Review of Transformations of the Graphs of Functions:

Vertical Shift

Horizontal Shift

Reflection about the *x*-axis

Reflection about the *y*-axis

Vertical Stretch/Compress

Horizontal Stretch/Compress

Vertical Shift:

For $c > 0$**,**

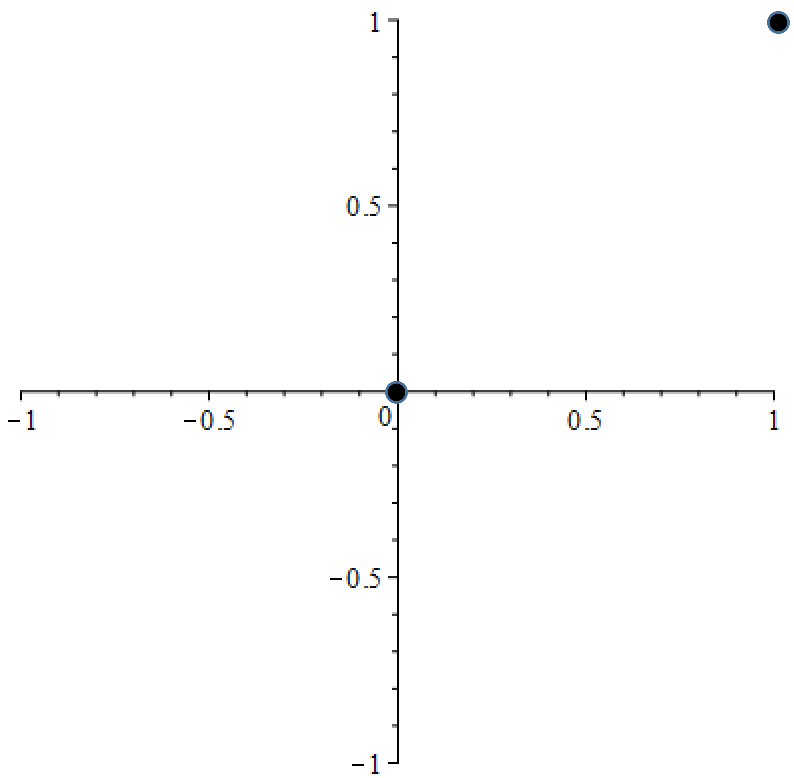
The graph of $g(x) = f(x) + c$**, is the graph of** $f(x)$ **shifted** ***c*** **units up.**

The graph of $g(x) = f(x) - c$**, is the graph of** $f(x)$ **shifted** ***c*** **units down.**

For a vertical shift, the ***y*****-coordinates change, but the** ***x*****-coordinates remain the same.**

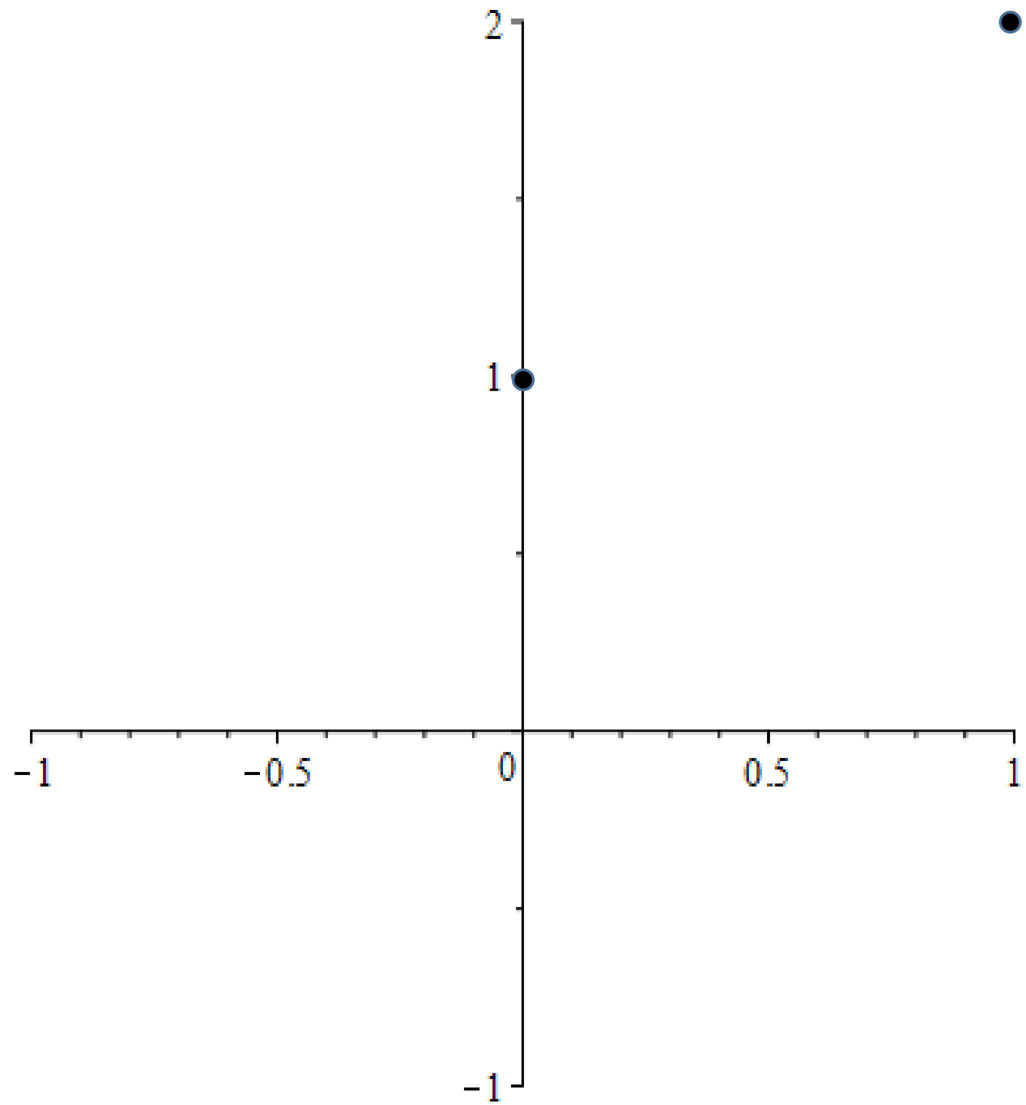
Original Function

$f = \{(0,0),(1,1)\}$



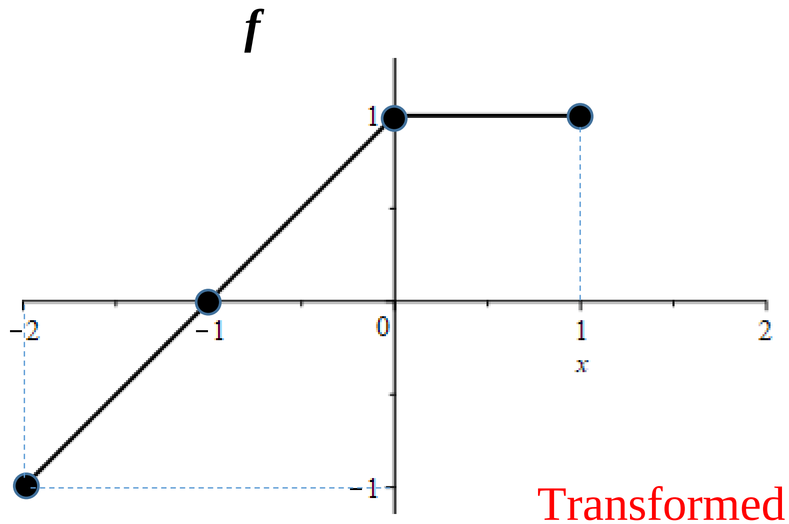
Transformed Function

$g(x) = f(x) + 1,\ g = \{(0, \boxed{1}),(1, \boxed{2})\}$



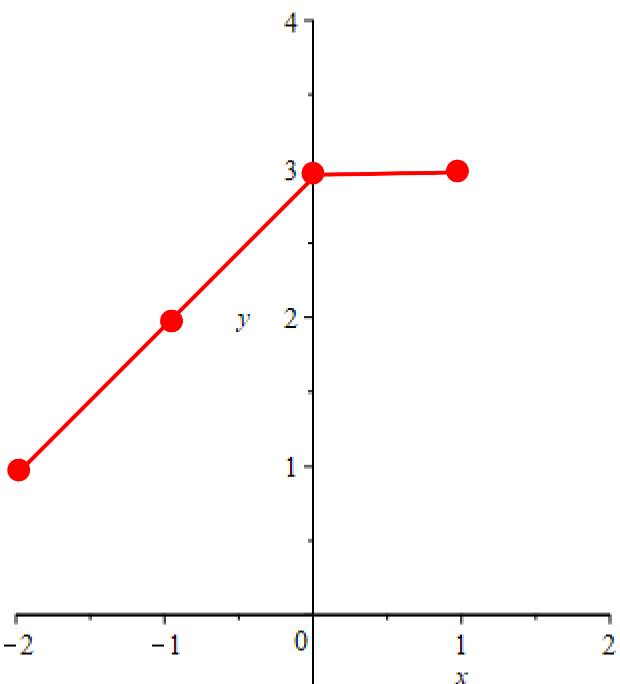
The original points have been shifted vertically up by 1 unit.

Original Function



Transformed Function

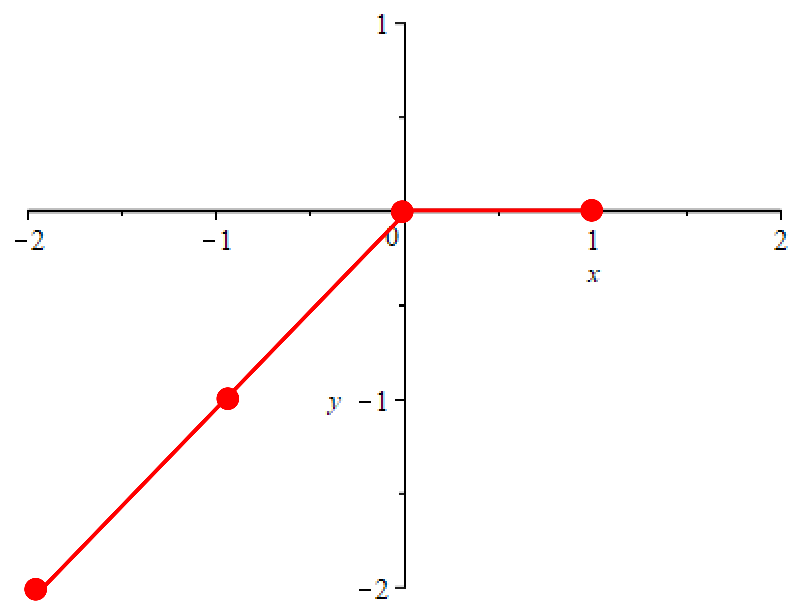
$g(x) = f(x) + 2$



The original graph has been shifted vertically up by 2 units.

Transformed Function

$h(x) = f(x) - 1$



The original points have been shifted vertically down by 1 unit.

Horizontal Shift:

For $c > 0$**,**

The graph of $g(x) = f(x - c)$**, is the graph of** $f(x)$ **shifted *c* units to the right.**

The graph of $g(x) = f(x + c)$**, is the graph of** $f(x)$ **shifted *c* units to the left.**

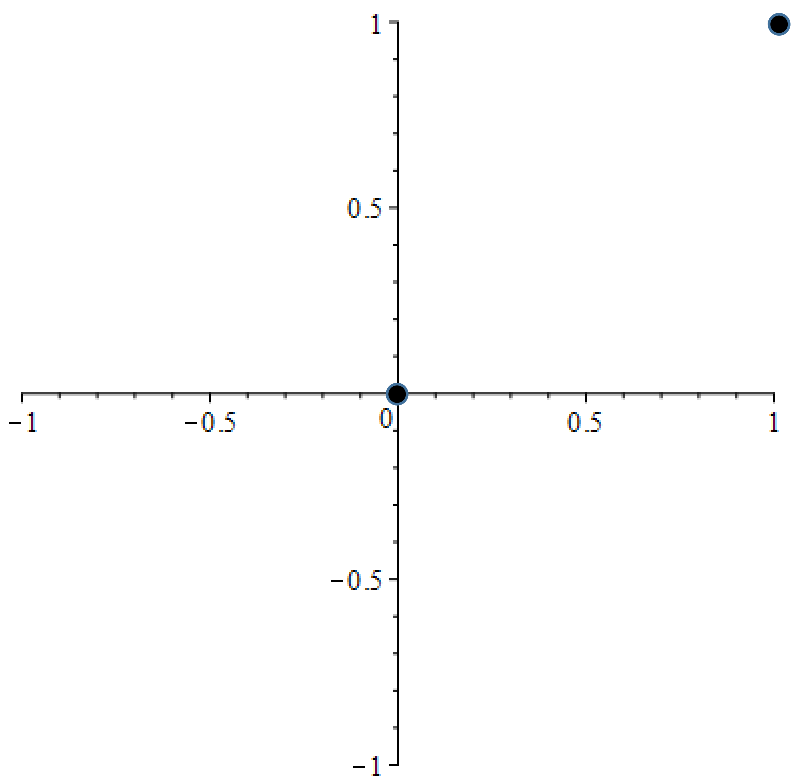
For a horizontal shift, the *x*-coordinates change, but the *y*-coordinates remain the same.

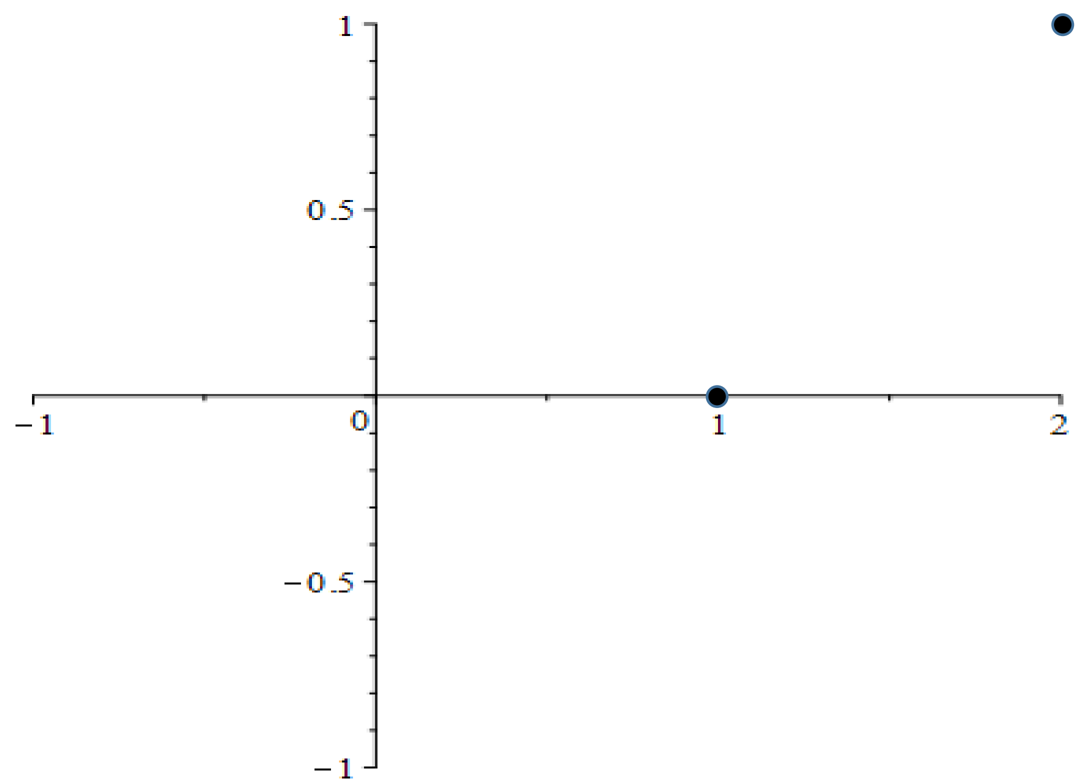
Original Function

$f=\{(0,0),(1,1)\}$

Transformed Function

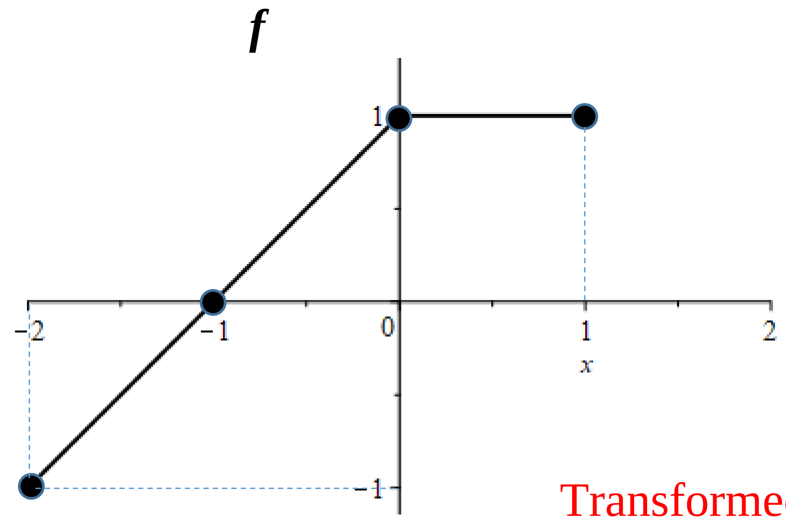
$g(x)=f(x-1),\ g=\{(\boxed{1},0),(\boxed{2},1)\}$





The original points have been shifted horizontally to the right by 1 unit.

Original Function

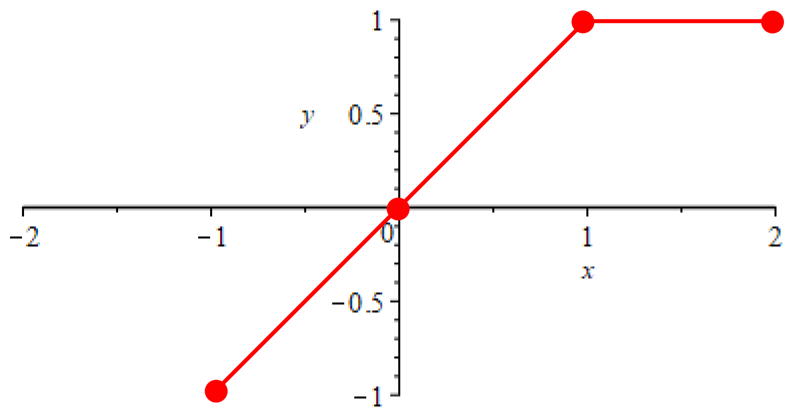


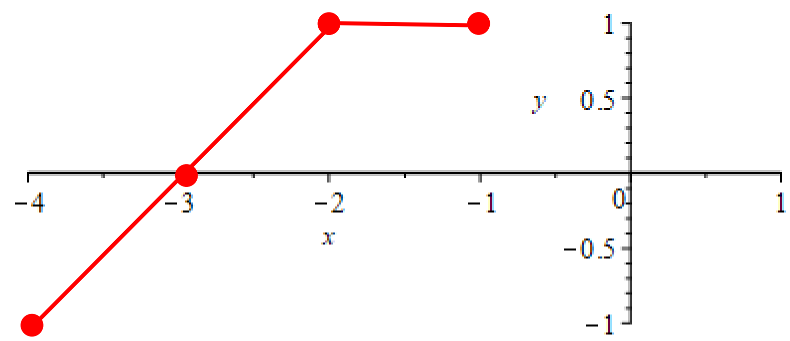
Transformed Function

$g(x) = f(x-1)$

Transformed Function

$h(x) = f(x+2)$





The original graph has been shifted horizontally to the right by 1 unit.

The original graph has been shifted horizontally to the left by 2 units.

Reflection about the x-axis:

The graph of $g(x) = -f(x)$ **is the graph of** $f(x)$ **reflected about the** ***x*****-axis.**

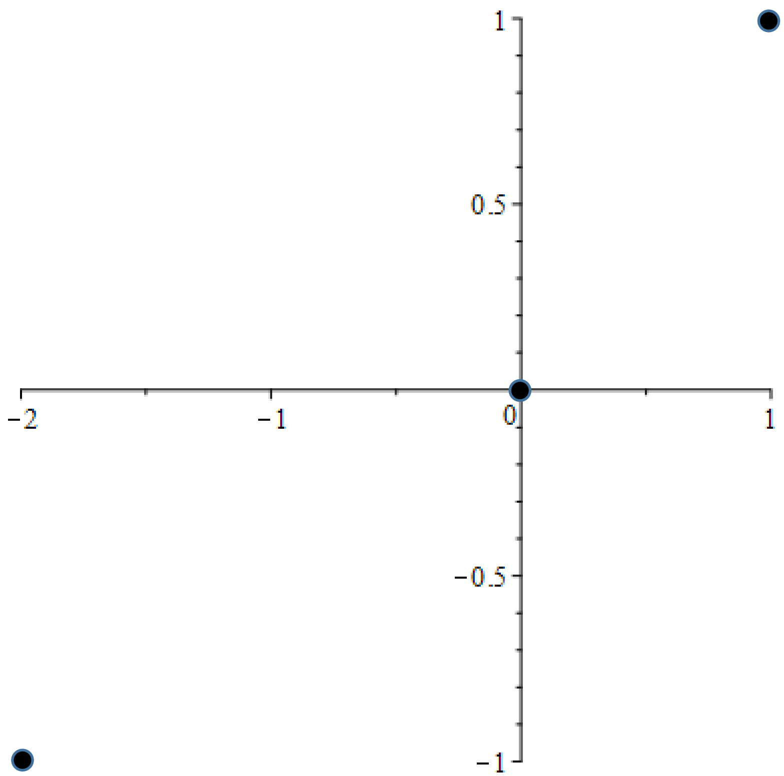
For reflection about the ***x*****-axis, the non-zero** ***y*****-coordinates change, but the** ***x*****-coordinates remain the same.**

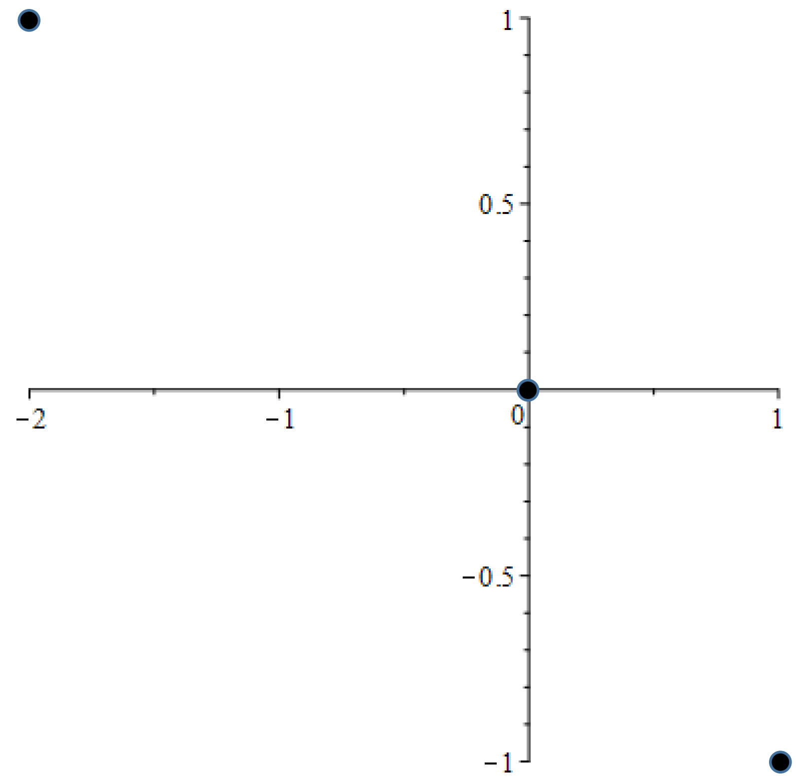
Original Function

$$f=\{(0,0),(1,1),(-2,-1)\}$$

Transformed Function

$$g(x)=-f(x),\ g=\{(0,\boxed{0}),(1,\boxed{-1}),(-2,\boxed{2})\}$$





The original points have been reflected across the *x*-axis.

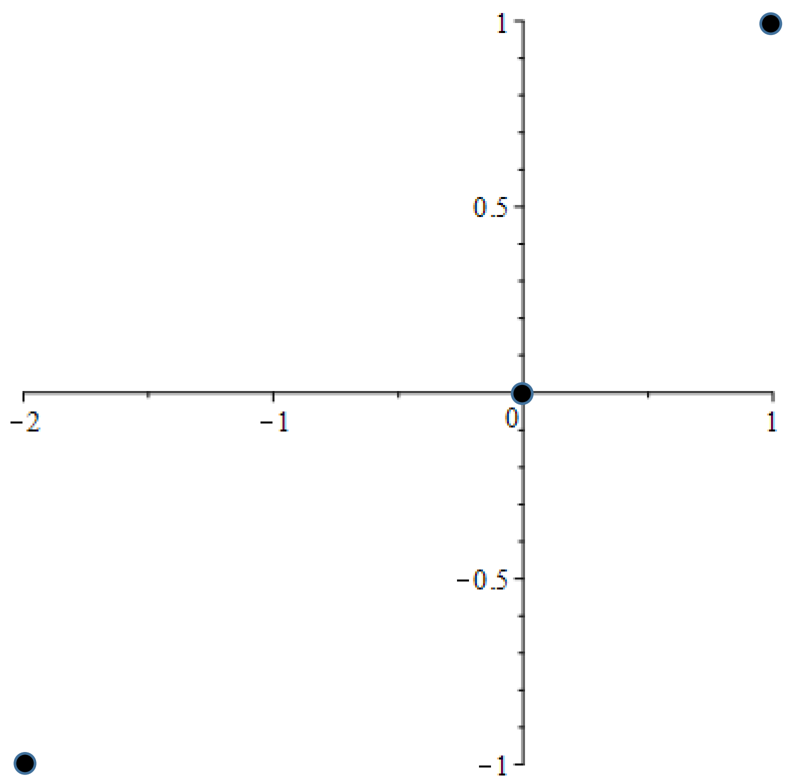
Reflection about the y-axis:

The graph of $g(x) = f(-x)$ **is the graph of** $f(x)$ **reflected about the *y*-axis.**

For reflection about the *y*-axis, the non-zero *x*-coordinates change, but the *y*-coordinates remain the same.

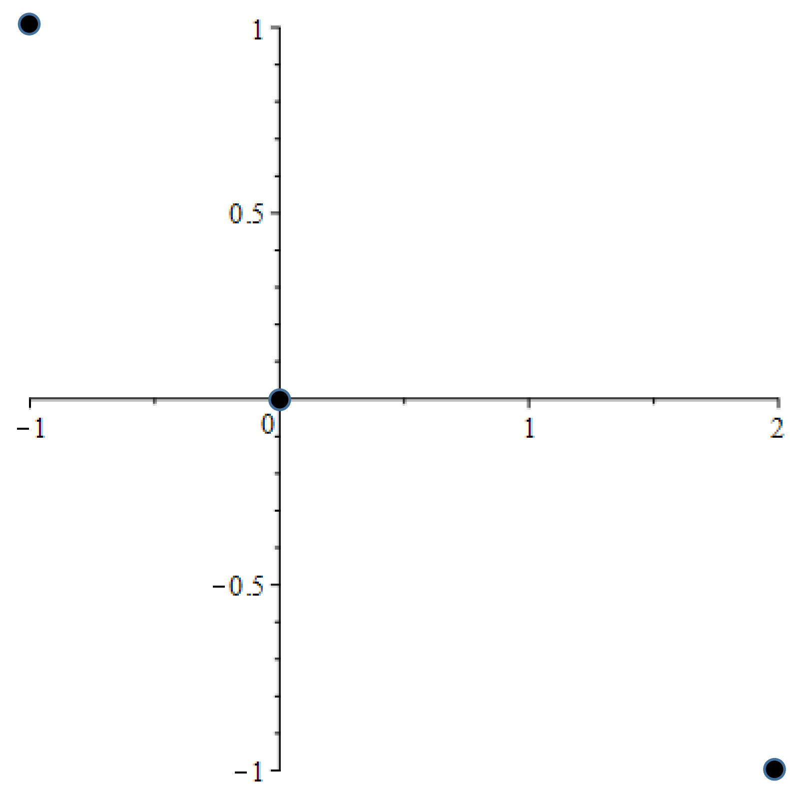
Original Function

$f=\{(0,0),(1,1),(-2,-1)\}$



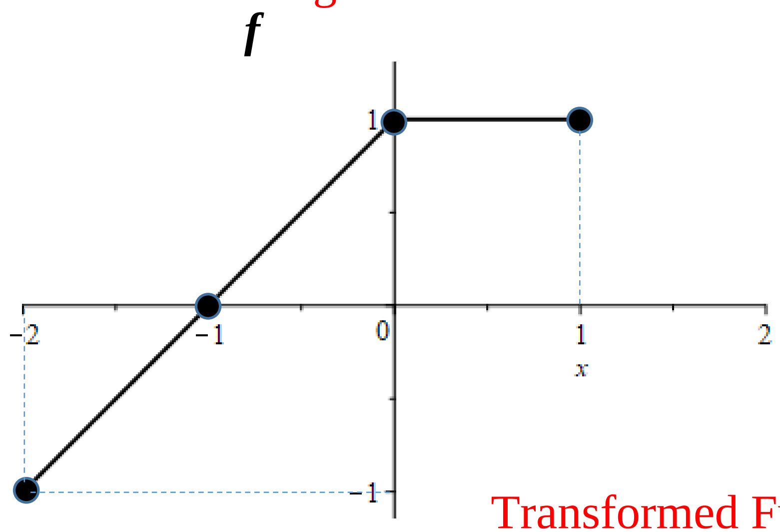
Transformed Function

$g(x)=f(-x),\ g=\{(\boxed{0},0),(\boxed{-1},1),(\boxed{2},-1)\}$



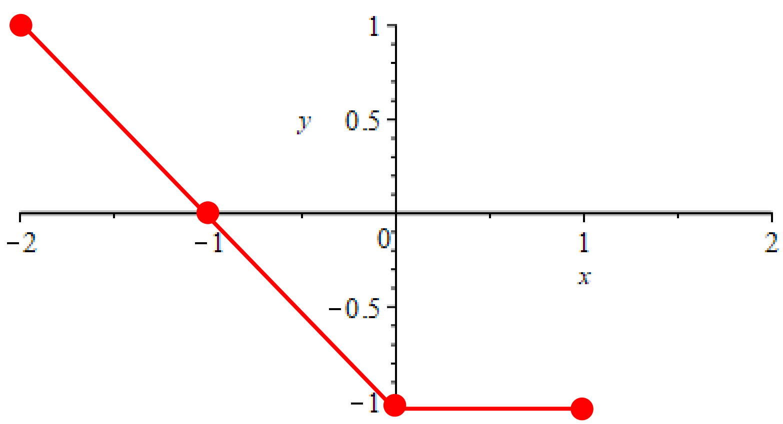
The original points have been reflected across the *y*-axis.

Original Function



Transformed Function

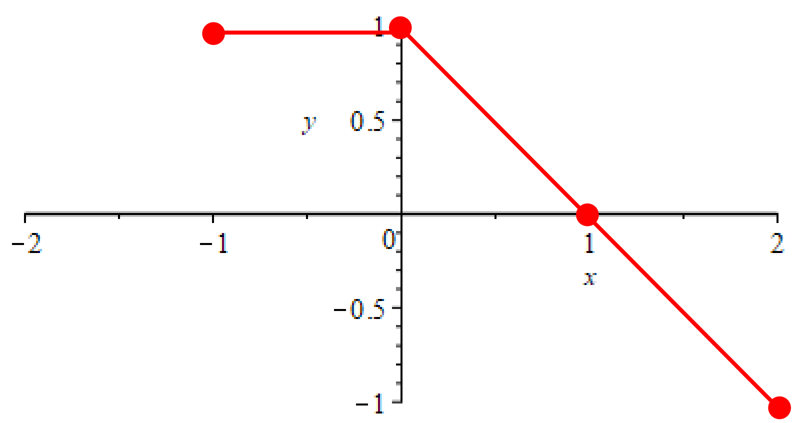
$g(x) = -f(x)$



The original graph has been reflected across the *x*-axis.

Transformed Function

$h(x) = f(-x)$



The original graph has been reflected across the *y*-axis.

Vertical Stretch/Compress:

For $c > 1$, the graph of $g(x) = cf(x)$ is the graph of $f(x)$ *stretched* away from the *x*-axis by a factor of *c*.

For $0 < c < 1$, the graph of $g(x) = cf(x)$ is the graph of $f(x)$ *compressed* toward the *x*-axis by a factor of *c*.

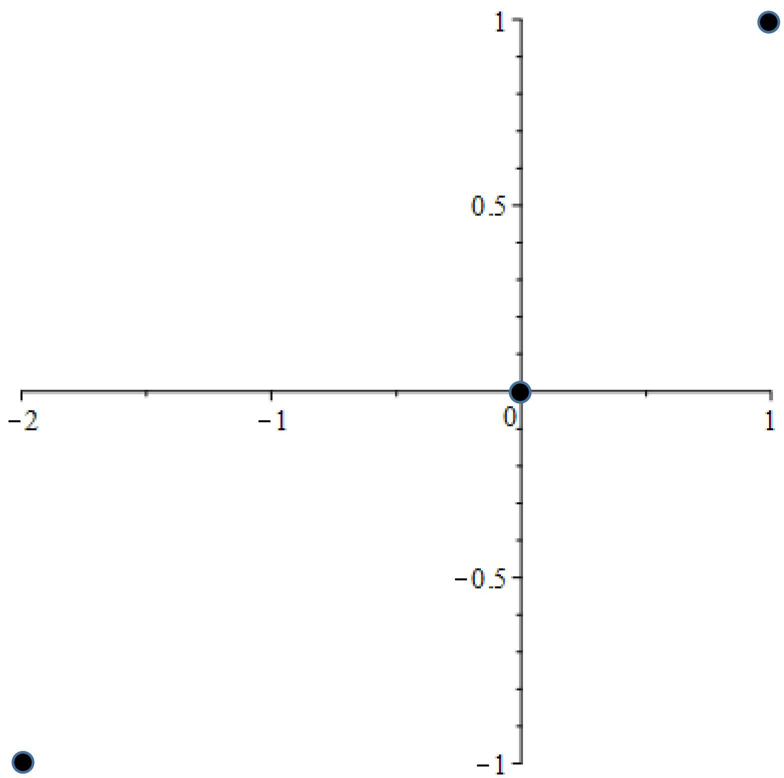
For a vertical stretch/compress, the non-zero *y*-coordinates change, but the *x*-coordinates remain the same.

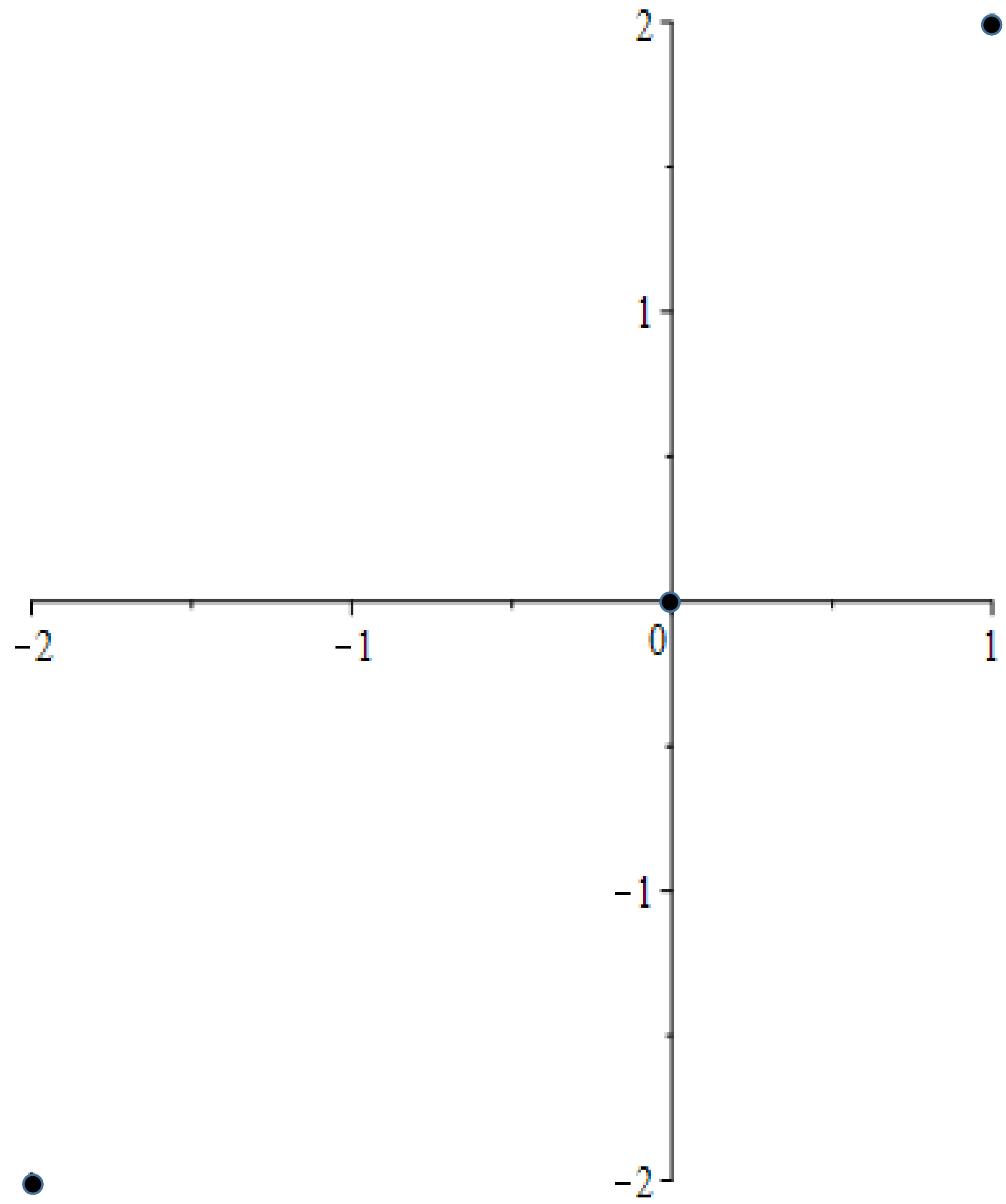
Original Function

$f = \{(0,0),(1,1),(-2,-1)\}$

Transformed Function

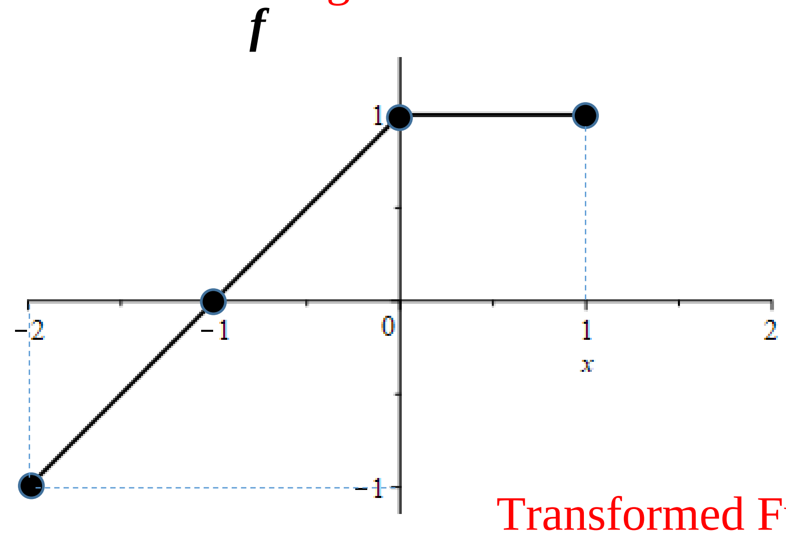
$g(x) = 2f(x),\ g = \{(0,\boxed{0}),(1,\boxed{2}),(-2,\boxed{-2})\}$





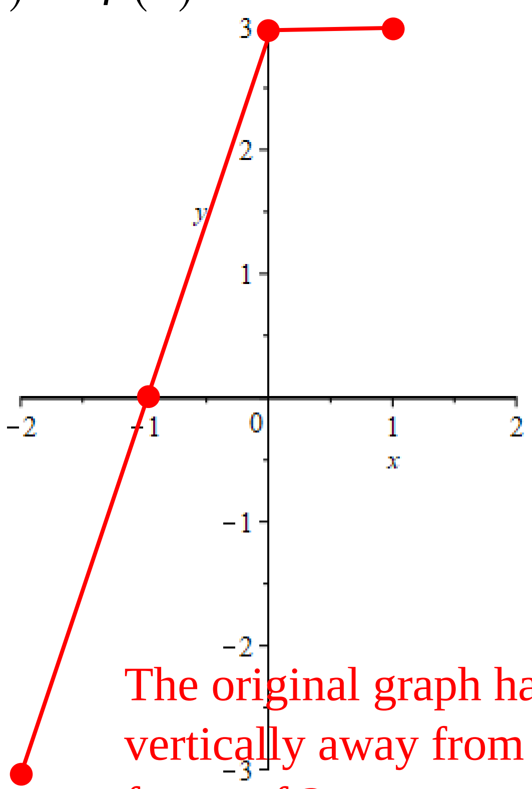
The original points have been stretched vertically away from the *x*-axis by a factor of 2.

Original Function



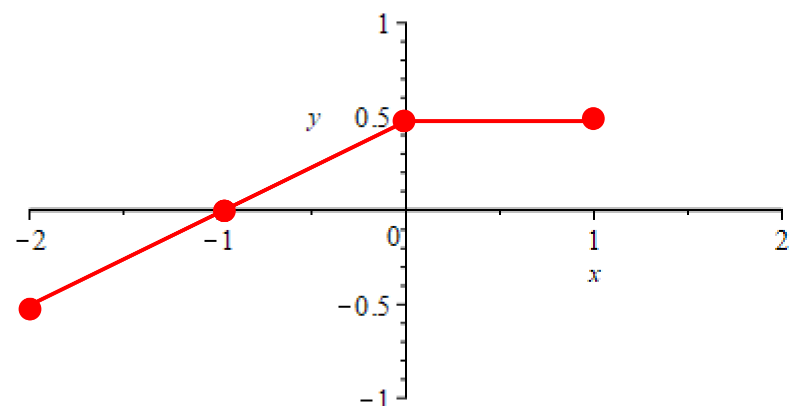
Transformed Function

$g(x) = 3f(x)$



Transformed Function

$h(x) = \frac{1}{2}f(x)$



The original graph has been stretched vertically away from the *x*-axis by a factor of 3.

The original graph has been compressed vertically toward the *x*-axis by a factor of $\frac{1}{2}$.

<u>Horizontal Stretch/Compress:</u>

For $c > 1$**, the graph of** $g(x) = f(cx)$ **is the graph of** $f(x)$ ***<u>compressed</u>*** **toward the *y*-axis by a factor of** $\frac{1}{c}$**.**

For $0 < c < 1$**, the graph of** $g(x) = f(cx)$ **is the graph of** $f(x)$ ***<u>stretched</u>*** **away from the *y*-axis by a factor of** $\frac{1}{c}$**.**

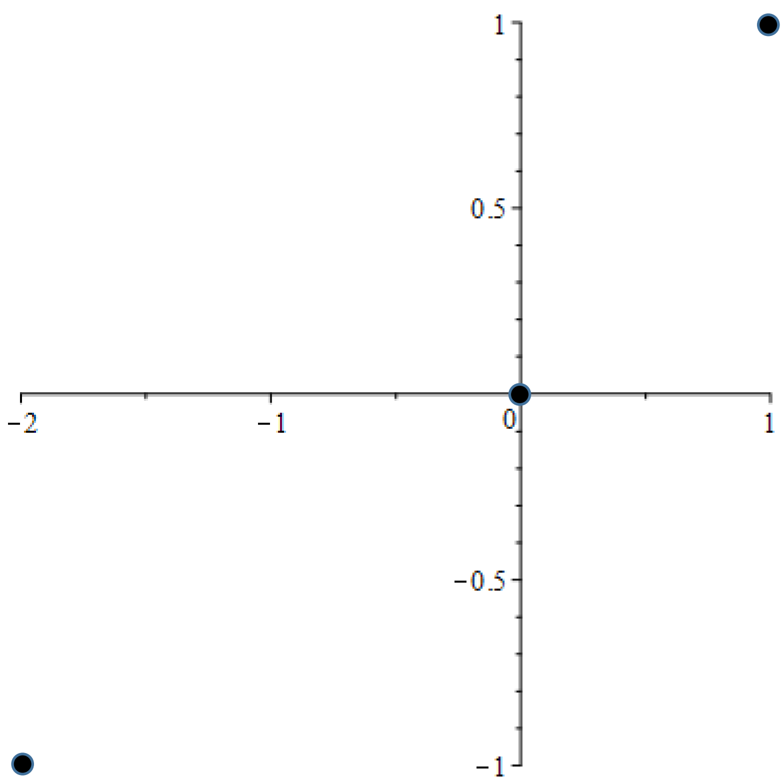
For a horizontal stretch/compress, the non-zero *x*-coordinates change, but the *y*-coordinates remain the same.

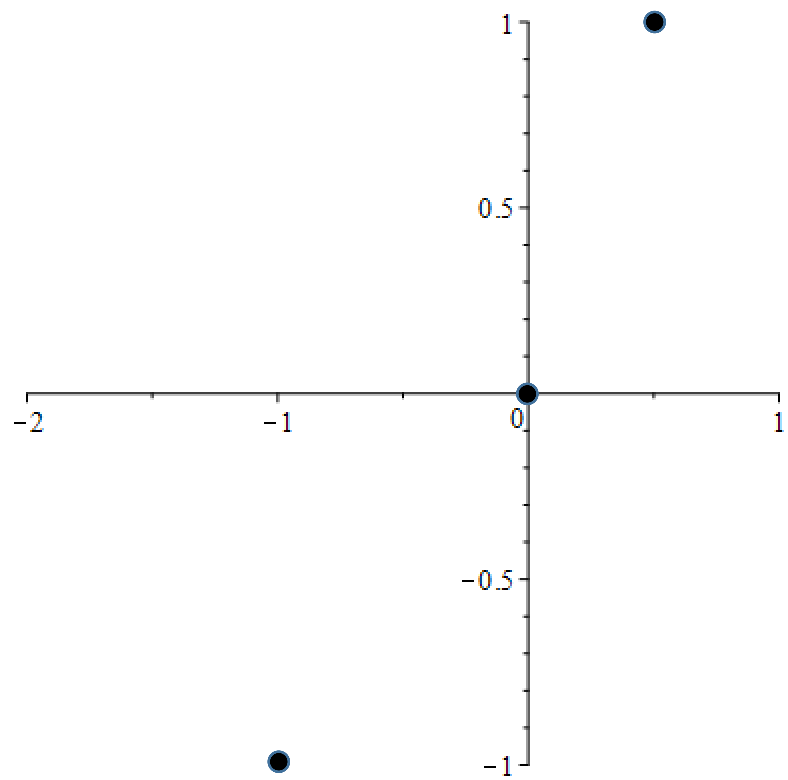
Original Function

$f = \{(0,0),(1,1),(-2,-1)\}$

Transformed Function

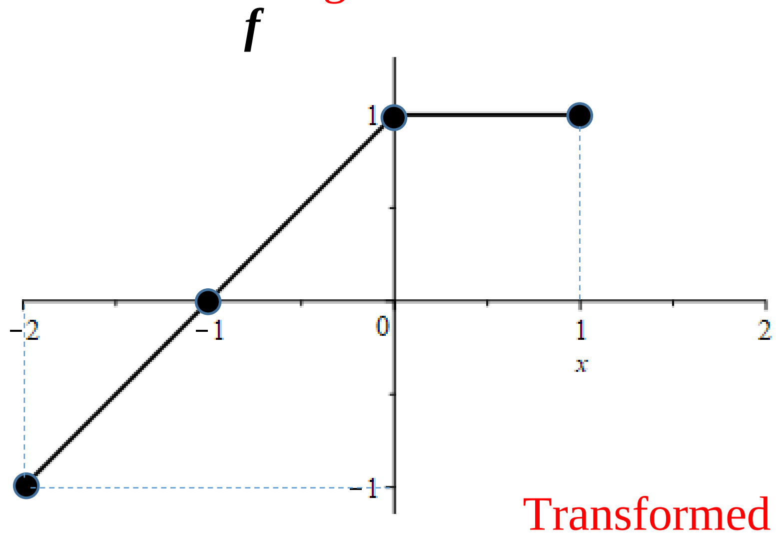
$g(x) = f(2x),\ g = \{(\boxed{0},0),(\boxed{\tfrac{1}{2}},1),(\boxed{-1},-1)\}$





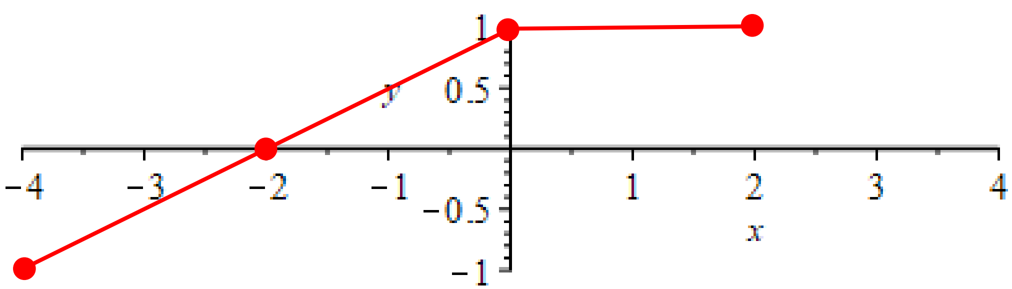
The original points have been compressed horizontally toward the *y*-axis by a factor of $\frac{1}{2}$.

Original Function



Transformed Function

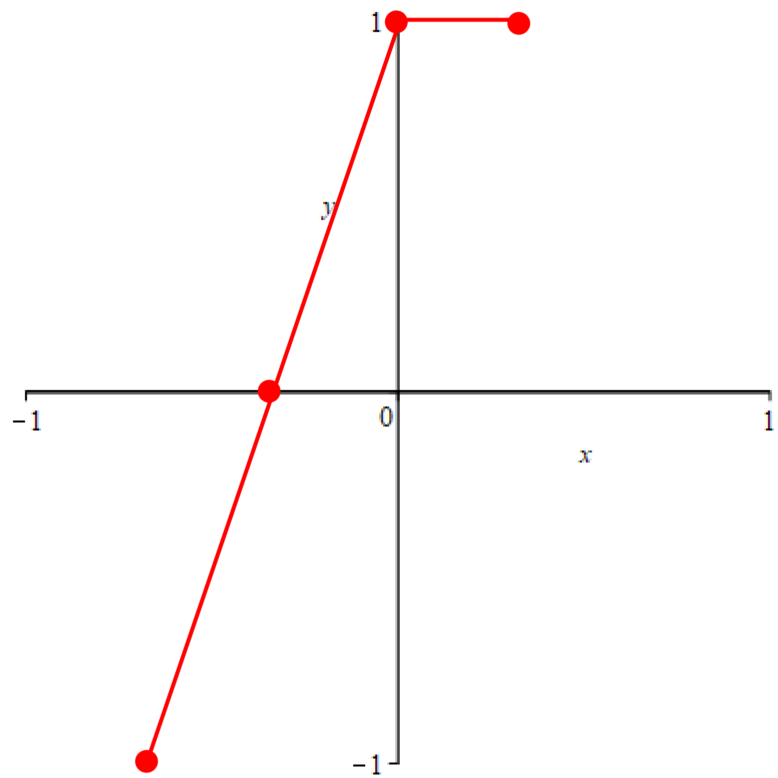
$g(x) = f\left(\frac{1}{2}x\right)$



Stretched horizontally by a factor of 2.

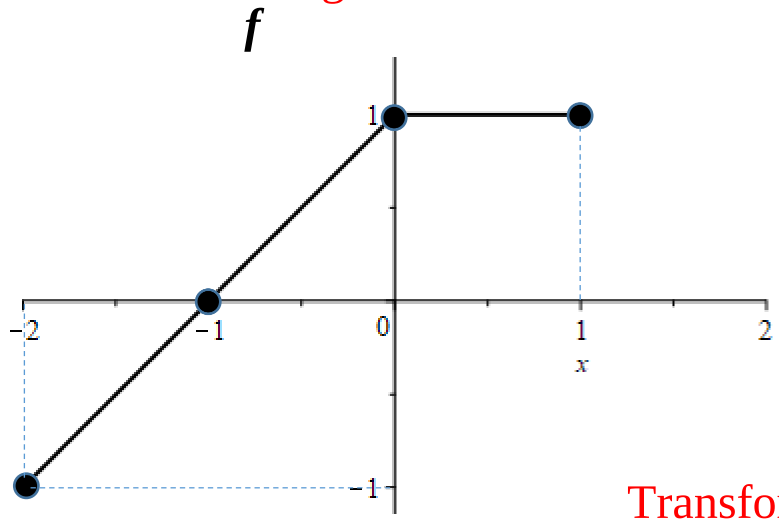
Transformed Function

$h(x) = f(3x)$



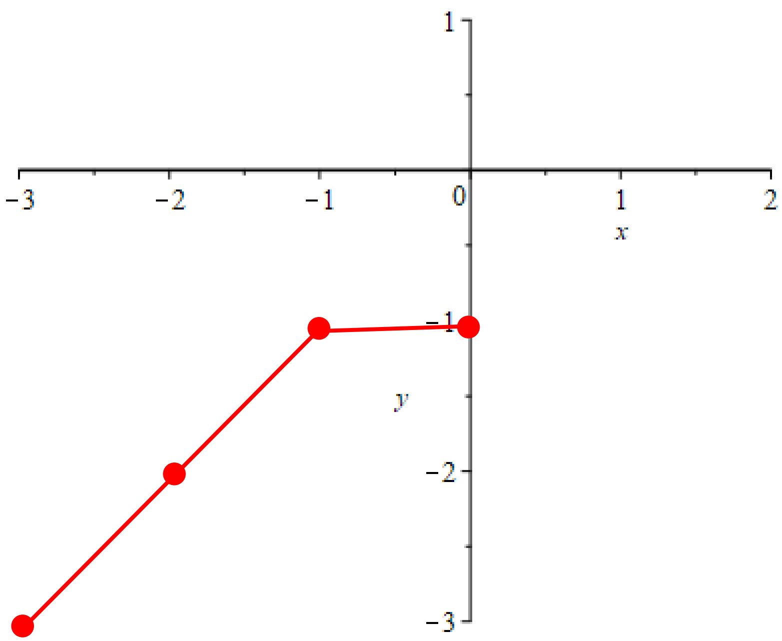
Compressed horizontally by a factor of $\frac{1}{3}$.

Original Function



Transformed Function

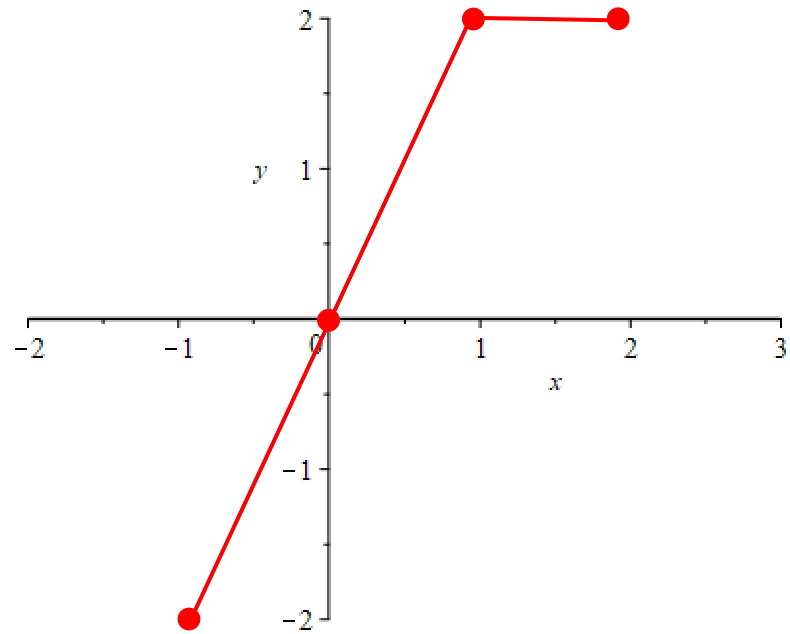
$g(x) = f(x+1) - 2$



Shifted left 1 unit and down 2 units.

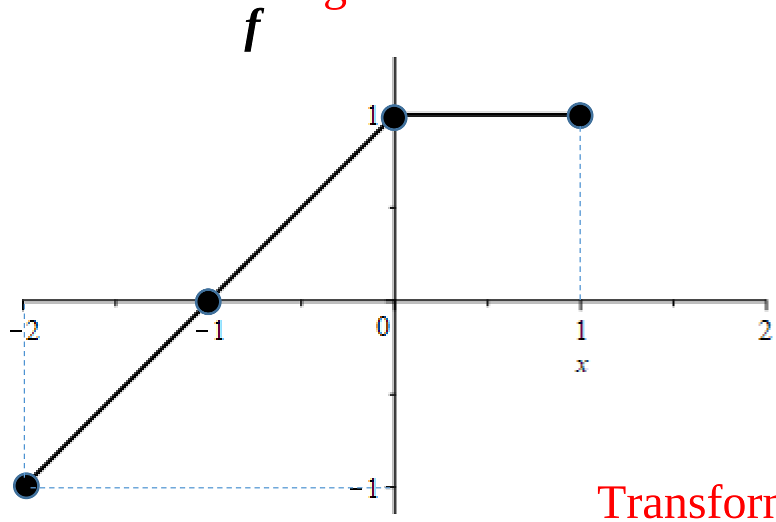
Transformed Function

$h(x) = 2f(x-1)$



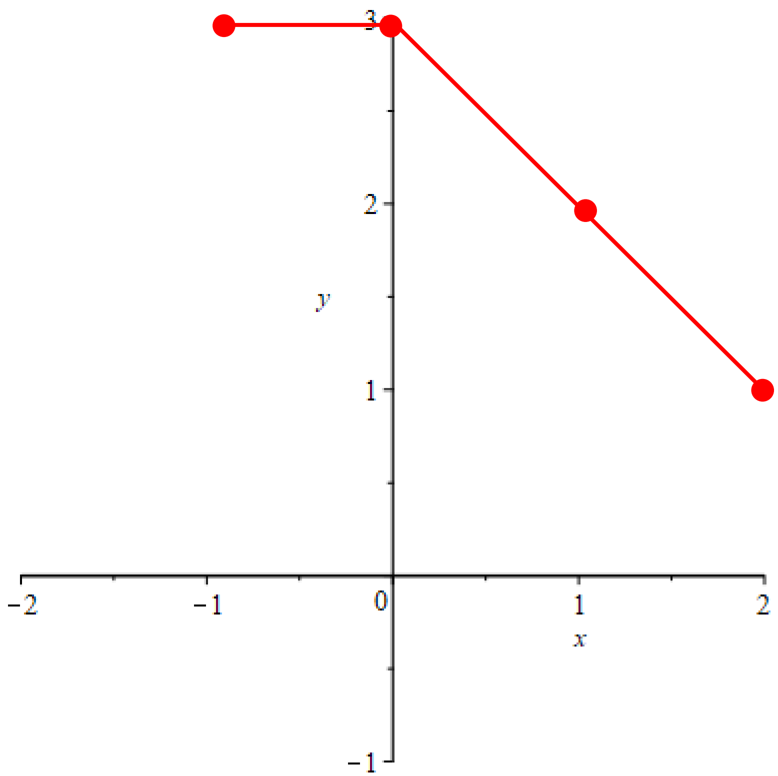
Shifted right 1 unit and stretched vertically by a factor of 2.

Original Function



Transformed Function

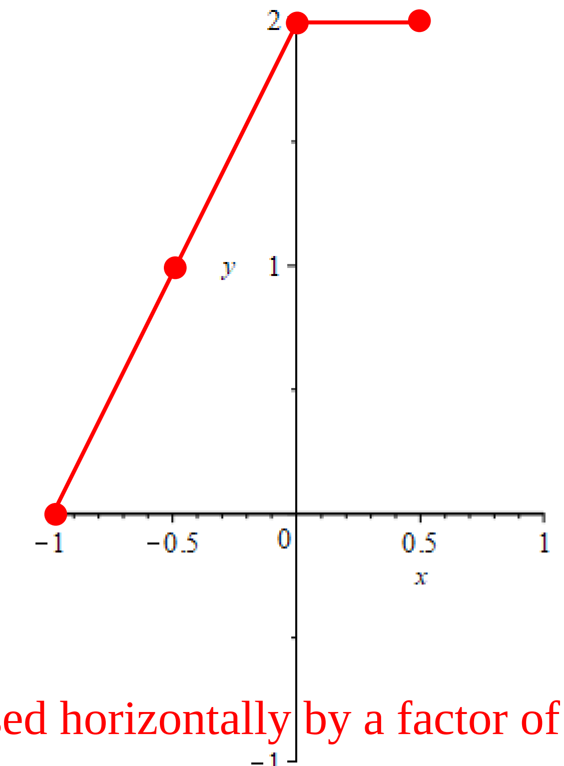
$g(x) = f(-x) + 2$



Reflected about the y-axis and shifted up 2 units.

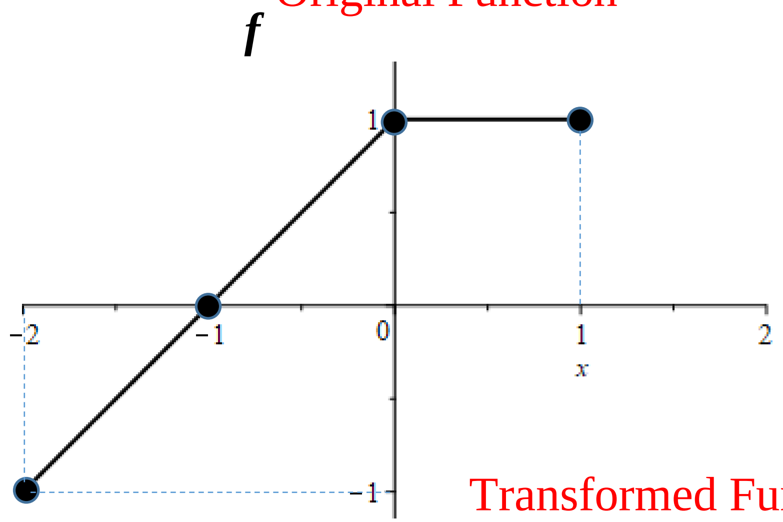
Transformed Function

$h(x) = f(2x) + 1$



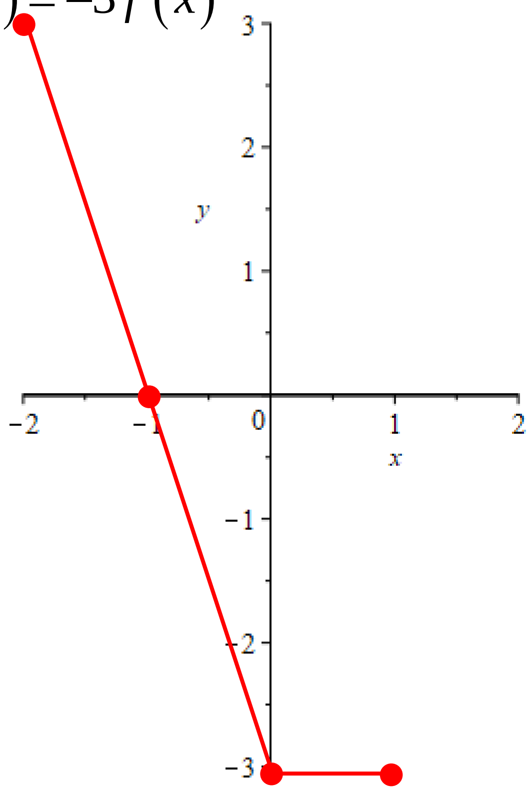
Compressed horizontally by a factor of $\frac{1}{2}$ and shifted up 1 unit.

Original Function



Transformed Function

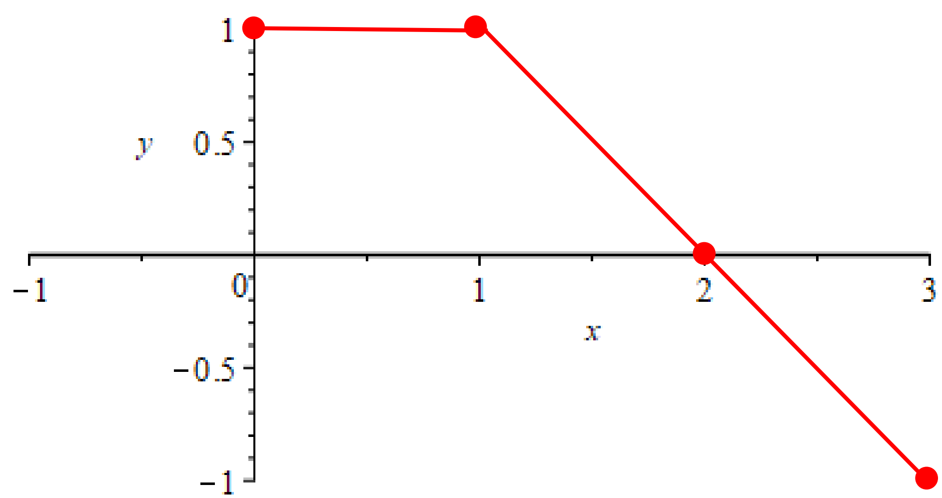
$$g(x) = -3f(x)$$



Stretched vertically by a factor of 3 and reflected about the *x*-axis.

Transformed Function

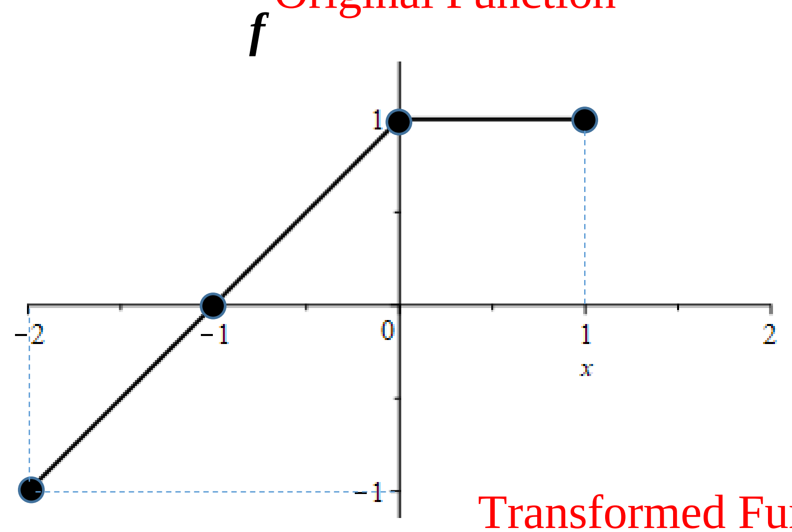
$$h(x) = f(1-x) = f(-x+1) \text{ or } f(-(x-1))$$



Shifted left 1 unit and reflected about the *y*-axis.

Or, reflected about the *y*-axis and shifted right 1 unit.

Original Function

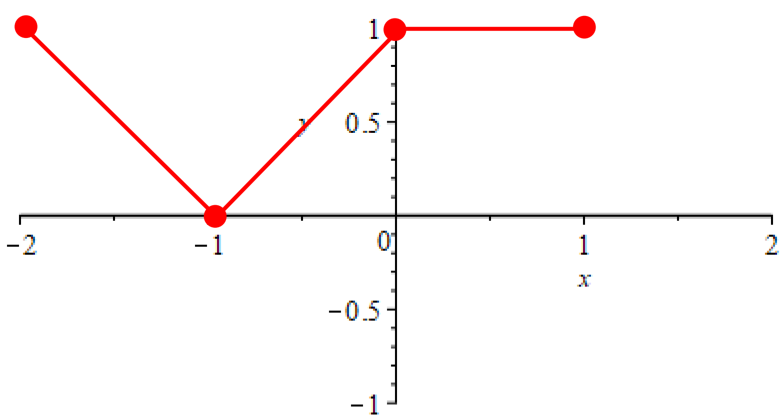


Transformed Function

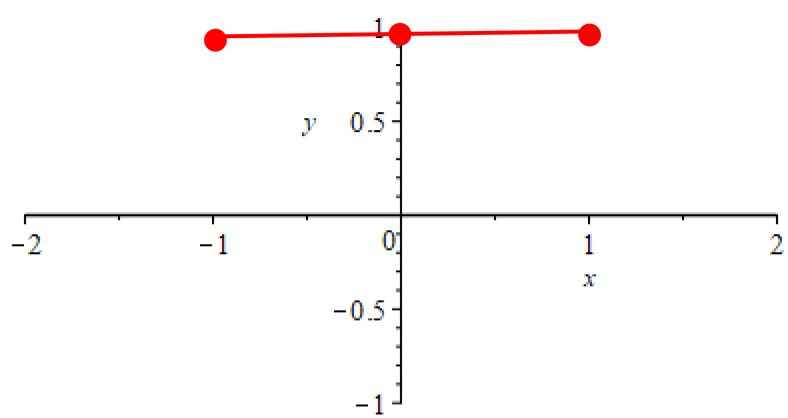
$g(x) = |f(x)|$

Transformed Function

$h(x) = f(|x|)$

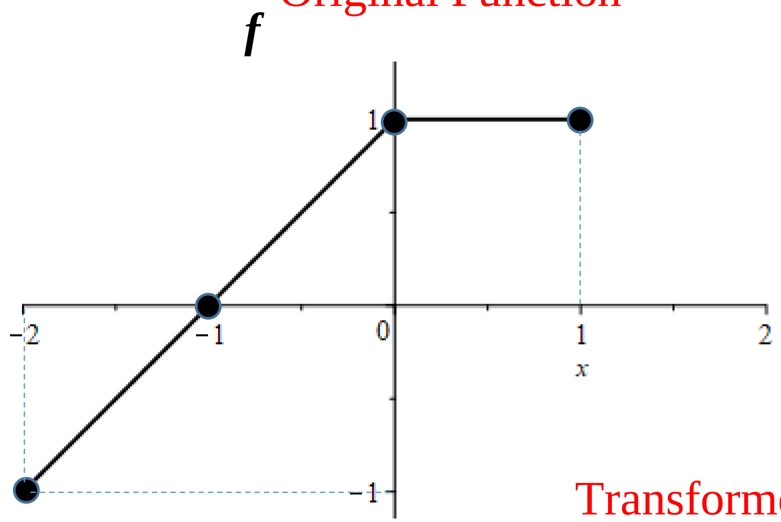


Points on or above the *x*-axis are unchanged, but points below are reflected above.



Points on or to the right of the *y*-axis are unchanged, but points to the left are reflections from the right.

Original Function

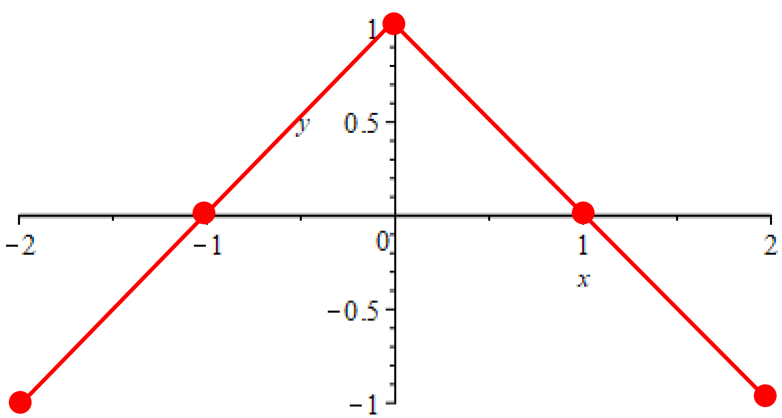


Transformed Function

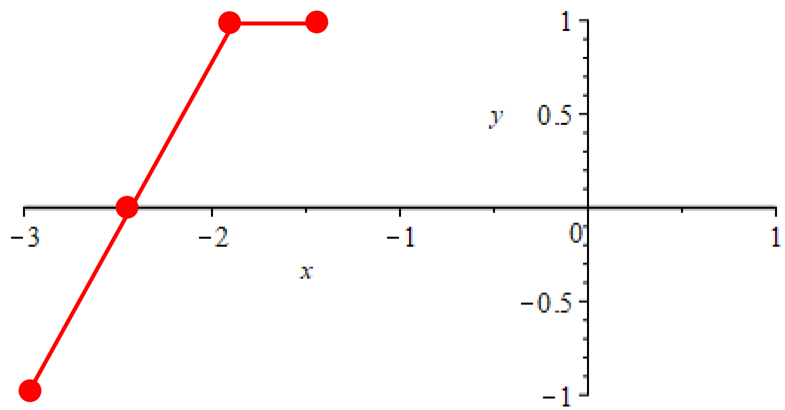
$$g(x) = f(-|x|)$$

Transformed Function

$$h(x) = f(2x+4) \text{ or } f(2(x+2))$$



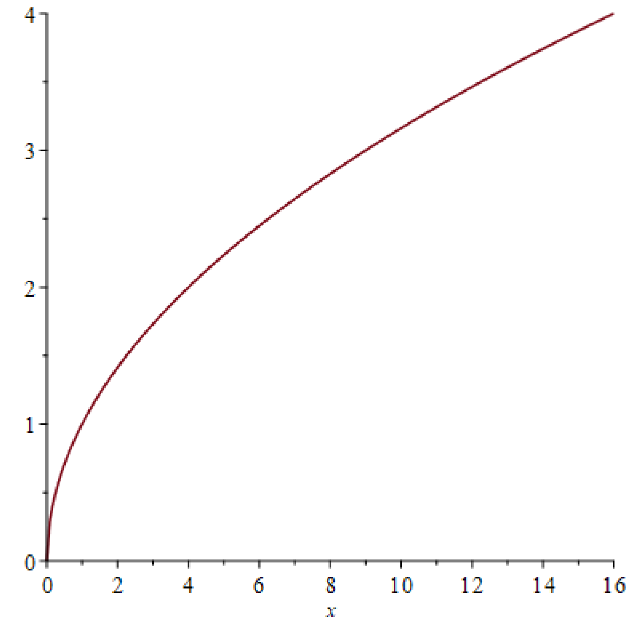
Points on or to the left of the *y*-axis are unchanged, but points to the right are reflections from the left.



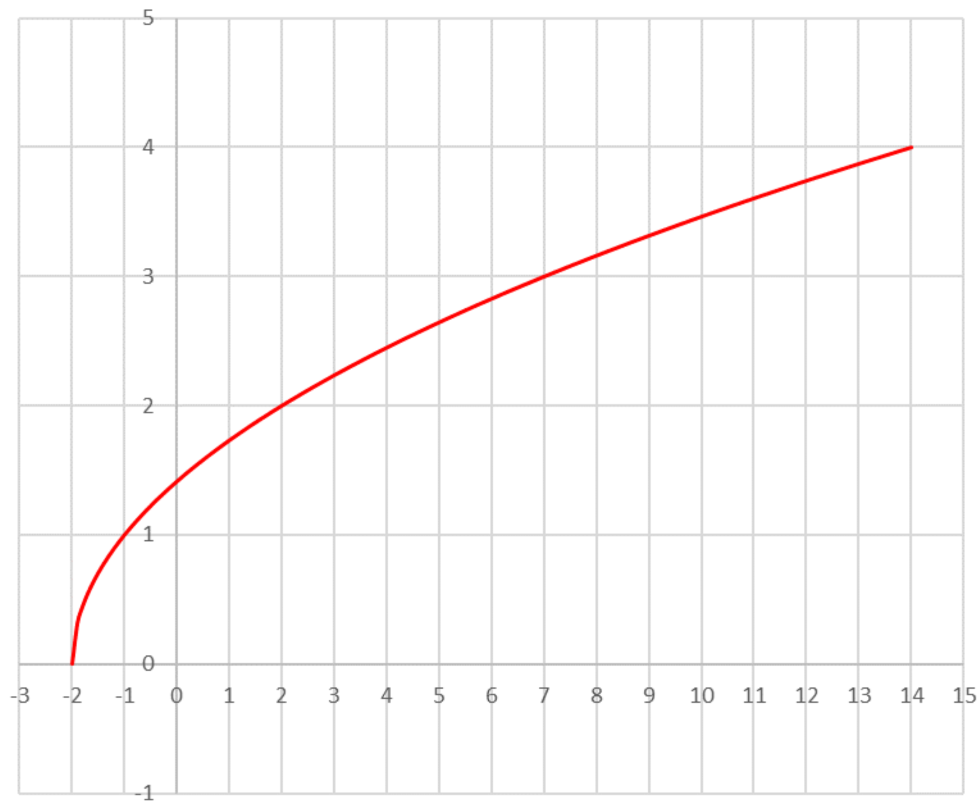
Shift 4 units to the left and compress horizontally by factor of $\frac{1}{2}$. Or, compress horizontally by a factor of $\frac{1}{2}$ and shift 2 units to the left.

$f(x)=\sqrt{x+2}$

Start with the graph of the square-root function.



Shift it to the left 2 units.

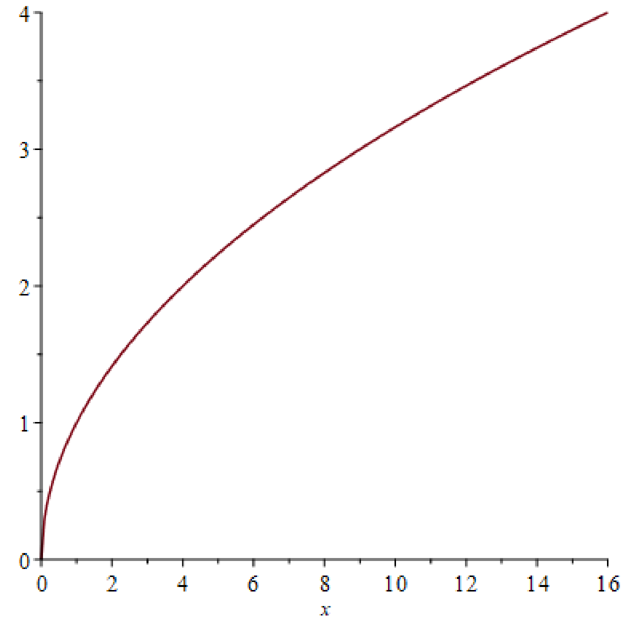


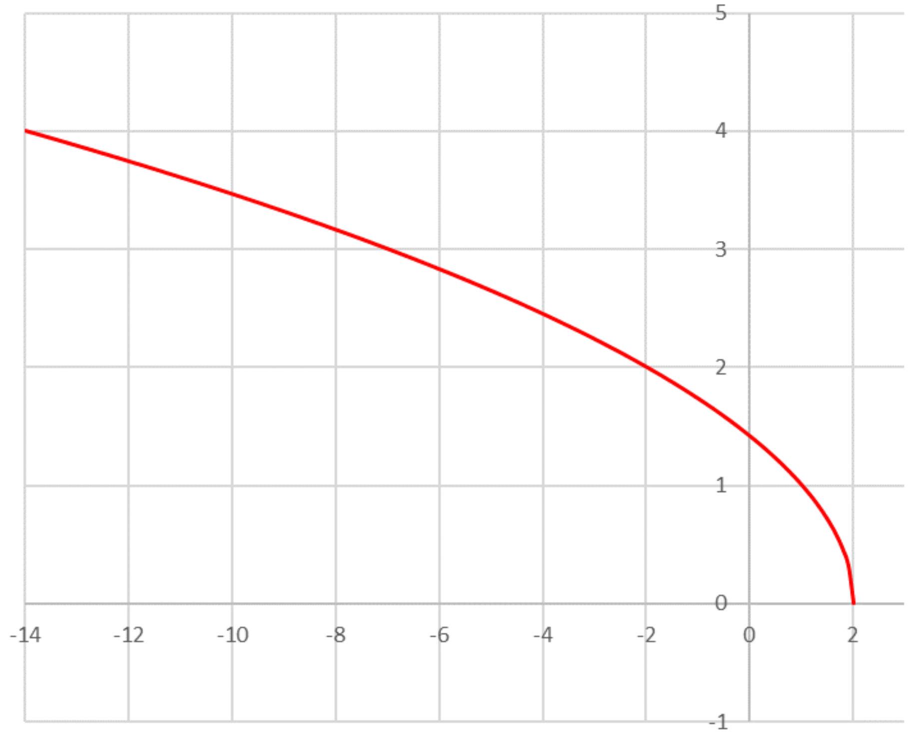
$$f(x) = \sqrt{2 - x} = \sqrt{-x + 2} \text{ or } \sqrt{-(x-2)}$$

Start with the graph of the square-root function.

Shift it 2 units to the left and then reflect about the *y*-axis,

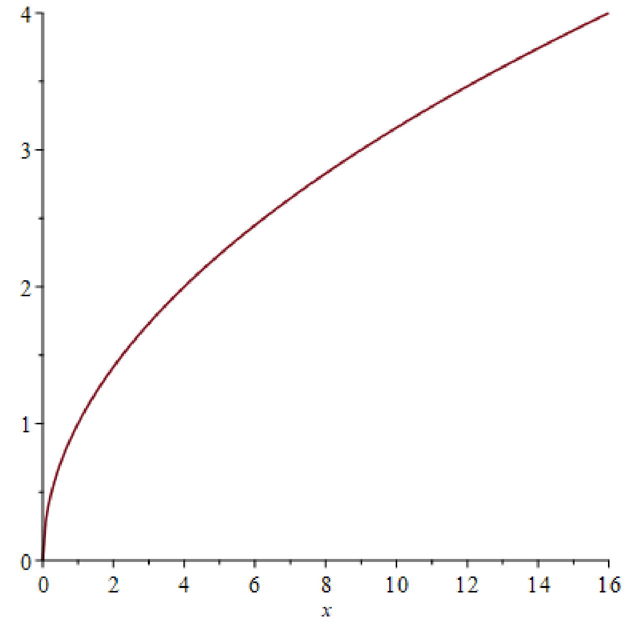
Or reflect it about the *y*-axis and then shift it 2 units to the right.



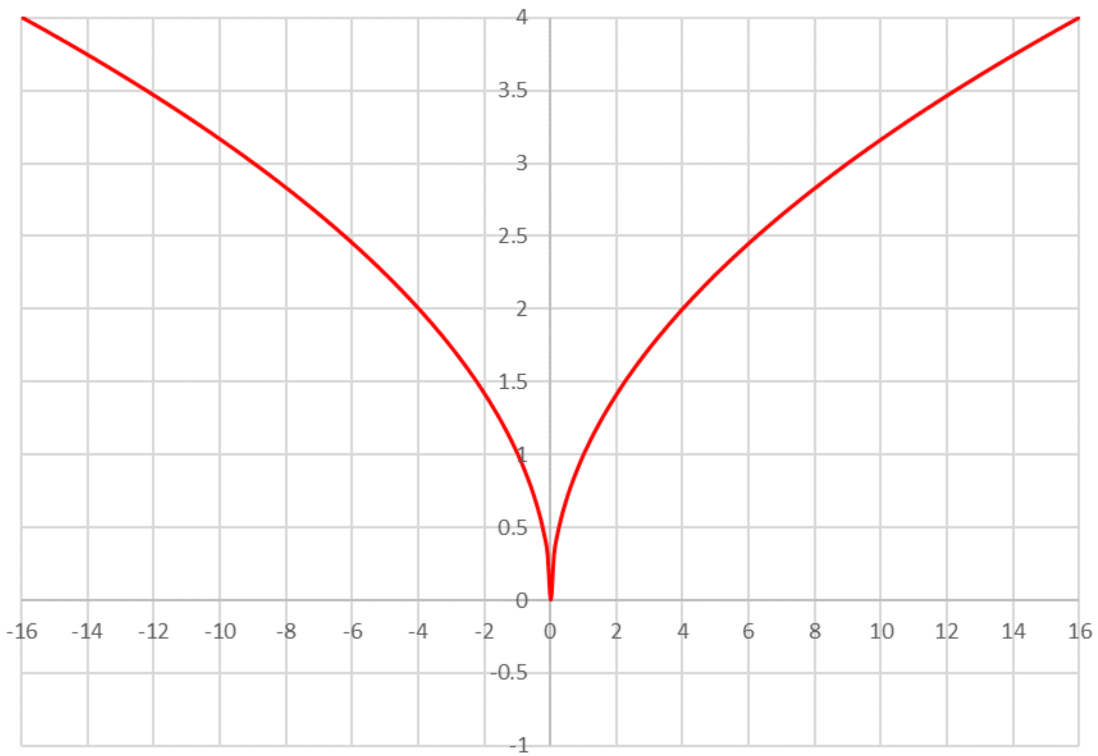


$f(x)=\sqrt{|x|}$

Start with the graph of the square-root function.



Leave the portion of the graph on and to the right of the *y*-axis alone, but also reflect it to the left side.



$f(x) = \left|\sqrt[3]{x}\right|$

Start with the graph of the cube-root function.

Leave the portion of the graph on and above the *x*-axis alone, and reflect the portion of the graph below the *x*-axis so that it's above the *x*-axis.