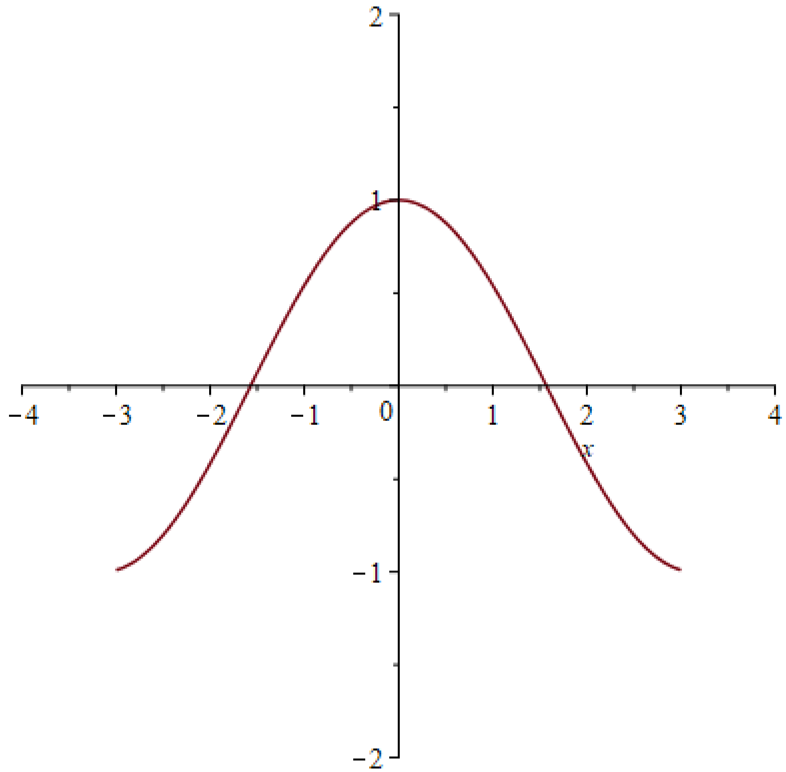


The graph has origin symmetry.

If zero is in the domain of an odd function, *f*, what must be the value of $f(0)$**?** 0

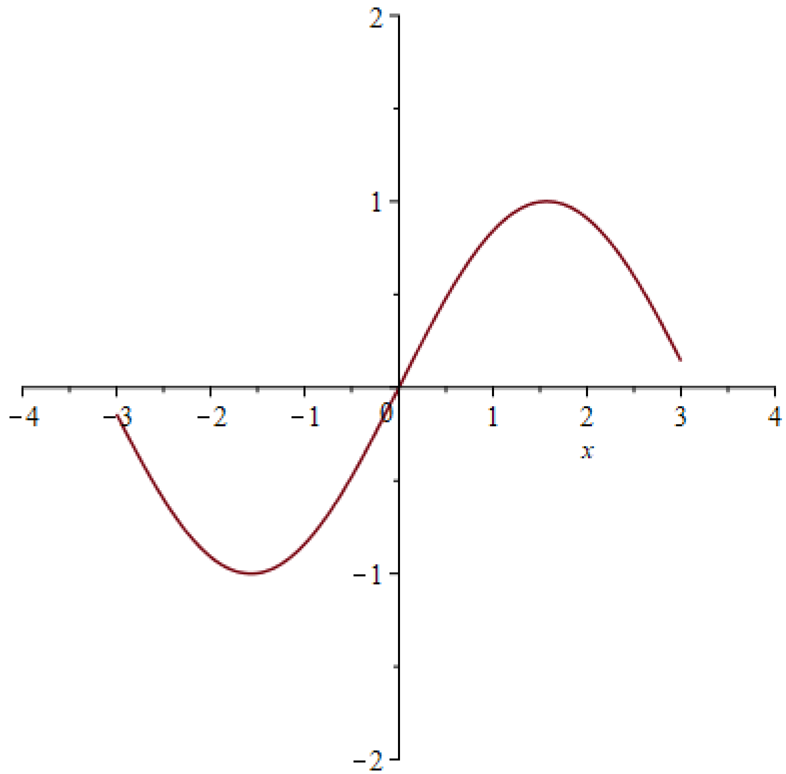
Determine if the following functions are odd, even, neither, or both.

1.



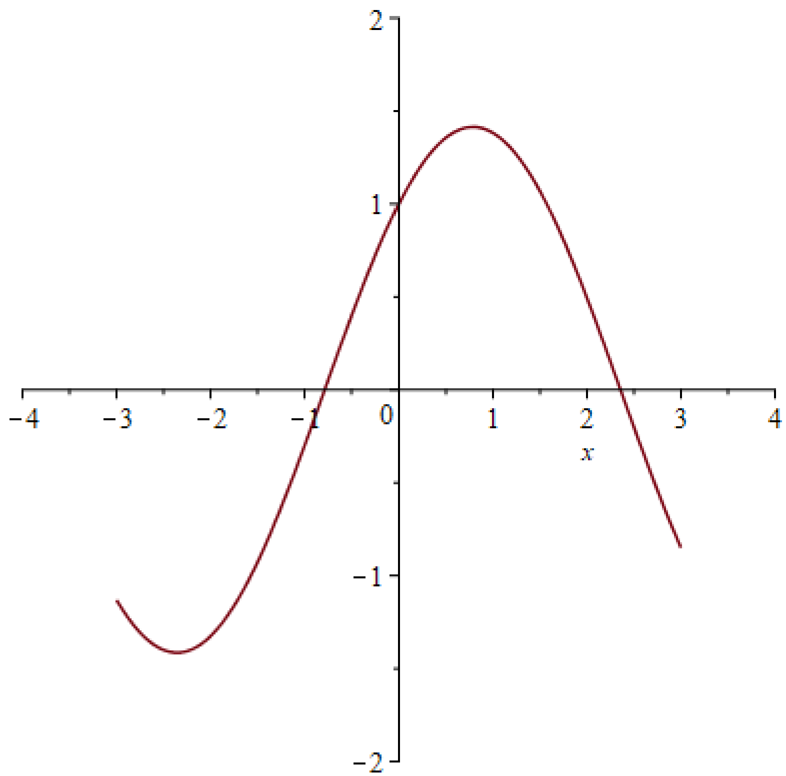
It's even since the graph has *y*-axis symmetry.

2.



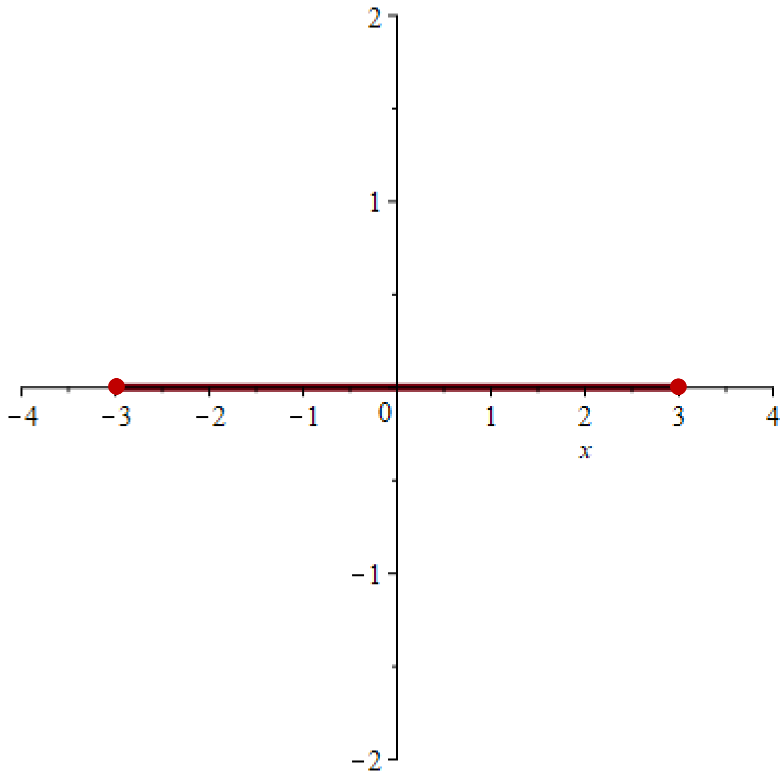
It's odd since the graph has origin symmetry.

3.



It’s neither since the graph has neither *y*-axis nor origin symmetry.

4.



It's both since the graph has both *y*-axis and origin symmetry.

5. $f(x) = 2x^4 - x^2$

$$f(-x) = 2(-x)^4 - (-x)^2$$
$$= 2x^4 - x^2 = f(x)$$

$\boxed{Even}$

6. $f(x) = x^3 + x$

$$f(-x) = (-x)^3 + (-x)$$
$$= -x^3 - x = -(x^3 + x) = -f(x)$$

$\boxed{Odd}$

7. $f(x) = x + x^2$

$$f(-x) = -x + x^2$$
$$\neq x + x^2 = f(x)$$
$$\neq -x - x^2 = -f(x)$$

$\boxed{Neither}$

8. $f(x) = (x+1)^2 - (x-1)^2 - 4x$ ***{Simplify the formula, first!}***

$$f(x) = x^2 + 2x + 1 - x^2 + 2x - 1 - 4x = 0$$
$$f(-x) = 0 = f(x)$$
$$f(-x) = 0 = -0 = -f(x)$$

$\boxed{Both}$

9.

$$f(x)=\begin{cases} x^3 & ;x\geq 0 \\ -x^3 & ;x\leq 0 \end{cases} \Rightarrow f(-x)=\begin{cases} (-x)^3 & ;-x\geq 0 \\ -(-x)^3 & ;-x\leq 0 \end{cases}=\begin{cases} -x^3 & ;x\leq 0 \\ x^3 & ;x\geq 0 \end{cases}$$

$$=\begin{cases} x^3 & ;x\geq 0 \\ -x^3 & ;x\leq 0 \end{cases}$$

$$= f(x)$$

$$\boxed{Even}$$

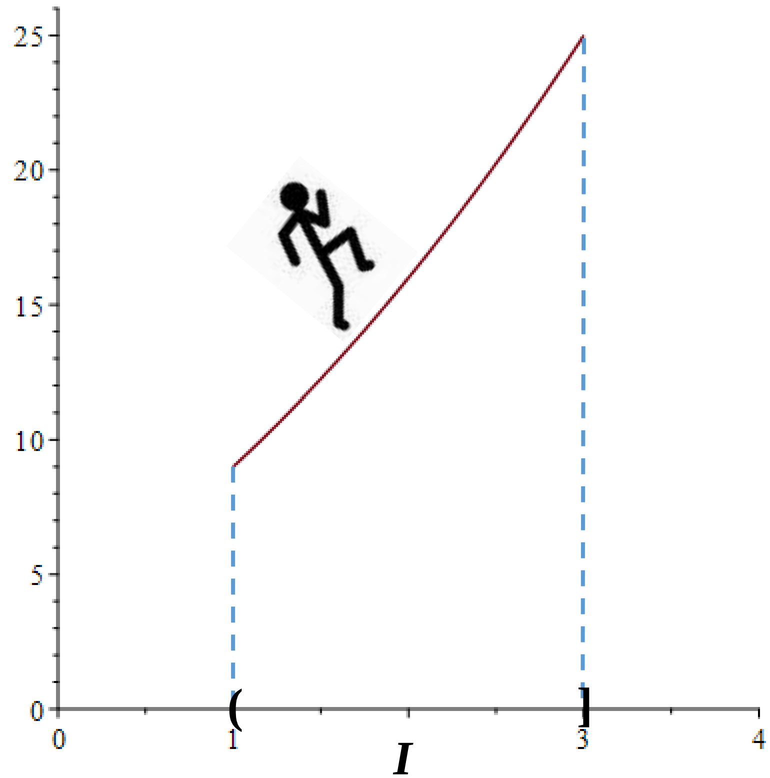
10. $$f(x)=\begin{cases} x-2;1\leq x\leq 3 \\ x+2;-3\leq x\leq -1 \end{cases} \Rightarrow f(-x)=\begin{cases} (-x)-2;1\leq -x\leq 3 \\ (-x)+2;-3\leq -x\leq -1 \end{cases}=\begin{cases} -x-2;-3\leq x\leq -1 \\ -x+2;1\leq x\leq 3 \end{cases}$$

$$=\begin{cases} -(x+2);-3\leq x\leq -1 \\ -(x-2);1\leq x\leq 3 \end{cases}=-f(x)$$

$$\boxed{Odd}$$

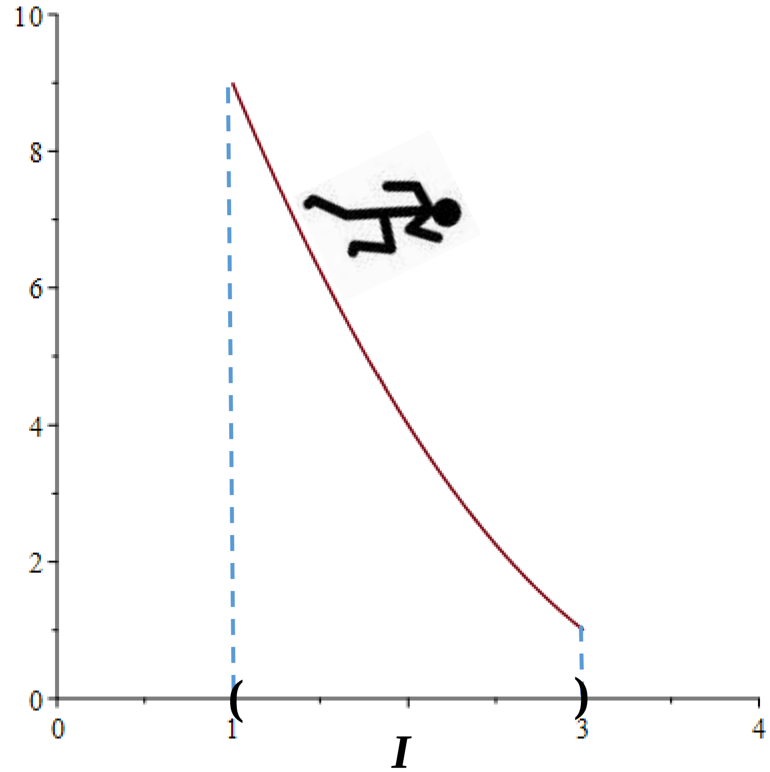
Increasing, Decreasing, Constant:

A function *f* is increasing on an interval *I*, if for x, y in *I* with $x < y$, then $f(x) < f(y)$.



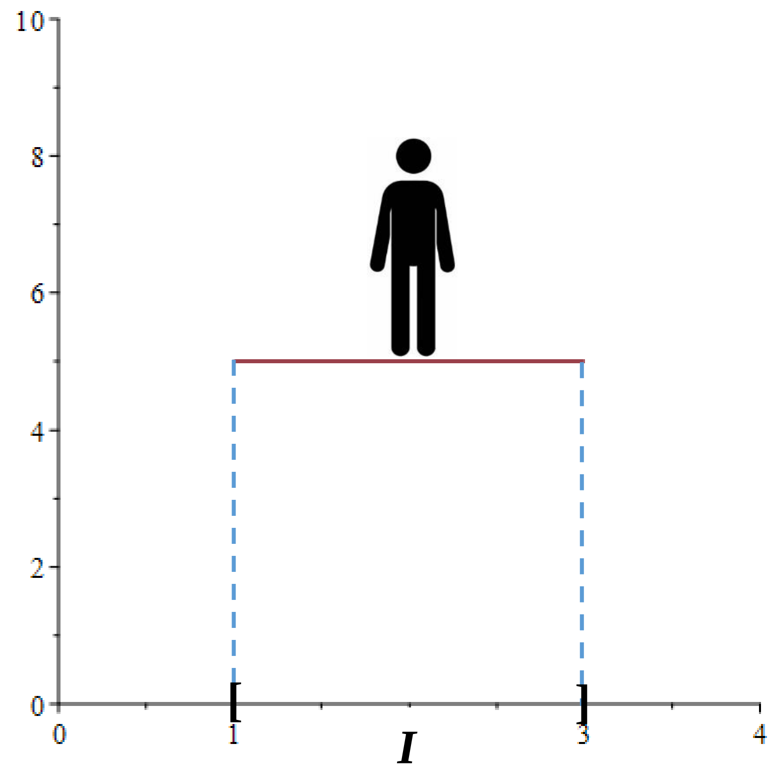
Moving up from left to right!

A function *f* is decreasing on an interval *I*, if for x, y **in *I* with** $x < y$**, then** $f(x) > f(y)$**.**



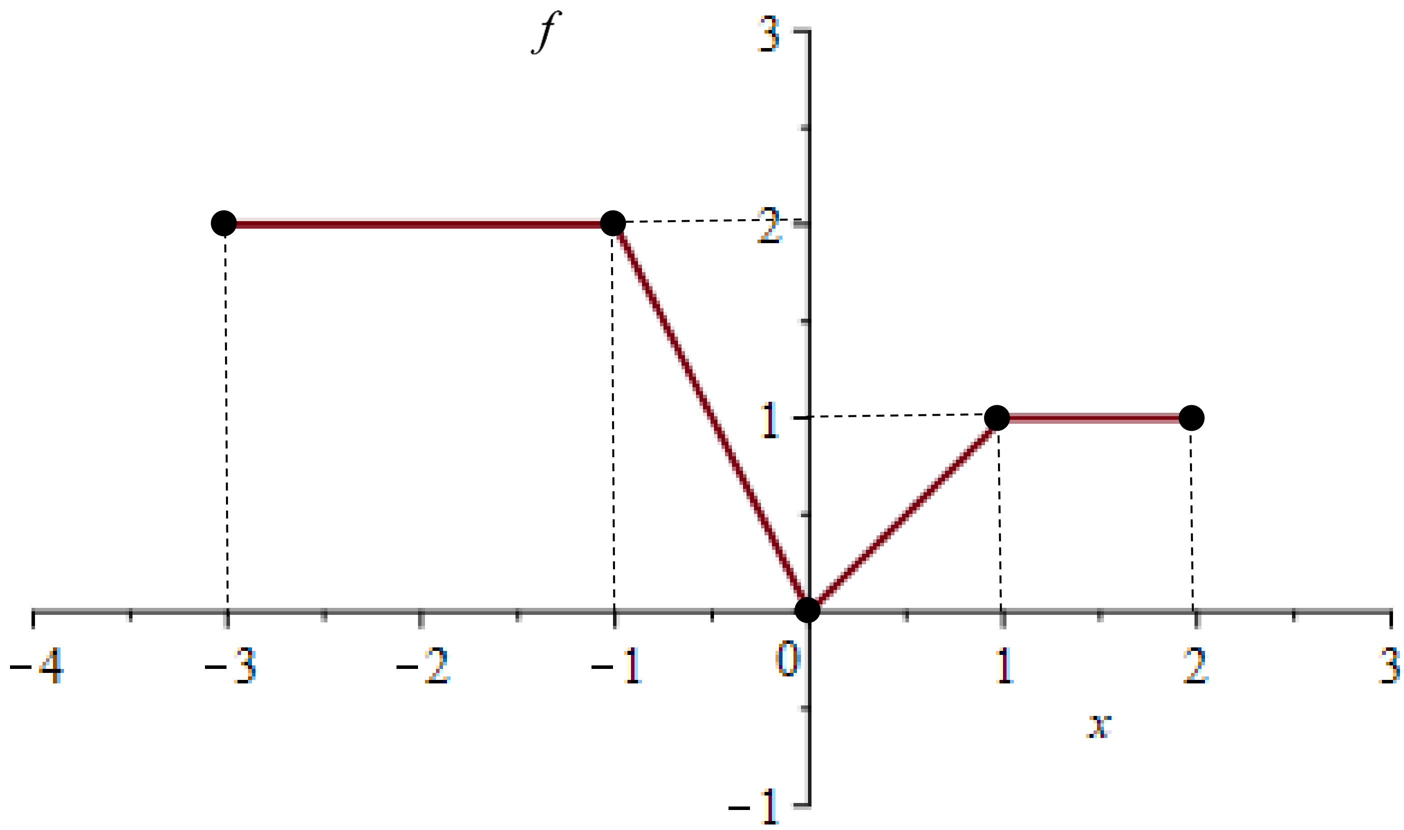
Moving down from left to right!

A function *f* is constant on an interval *I*, if for x, y in *I*, then $f(x) = f(y)$.



Level ground!

Determine the intervals where *f* is increasing, decreasing, and constant.

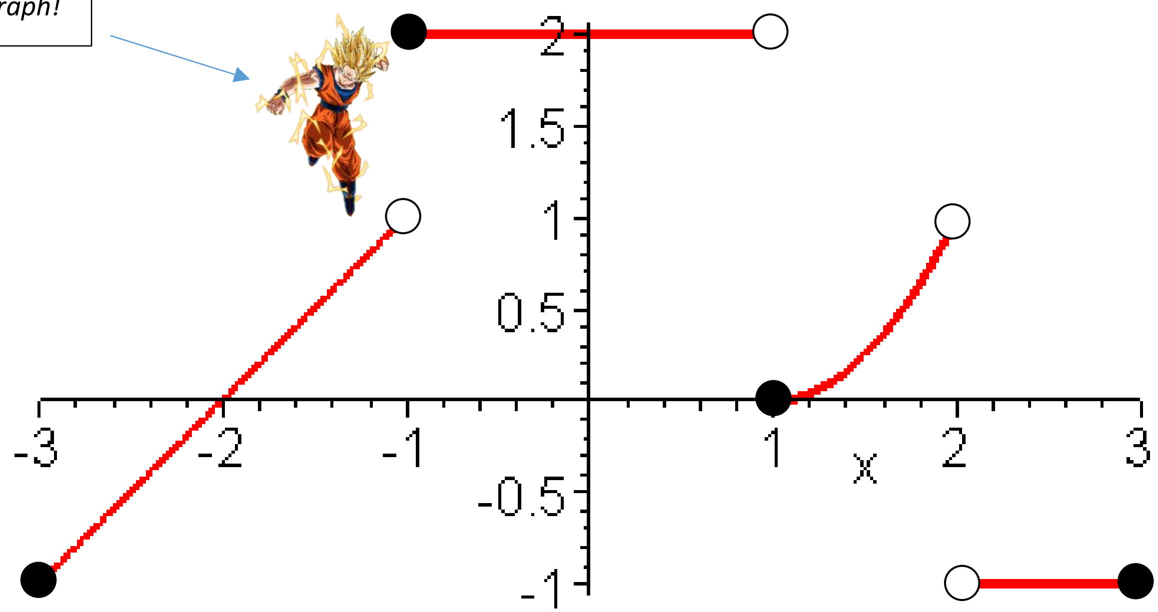


Increasing	**Decreasing**	**Constant**
$[0,1]$	$[-1,0]$	$[-3,-1]\cup[1,2]$

Determine the intervals where *g* is increasing, decreasing, and constant.

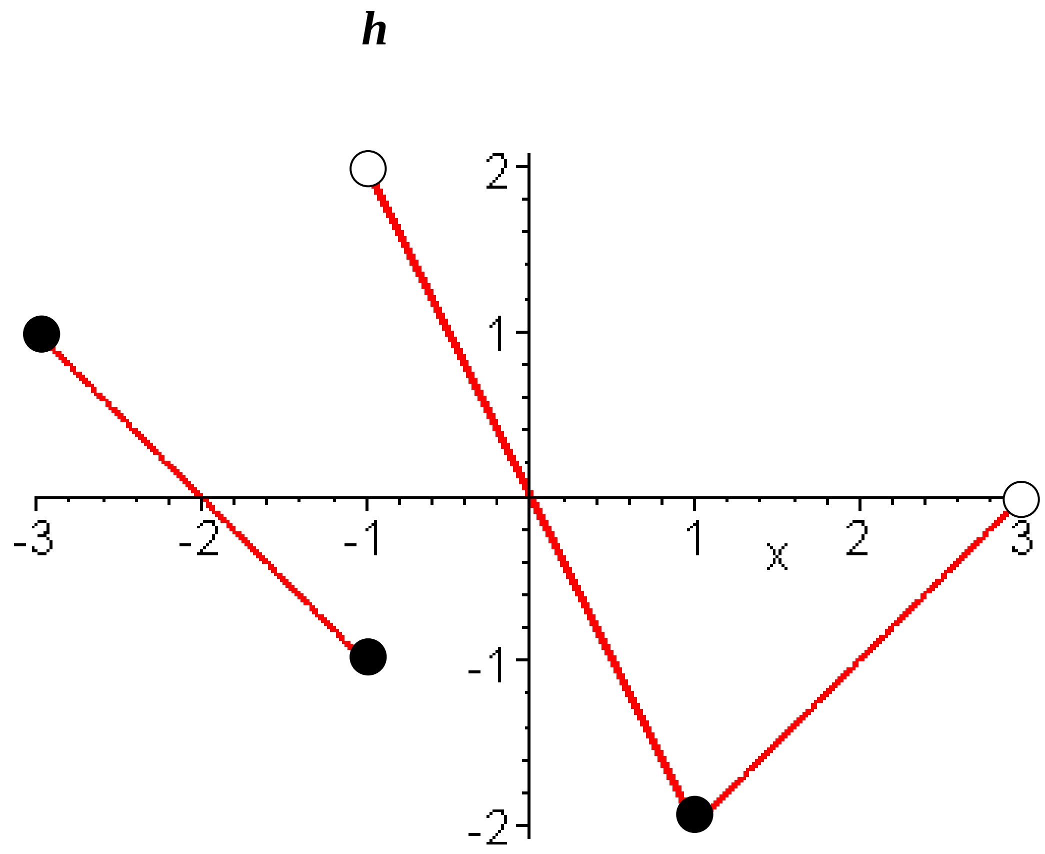
g

Note: Goku is not part of the graph!



Increasing	**Decreasing**	**Constant**
$[-3,-1]\cup[1,2)$	nowhere	$[-1,1)\cup(2,3]$

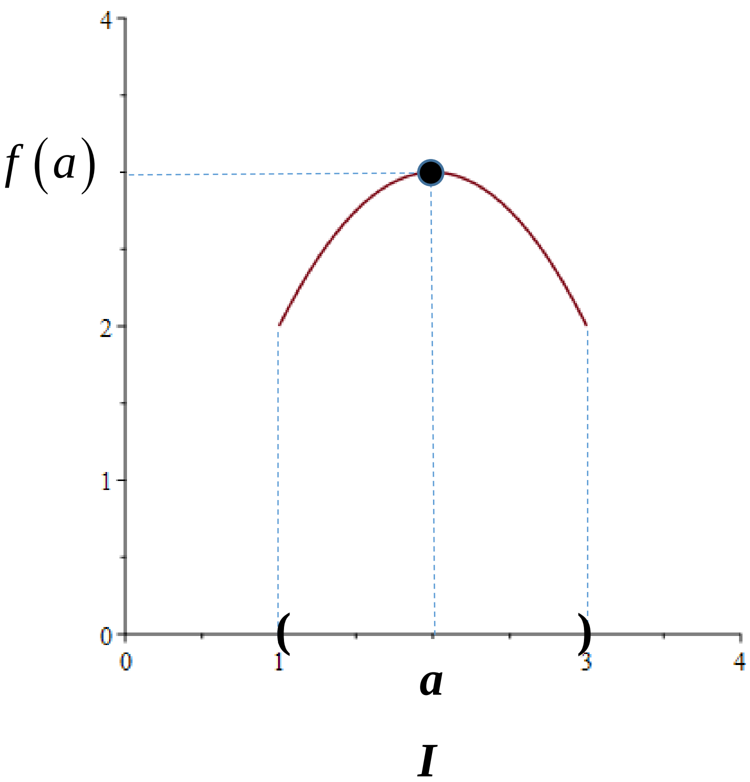
Determine the intervals where *h* is increasing, decreasing, and constant.

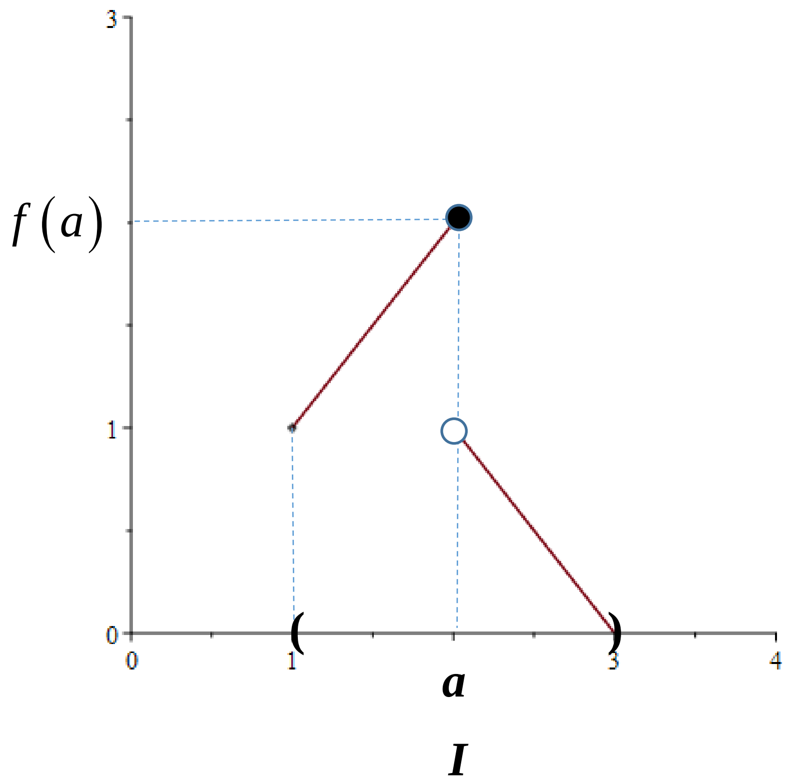


Increasing	**Decreasing**	**Constant**
$[1,3)$	$[-3,-1]\cup(-1,1]$	nowhere

Local Extrema:

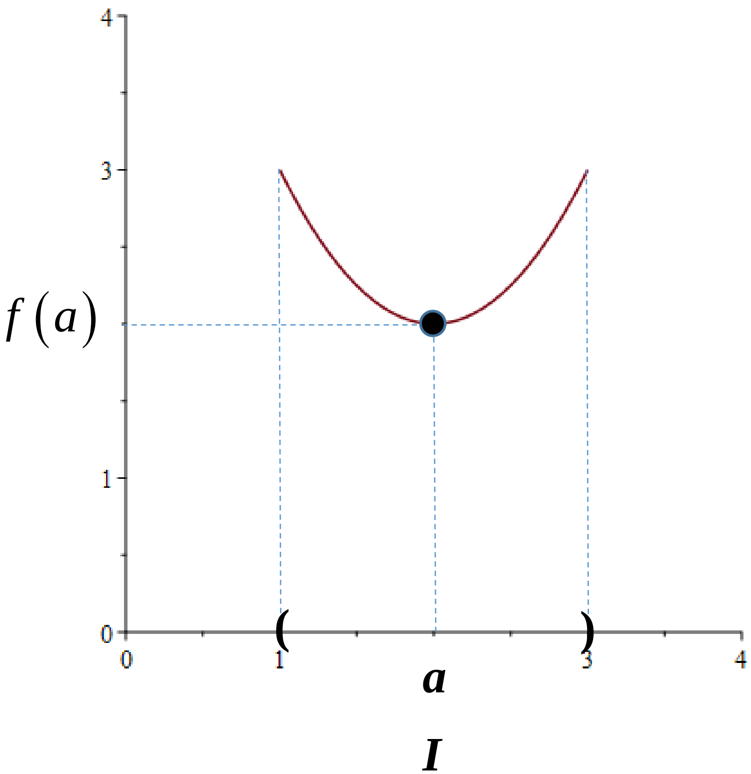
A function *f* has a local maximum at *a*, if there is an open interval *I* containing *a* with $f(x) < f(a)$ **for all *x* in *I* with** $x \neq a$**.**

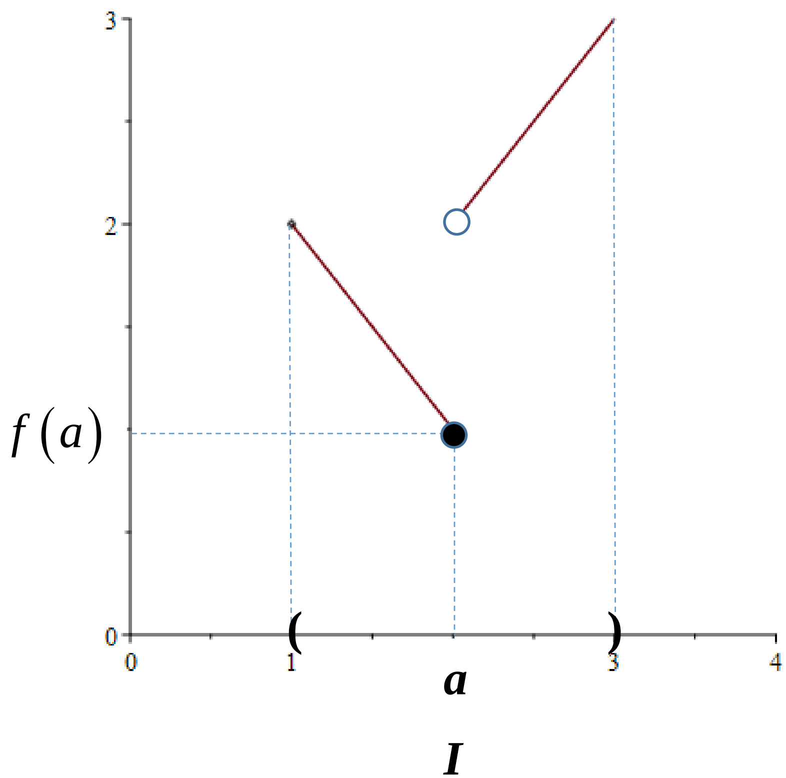




A local maximum corresponds to a high spot in the graph of the function!

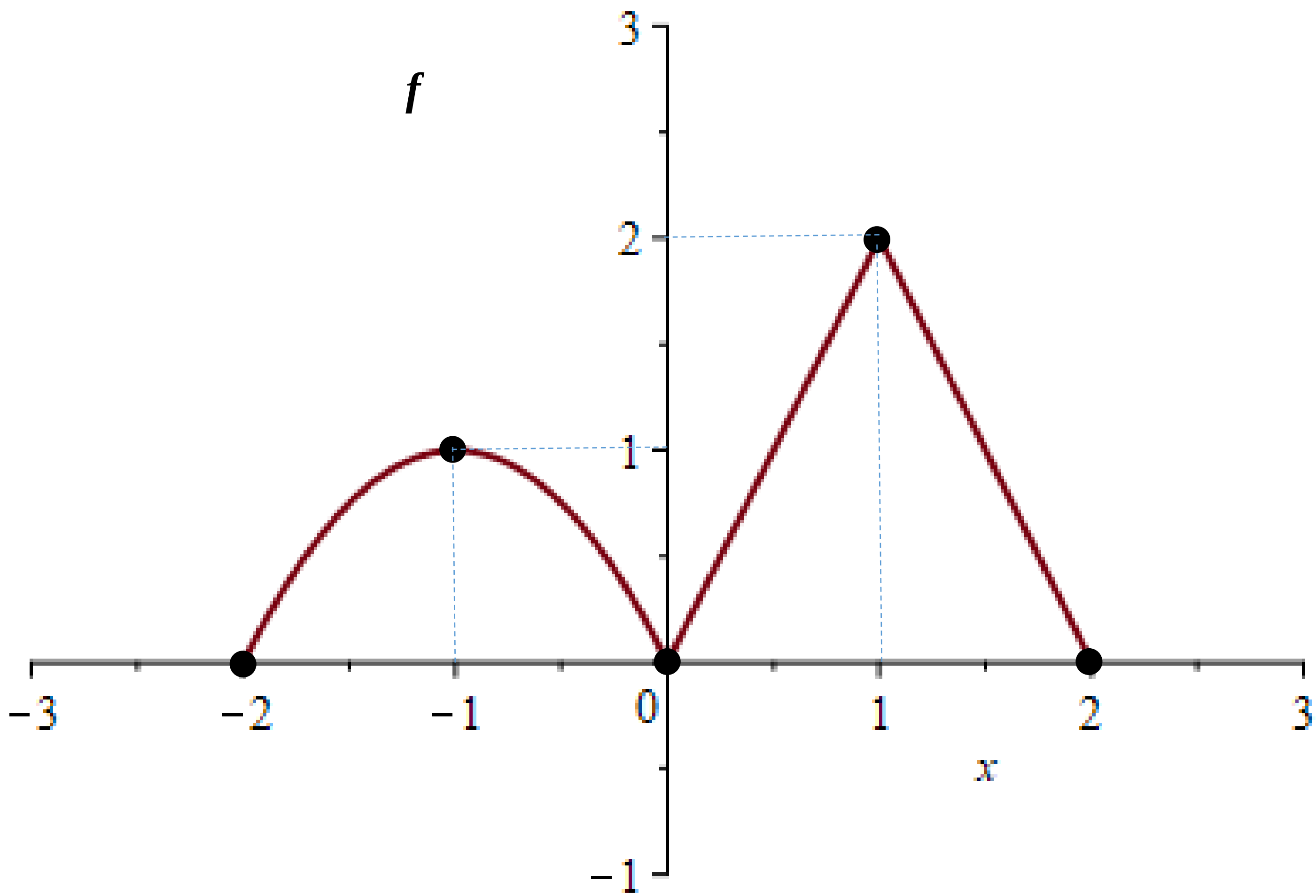
A function *f* has a local minimum at *a*, if there is an open interval *I* containing *a* with $f(x) > f(a)$ for all *x* in *I* with $x \neq a$.





A local minimum corresponds to a low spot in the graph of the function!

Find all the local extrema of the function *f*.



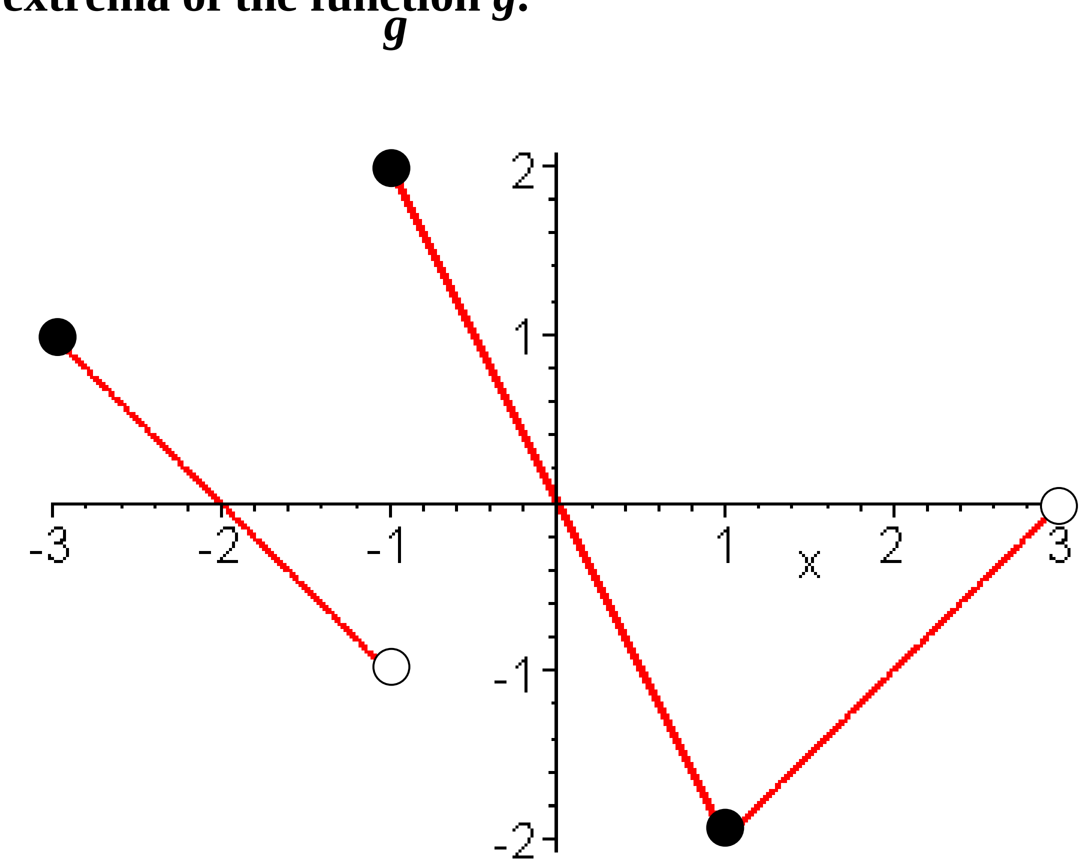
<u>Local Maxima</u>

$x = -1, 1$

<u>Local Minima</u>

$x = 0$

Find all the local extrema of the function *g*.



<u>Local Maxima</u>

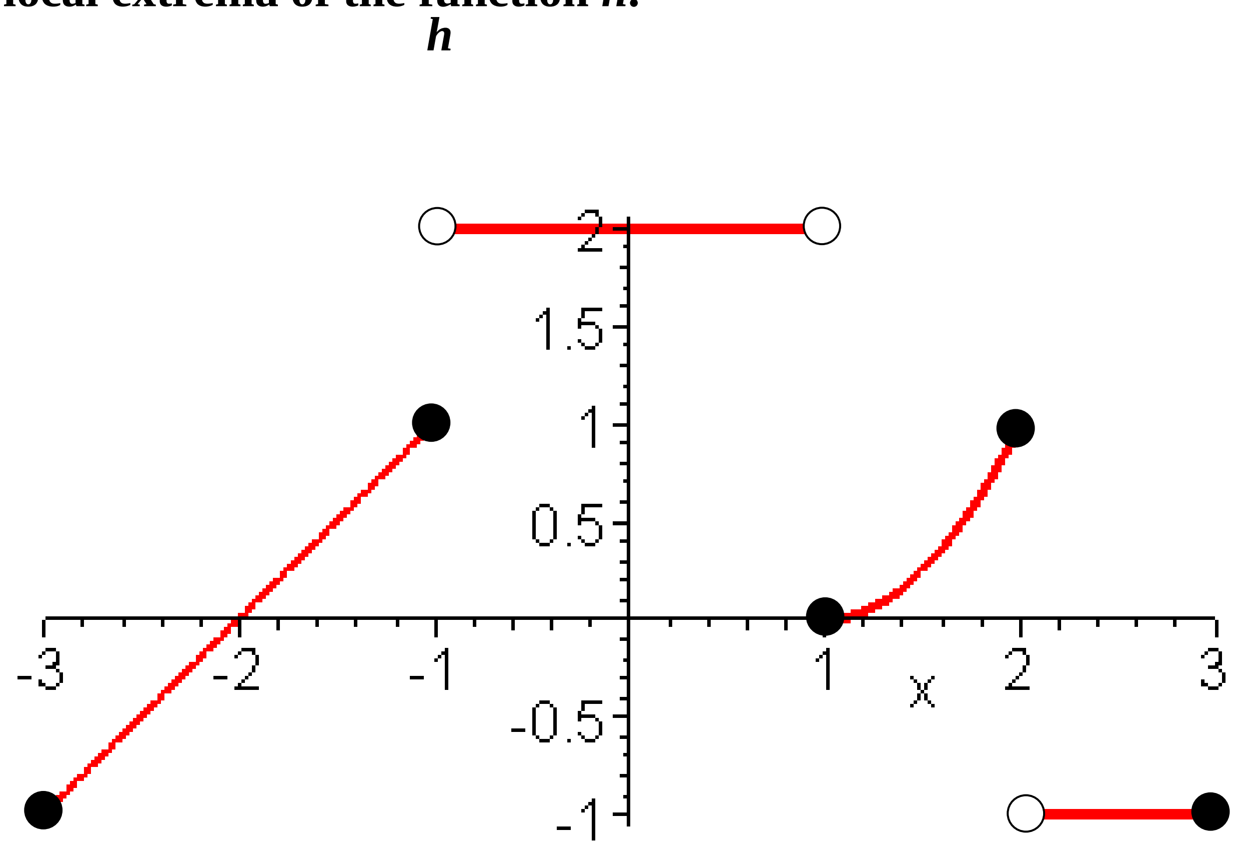
$x = -1$

<u>Local Minima</u>

$x = 1$

There's no local maximum at $x = -3$ because there's no portion of the graph immediately to the left of the point with *x*-coordinate of -3.

Find all the local extrema of the function *h*.



<u>Local Maxima</u>

$x = 2$

<u>Local Minima</u>

$x = 1$

There's no local minimum at $x = -3$ because there's no portion of the graph immediately to the left of the point with *x*-coordinate of -3.

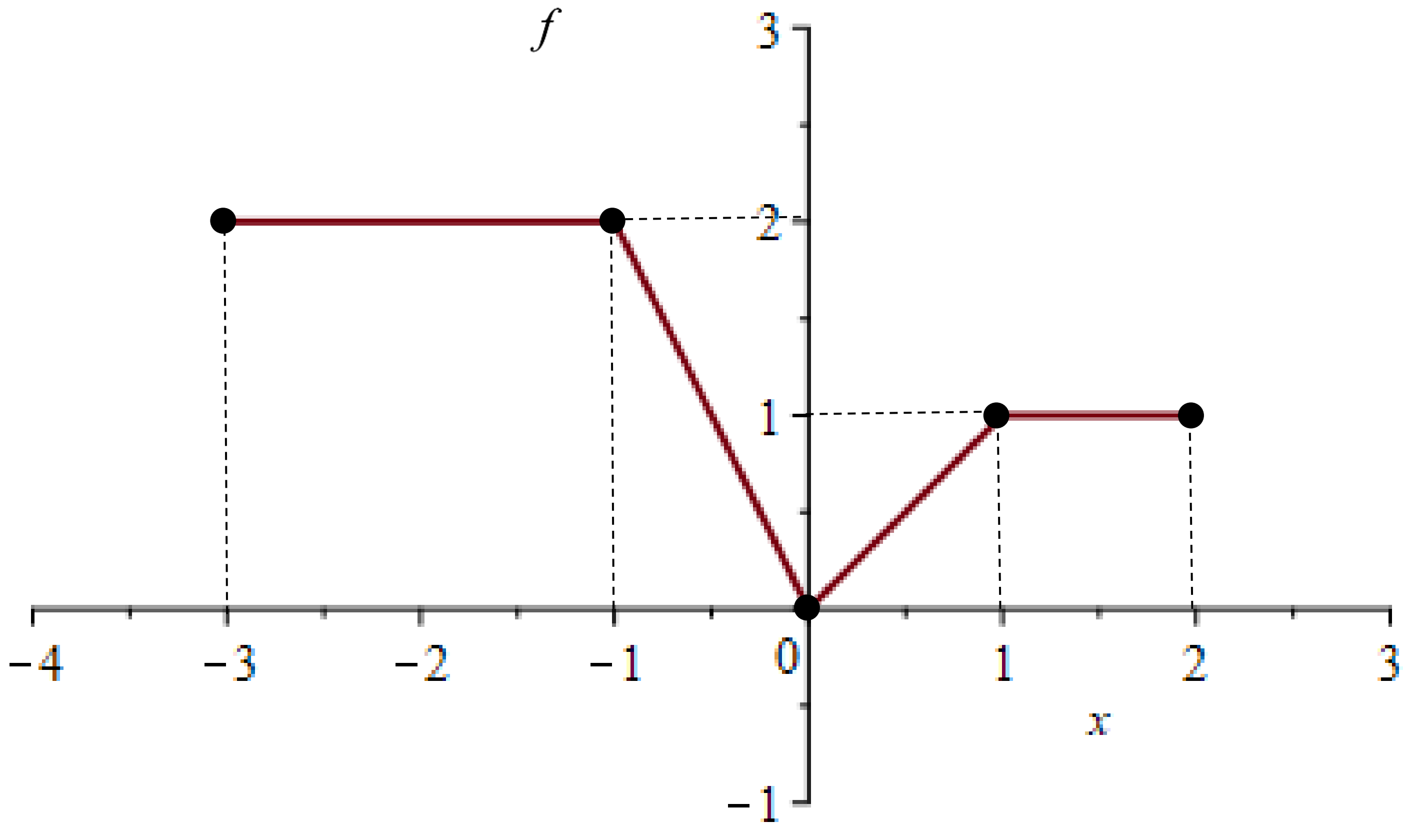
Absolute Extrema:

A function *f* has an absolute maximum at *a*, if $f(x) \le f(a)$ **for all *x* in the domain of *f*. In this case,** $f(a)$ **is called the absolute maximum value of the function.**

If the graph of a function has a highest point, then it corresponds to an absolute maximum. The absolute maximum can occur at more than one point, but its value is unique.

A function *f* has an absolute minimum at *a*, if $f(x) \ge f(a)$ **for all *x* in the domain of *f*. In this case,** $f(a)$ **is called the absolute minimum value of the function.**

If the graph of a function has a lowest point, then it corresponds to an absolute minimum. The absolute minimum can occur at more than one point, but its value is unique.

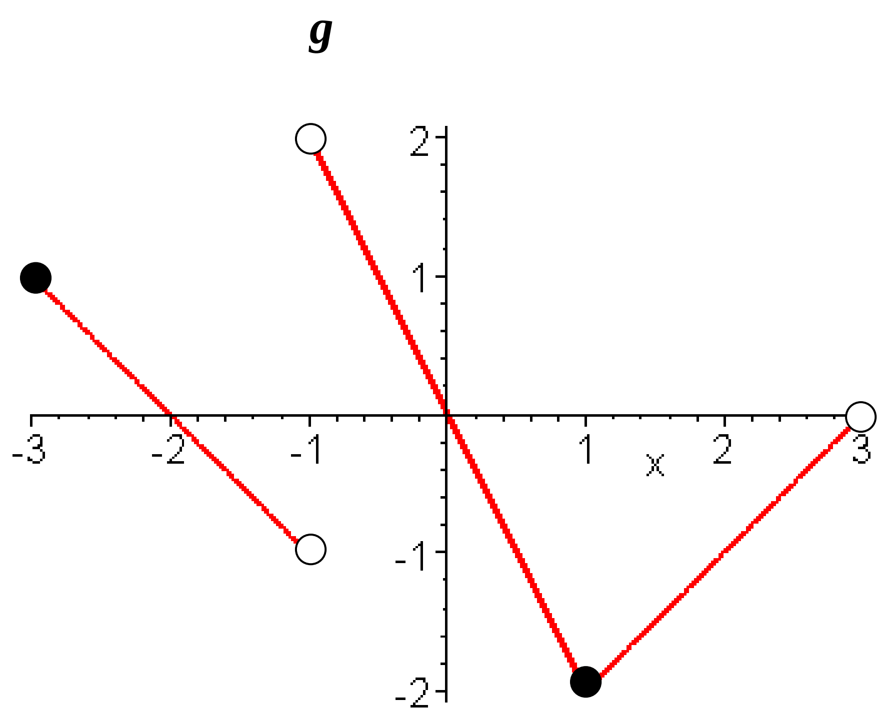


What is the absolute maximum value of the function *f*?

2

What is the absolute minimum value of the function *f*?

0



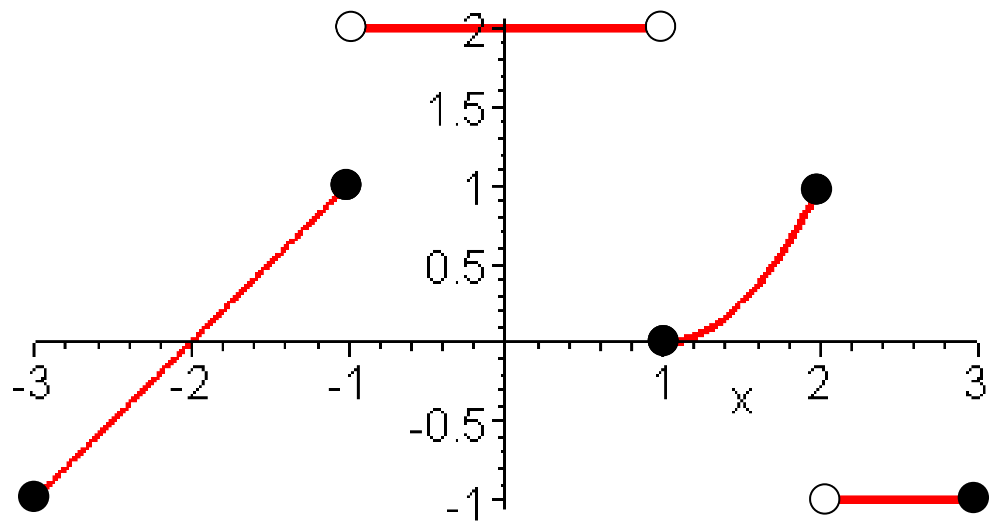
What is the absolute maximum value of the function *g*?

Does not exist.

What is the absolute minimum value of the function *g*?

-2

h

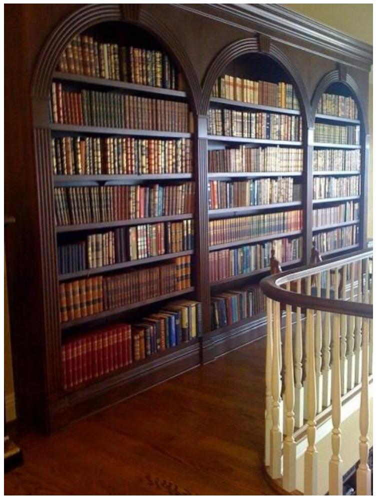


What is the absolute maximum value of the function *h*?

2

What is the absolute minimum value of the function *h*?

-1



Library of Common Functions:

1. Linear Function:

$f(x) = mx + b$

Graph?

Line with y-intercept of b and slope of m.

Domain/Range?

Domain: $(-\infty, \infty)$

Range: If $m \neq 0$, $(-\infty, \infty)$, otherwise $\{b\}$.

Increasing/decreasing/constant?

$m > 0$, increasing

$m < 0$, decreasing

$m = 0$, constant

Local/Absolute extrema?

Local extrema: nowhere

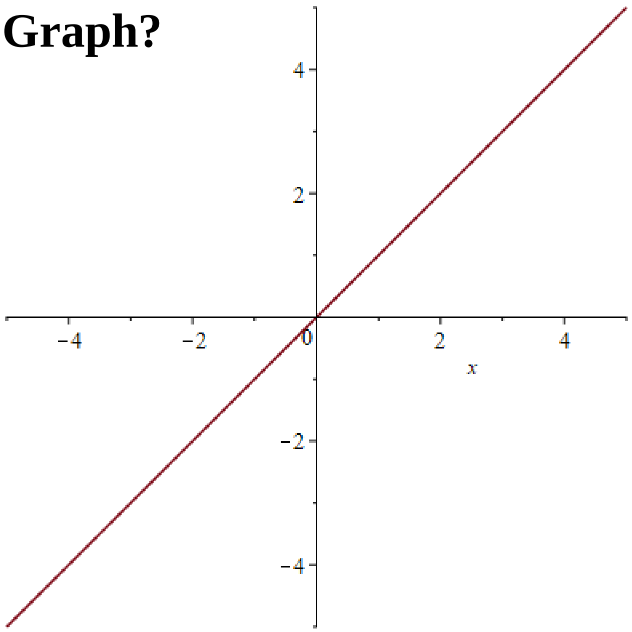
$m = 0$, Absolute max/min: b

$m \neq 0$, Absolute extrema: none

2. Identity Function:

$f(x) = x$

Graph?



Domain/Range?

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing/decreasing?

increasing: $(-\infty, \infty)$

decreasing: nowhere

Local/Absolute extrema?

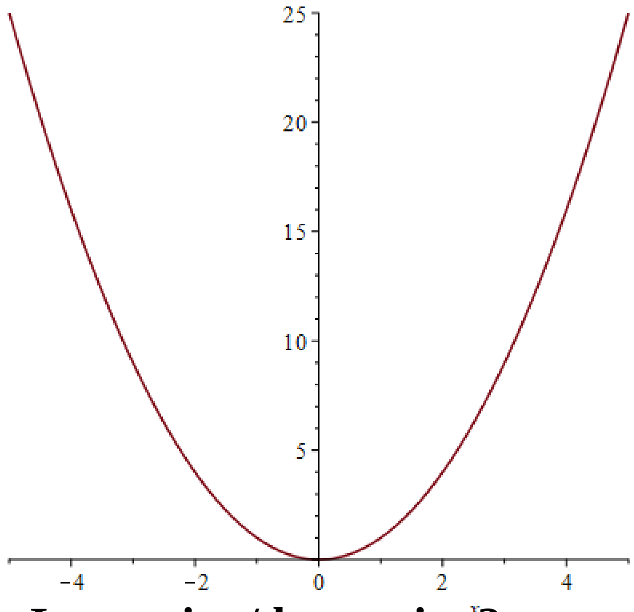
Local extrema: nowhere

Absolute extrema: none

3. Squaring Function:

$f(x) = x^2$

Graph?



Domain/Range?

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Increasing/decreasing?

increasing: $[0, \infty)$

decreasing: $(-\infty, 0]$

Local/Absolute extrema?

Local minimum: $x = 0$

Local maximum: nowhere

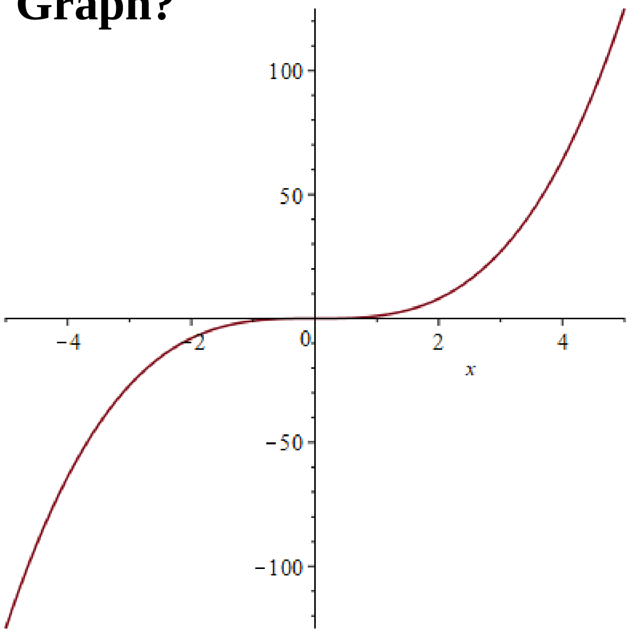
Absolute minimum: 0

Absolute maximum: none

4. Cubing Function:

$f(x) = x^3$

Graph?



Domain/Range?

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing/decreasing?

increasing: $(-\infty, \infty)$

decreasing: nowhere

Local/Absolute extrema?

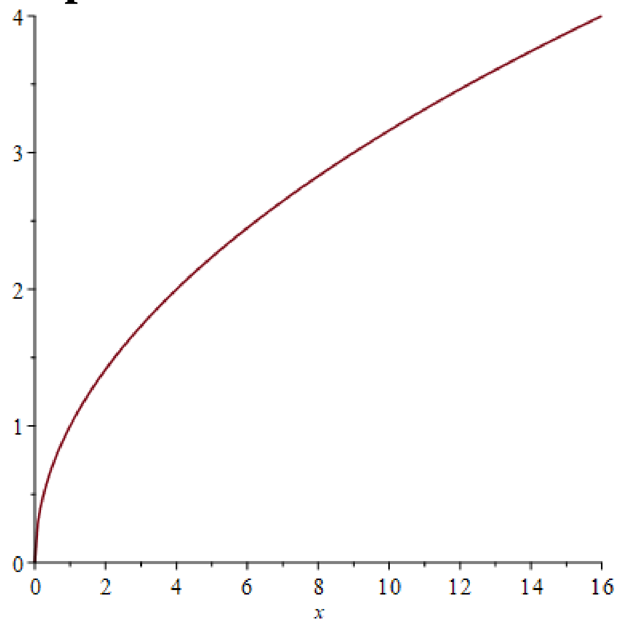
Local extrema: nowhere

Absolute extrema: none

5. Square Root Function:

$f(x) = \sqrt{x}$

Graph?



Domain/Range?

Domain: $[0, \infty)$

Range: $[0, \infty)$

Increasing/decreasing?

increasing: $[0, \infty)$

decreasing: nowhere

Local/Absolute extrema?

Local extrema: nowhere

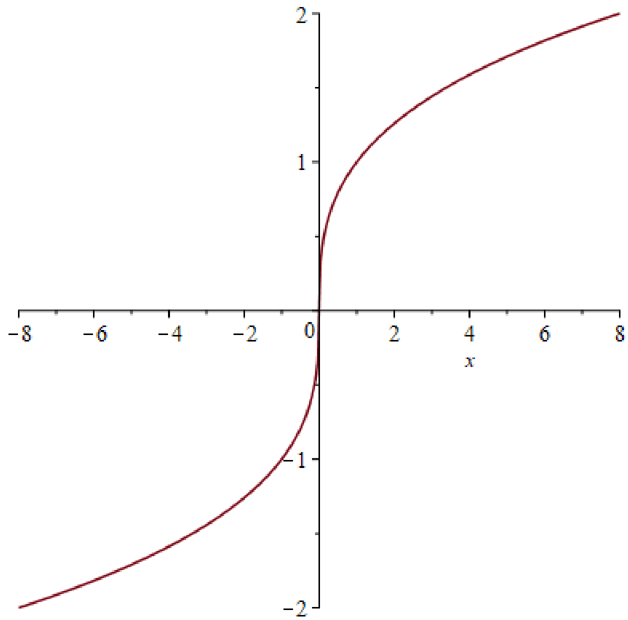
Absolute minimum: 0

Absolute maximum: none

6. Cube Root Function:

$f(x) = \sqrt[3]{x}$

Graph?



Domain/Range?

Domain: $(-\infty, \infty)$

Range: $(-\infty, \infty)$

Increasing/decreasing?

increasing: $(-\infty, \infty)$

decreasing: nowhere

Local/Absolute extrema?

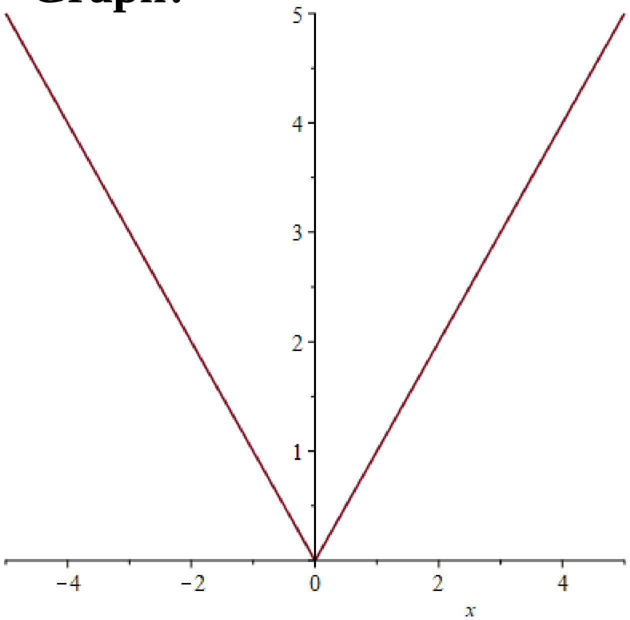
Local extrema: nowhere

Absolute extrema: none

7. Absolute Value Function:

$f(x) = |x|$

Graph?



Domain/Range?

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

Increasing/decreasing?

increasing: $[0, \infty)$

decreasing: $(-\infty, 0]$

Local/Absolute extrema?

Local minimum: $x = 0$

Local maximum: nowhere

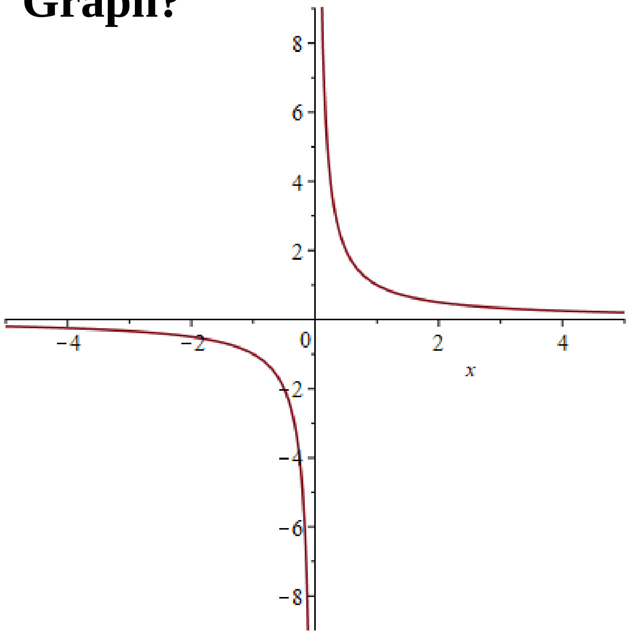
Absolute minimum: 0

Absolute maximum: none

8. Reciprocal Function:

$$f(x) = \frac{1}{x}$$

Graph?



Domain/Range?

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

Increasing/decreasing?

increasing: nowhere

decreasing: $(-\infty, 0) \cup (0, \infty)$

Local/Absolute extrema?

Local extrema: nowhere

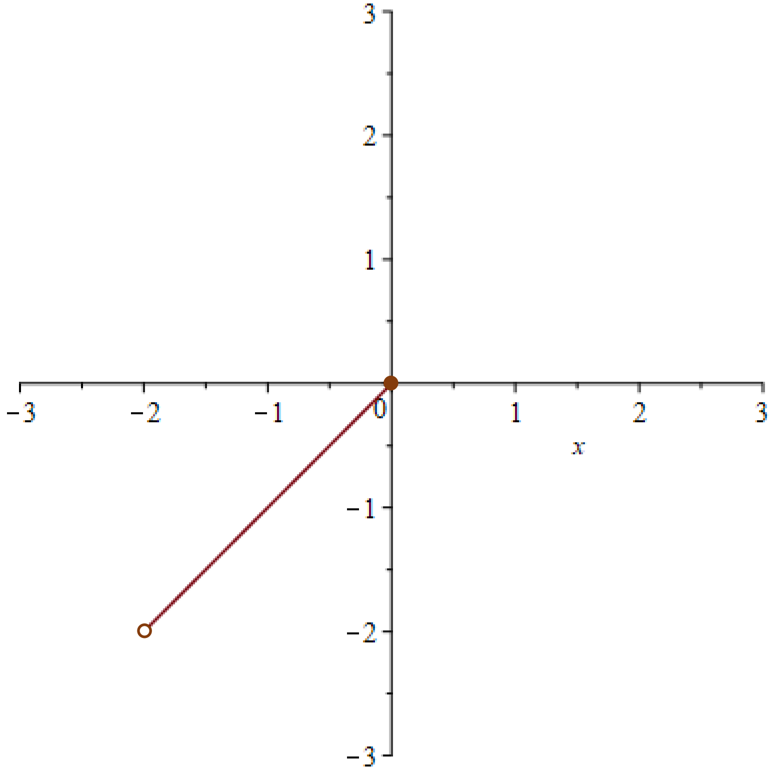
Absolute extrema: none

Graphing Piecewise-defined Functions Constructed From the Library Functions.

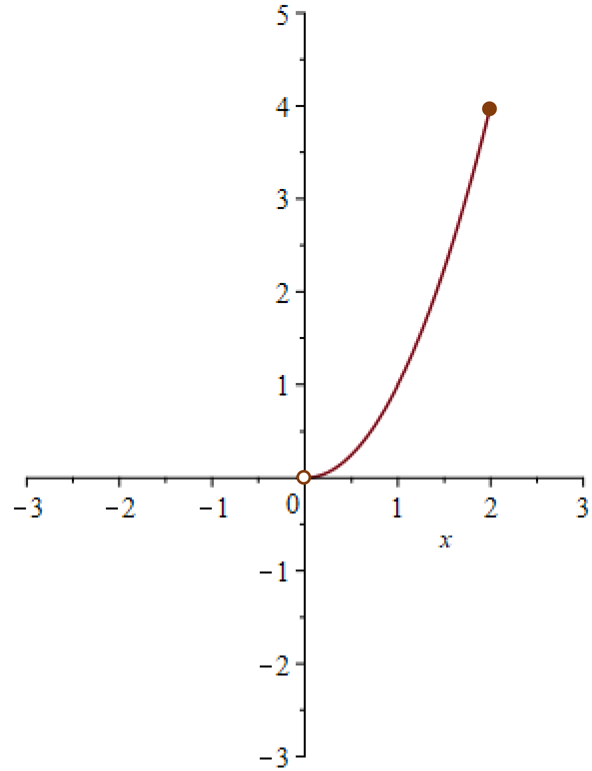
1. $f(x)=\begin{cases} x; -2 < x \le 0 \\ x^2; 0 < x \le 2 \end{cases}$

Graph?

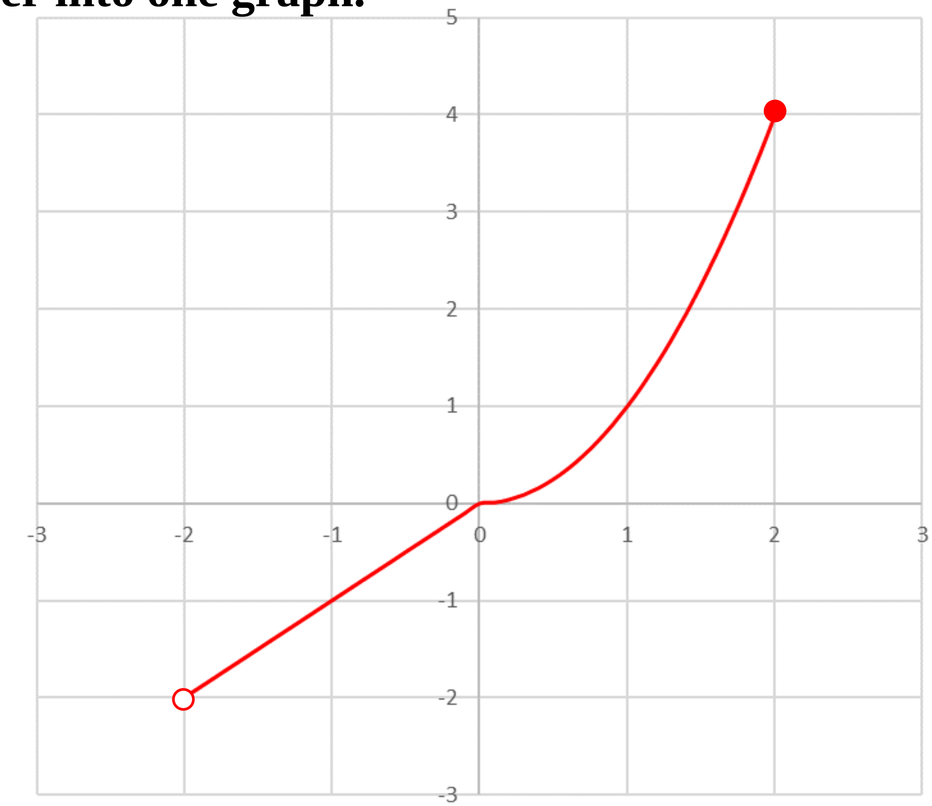
$x; -2 < x \le 0$



$x^2; 0 < x \le 2$



Now put them together into one graph.



Domain?

$(-2,2]$

Range?

$(-2,4]$

Increasing?

$(-2,2]$

Decreasing?

nowhere

Local extrema?

none

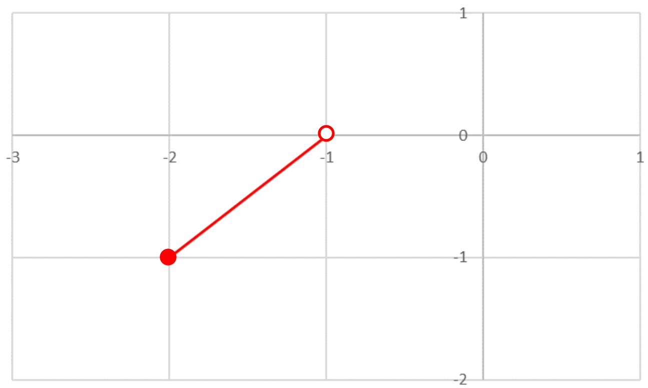
Absolute extrema?

Abs. max: 4 Abs. min.: DNE

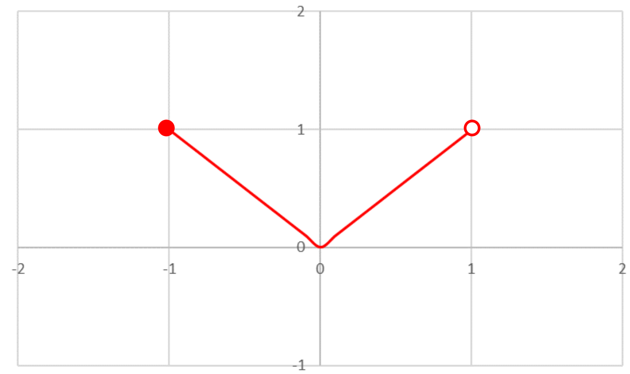
2. $g(x)=\begin{cases} x+1; -2\le x<-1 \\ |x| \;; -1\le x<1 \\ \sqrt{x}\;; 1\le x<4 \end{cases}$

Graph?

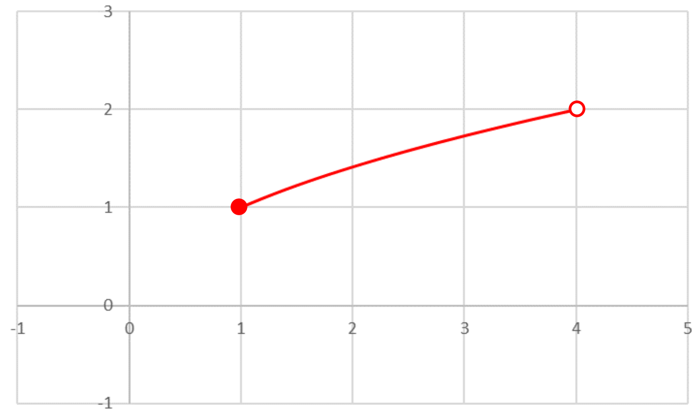
$x+1; -2\le x<-1$



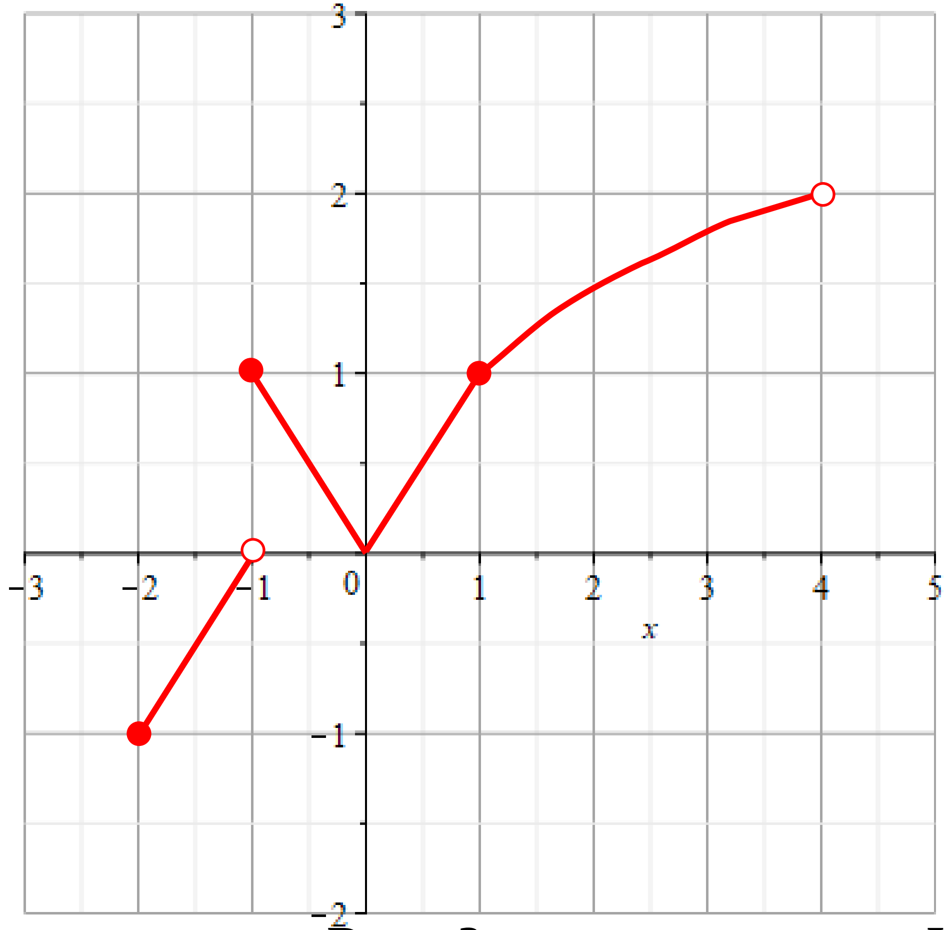
$|x|; -1\le x<1$



$\sqrt{x}; 1\le x<4$



Now put them together into one graph.



Domain?

$[-2,4)$

Range?

$[-1,2)$

Increasing?

$[-2,-1]\cup[0,4)$

Decreasing?

$[-1,0]$

Local extrema?

Local max.: $x=-1$

Local min. : $x=0$

Absolute extrema?

Abs. max.: DNE

Abs. min. : -1