Review of Exponential Functions:

A function of the form $f(x)=b^x$ **with** $b>0$ **and** $b\neq 1$ **is a called an exponential function with base *b*.**

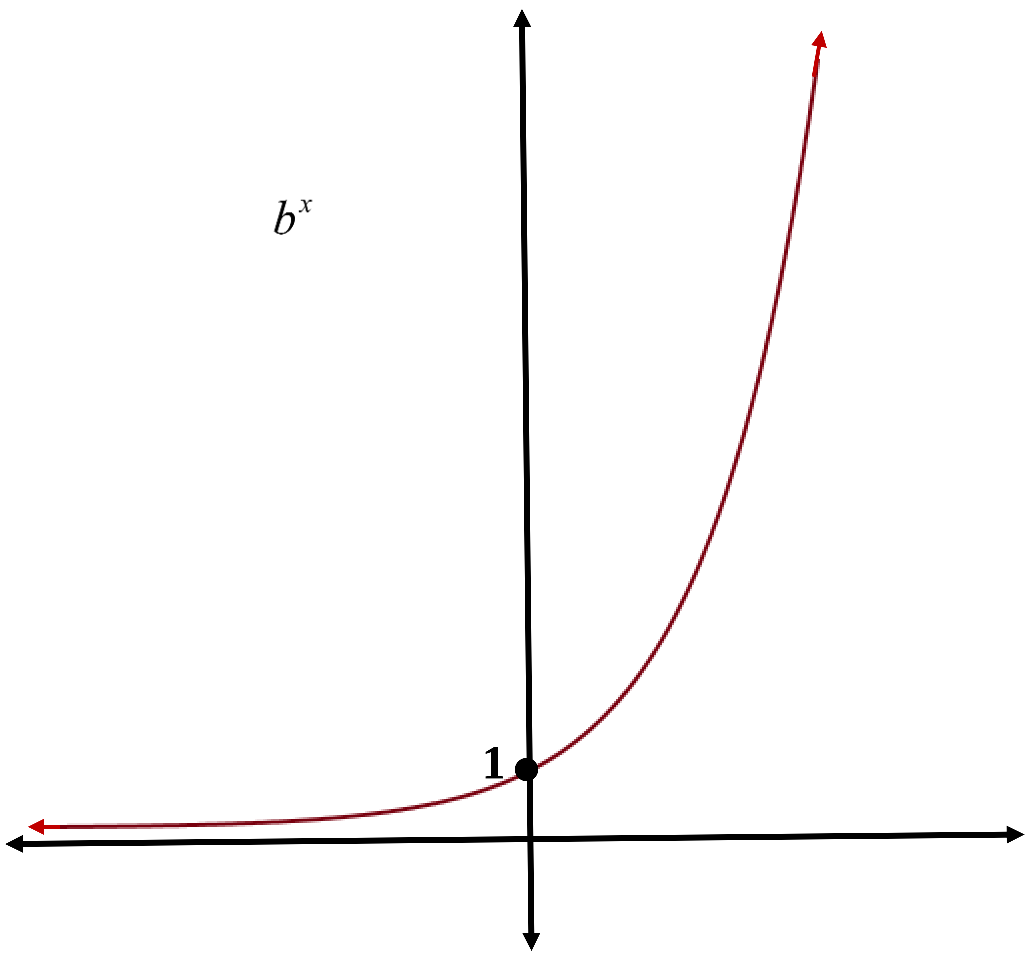
The bases naturally divide into two categories:

$$b>1$$

And

$$0<b<1$$

For $b > 1$**,**



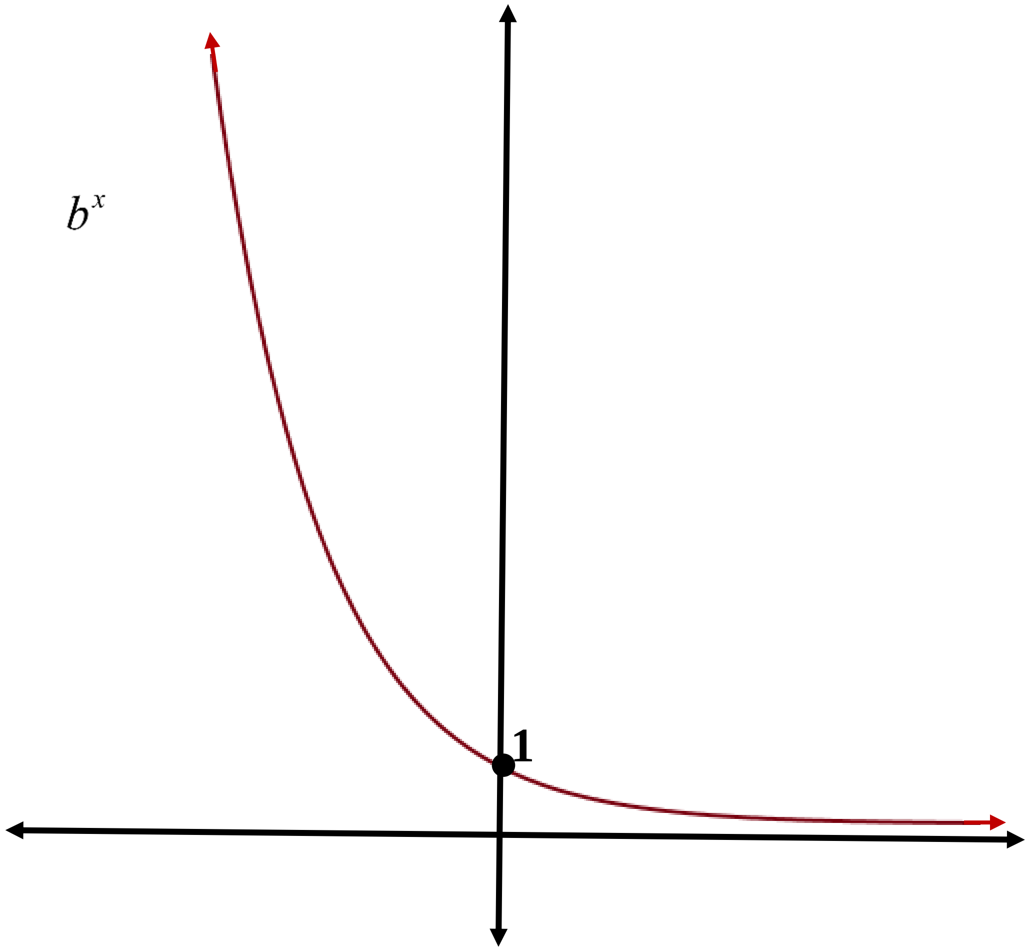
Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

Horizontal Asymptote: $y = 0$ **to the left**

Increasing: $(-\infty, \infty)$

For $0 < b < 1$**,**



Domain: $(-\infty, \infty)$

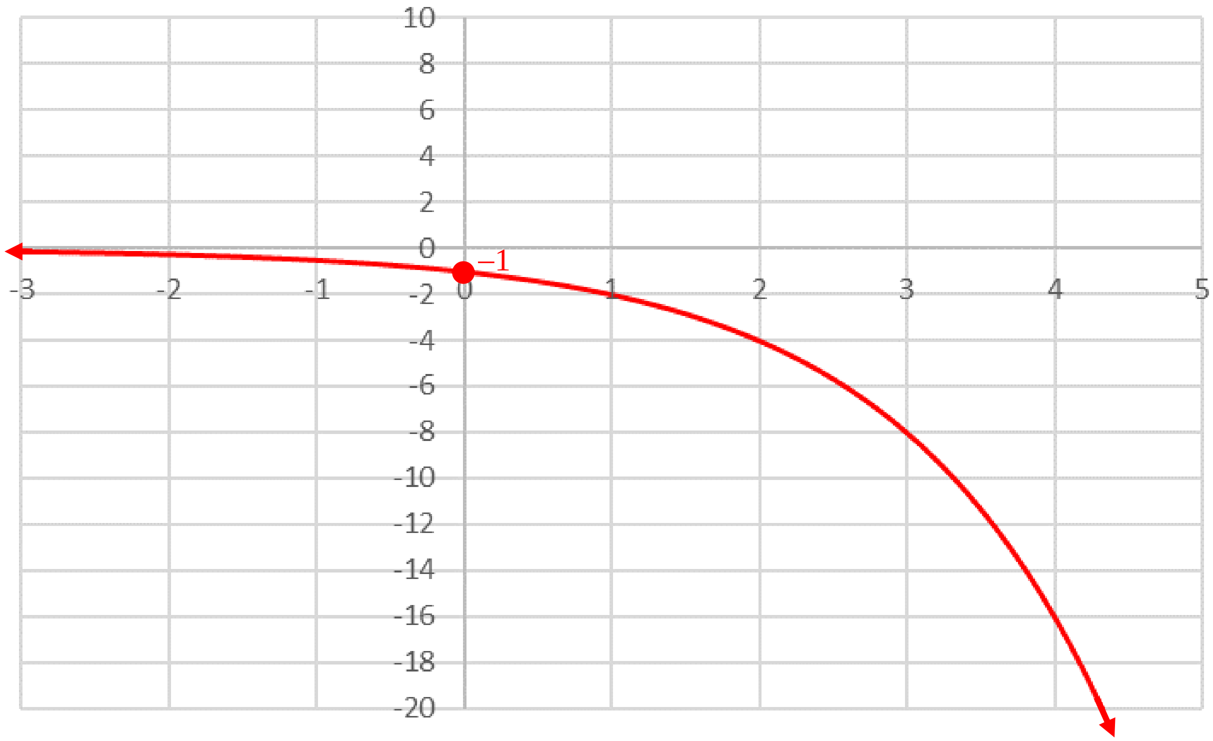
Range: $(0, \infty)$

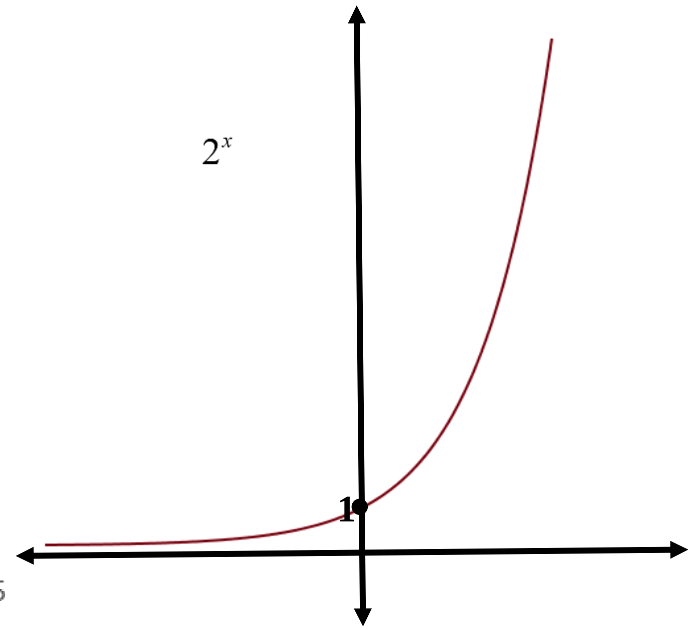
Horizontal Asymptote: $y = 0$ **to the right**

Decreasing: $(-\infty, \infty)$

Transformations of Exponential Functions:

1. $f(x) = -2^x$



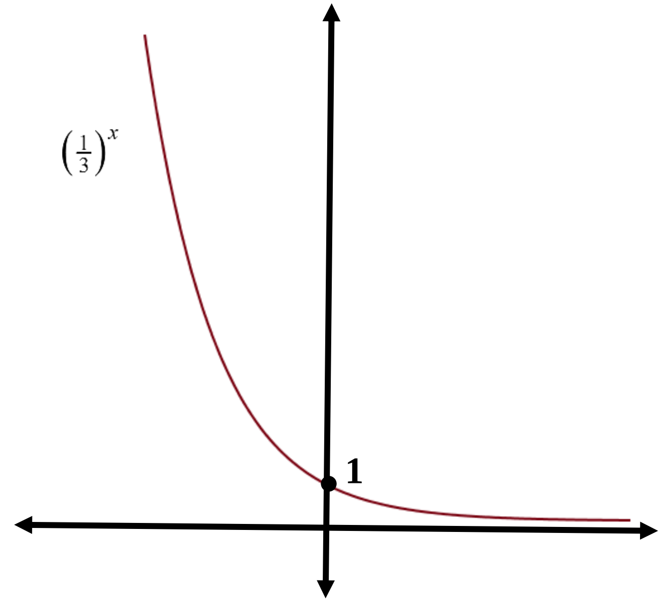


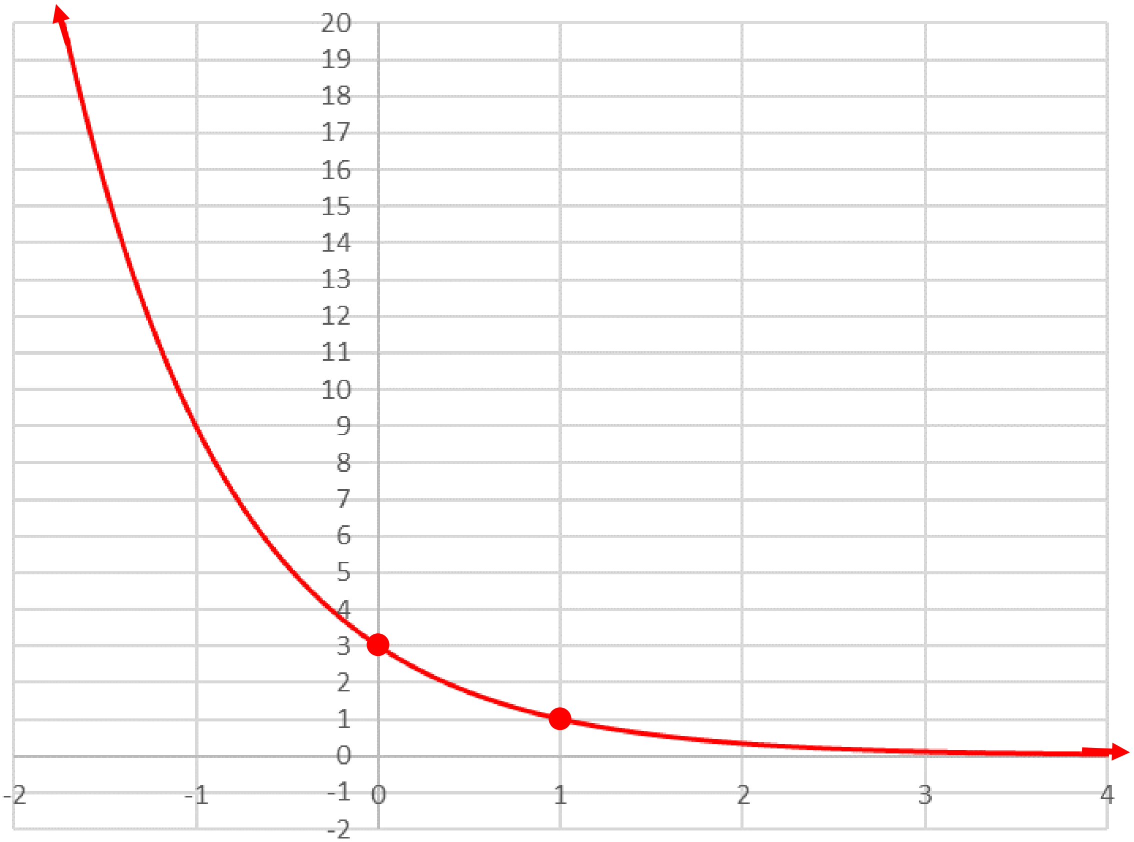
Reflect the original graph across the *x*-axis.

Domain: $(-\infty, \infty)$

Range: $(-\infty, 0)$

2. $f(x)=\left(\frac{1}{3}\right)^{x-1}$





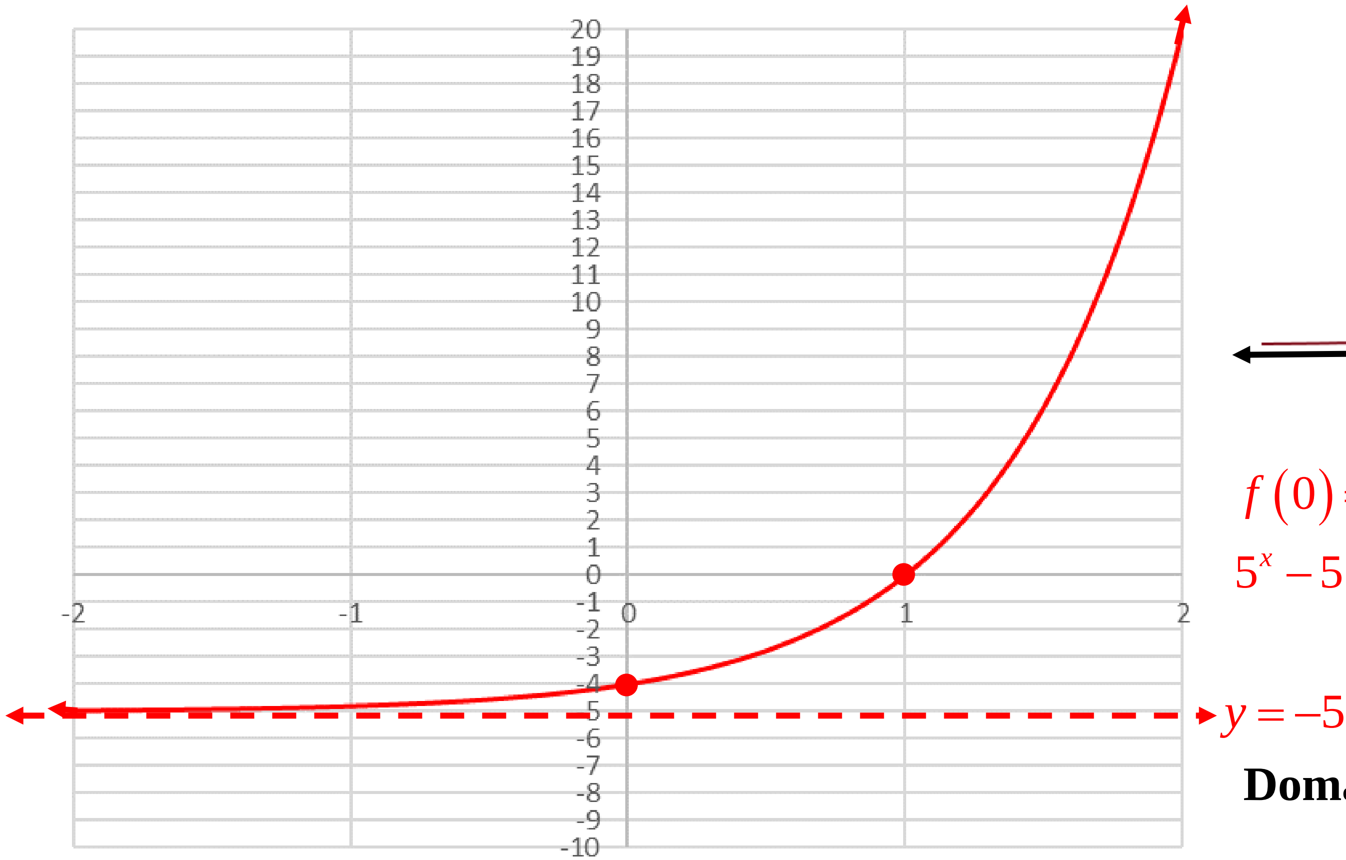
$f(0)=\left(\frac{1}{3}\right)^{-1}=3$

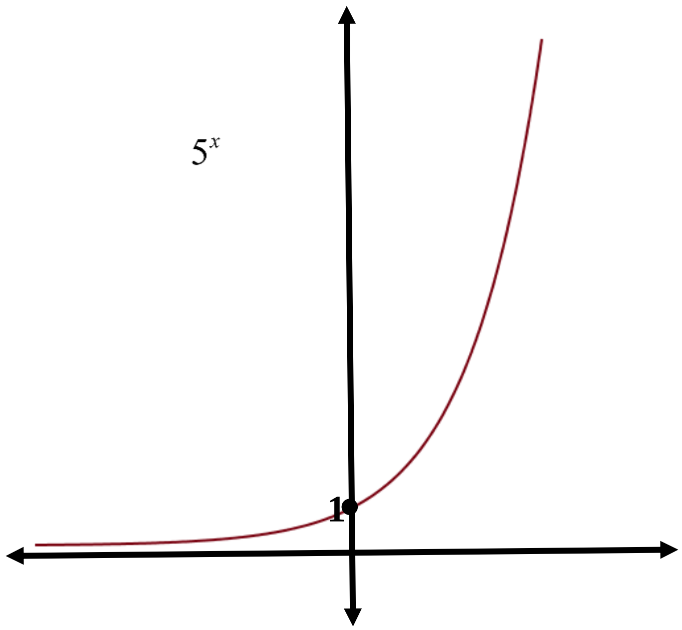
Domain: $(-\infty,\infty)$

Shift the original graph 1 unit to the right.

Range: $(0,\infty)$

3. $f(x) = 5^x - 5$





$f(0) = 1 - 5 = -4,$

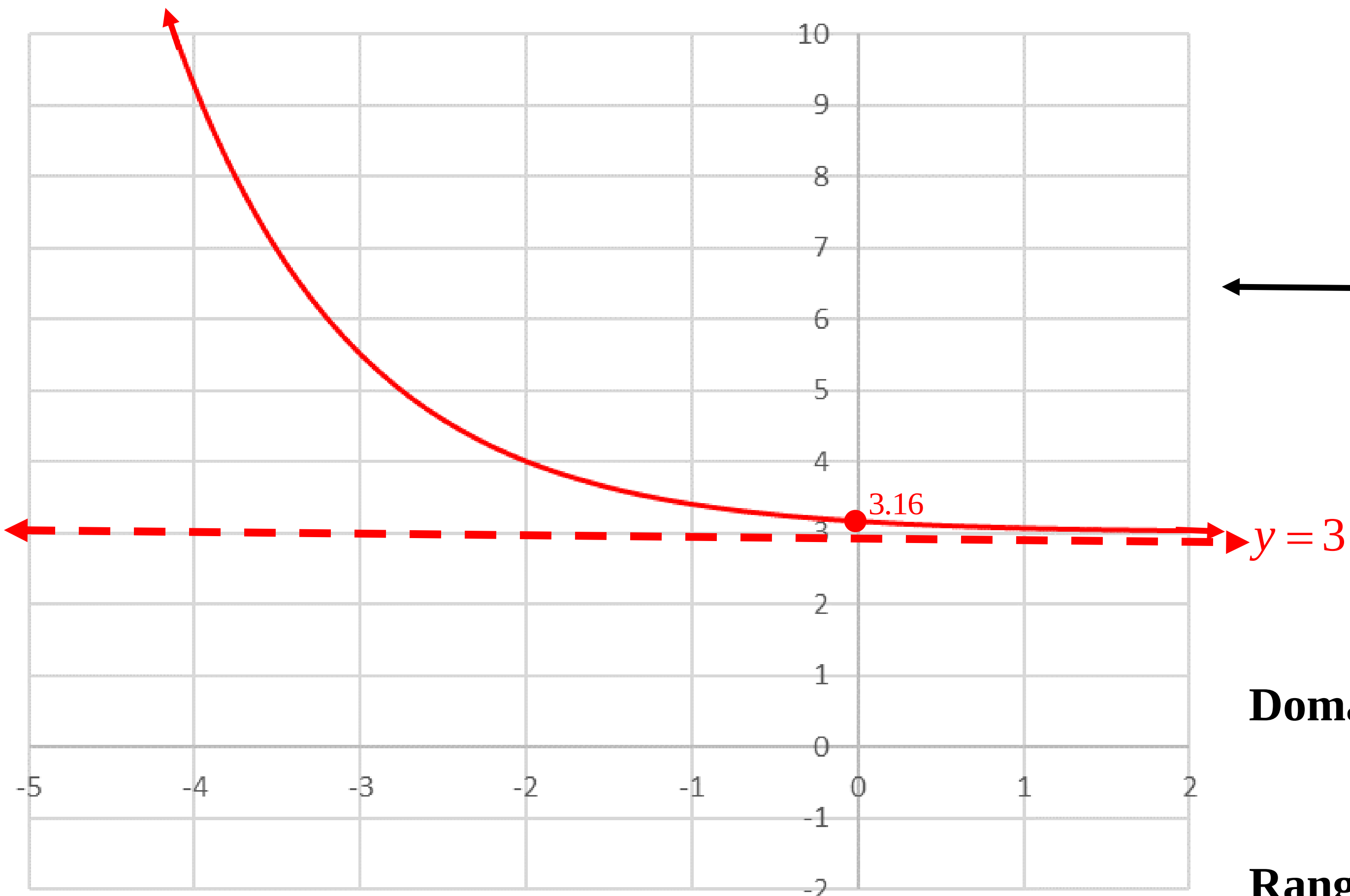
$5^x - 5 = 0 \Rightarrow 5^x = 5 \Rightarrow x = 1$

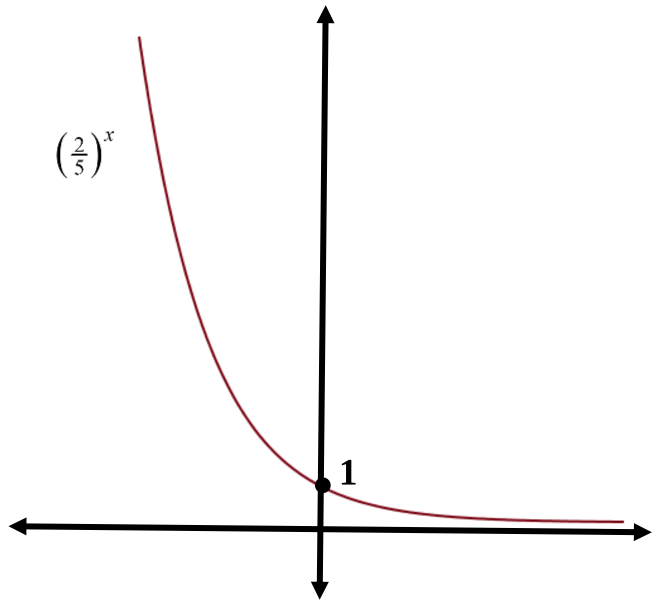
Domain: $(-\infty, \infty)$

Shift the original graph 5 units down.

Range: $(-5, \infty)$

4. $f(x)=\left(\frac{2}{5}\right)^{x+2}+3$





$f(0)=\left(\frac{2}{5}\right)^2+3=3\frac{4}{25}$

Domain: $(-\infty,\infty)$

Range: $(3,\infty)$

Shift the original graph 2 units to the left and 3 units up.

An Important Exponential Property:

If $b^x = b^y$**, then** $x = y$**.**

Solve the following basic exponential equations:

1. $5^x = 5^{-6}$

$$x = \boxed{-6}$$

2. $3^{-x} = 81$

$$3^{-x} = 3^4 \Rightarrow -x = 4 \Rightarrow x = \boxed{-4}$$

3. $4^{x^2} = 2^x$

$$\left(2^2\right)^{x^2} = 2^x \Rightarrow 2^{2x^2} = 2^x \Rightarrow 2x^2 = x \Rightarrow 2x^2 - x = 0$$

$$\Rightarrow x(2x-1) = 0 \Rightarrow x = \boxed{0, \frac{1}{2}}$$

4. $9^{-x+15} = 27^x$

$$\left(3^2\right)^{-x+15} = \left(3^3\right)^x \Rightarrow 3^{-2x+30} = 3^{3x} \Rightarrow -2x+30 = 3x$$

$$\Rightarrow 5x = 30 \Rightarrow x = \boxed{6}$$

5. $5^{x^2+8} = 125^{2x}$

$$5^{x^2+8} = \left(5^3\right)^{2x} \Rightarrow 5^{x^2+8} = 5^{6x} \Rightarrow x^2 + 8 = 6x$$

$$\Rightarrow x^2 - 6x + 8 = 0 \Rightarrow (x-2)(x-4) = 0 \Rightarrow x = \boxed{2,4}$$

6. If $5^{-x}=3$, then what's the value of 5^{3x}?

$$5^{-x}=3 \Rightarrow \left(5^{-x}\right)^{-3}=3^{-3} \Rightarrow 5^{3x}=\boxed{\frac{1}{27}}$$

7. If $2^{x}=3$, then what's the value of 4^{-x}?

$$2^{x}=3 \Rightarrow \left(2^{x}\right)^{-2}=3^{-2} \Rightarrow 4^{-x}=\boxed{\frac{1}{9}}$$

Review of Logarithmic Functions:

A function of the form $f(x) = log_b x$ **with** $b > 0$ **and** $b \neq 1$ **is a called a logarithmic function with base *b*. It is the inverse of the exponential function** b^x**, so we can get the graph of a logarithmic function by reflecting the graph of the corresponding exponential function about the line** $y = x$**.**

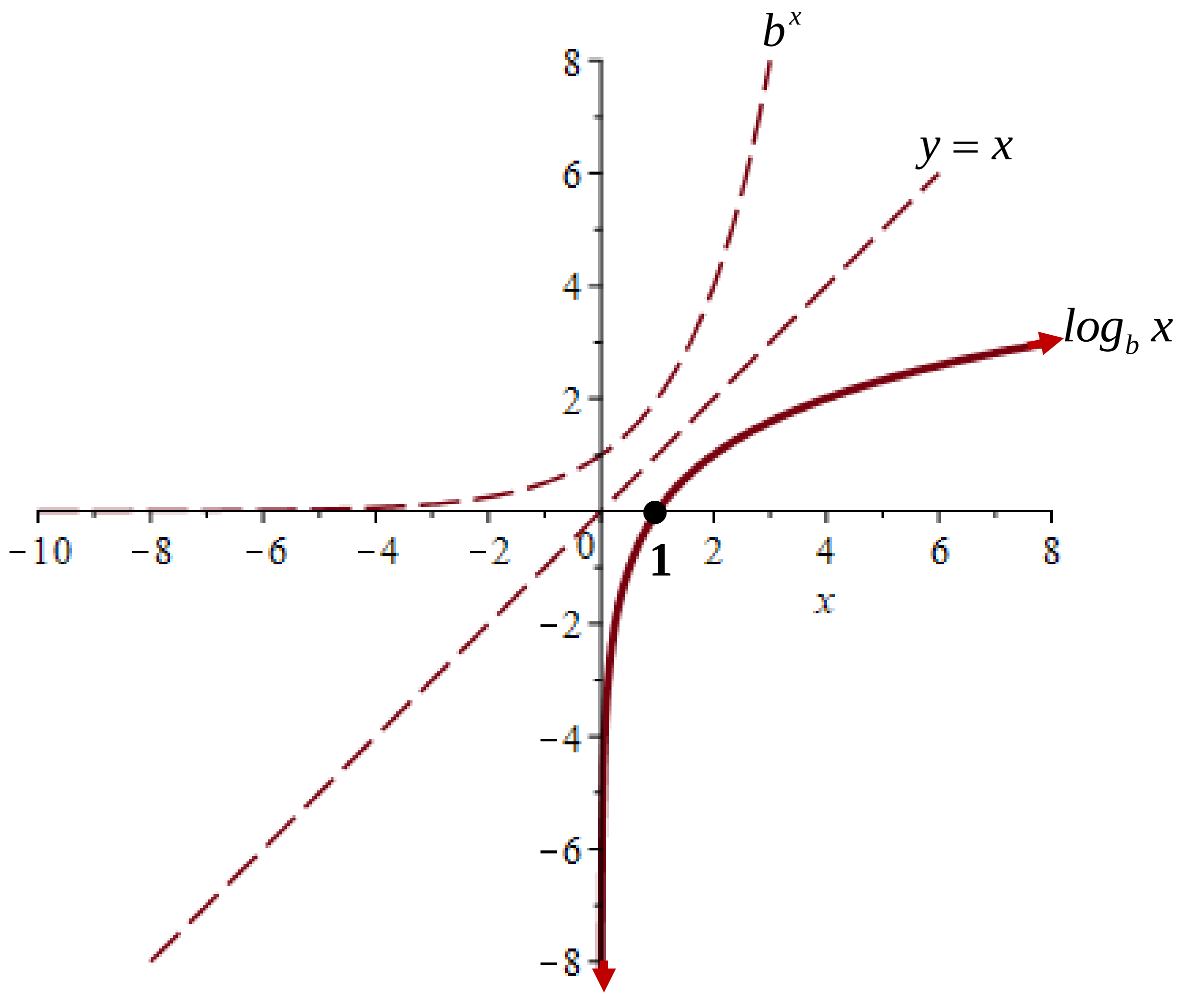
Just like the exponential functions, the bases separate into two categories:

$$b > 1$$

And

$$0 < b < 1$$

For $b > 1$**,**



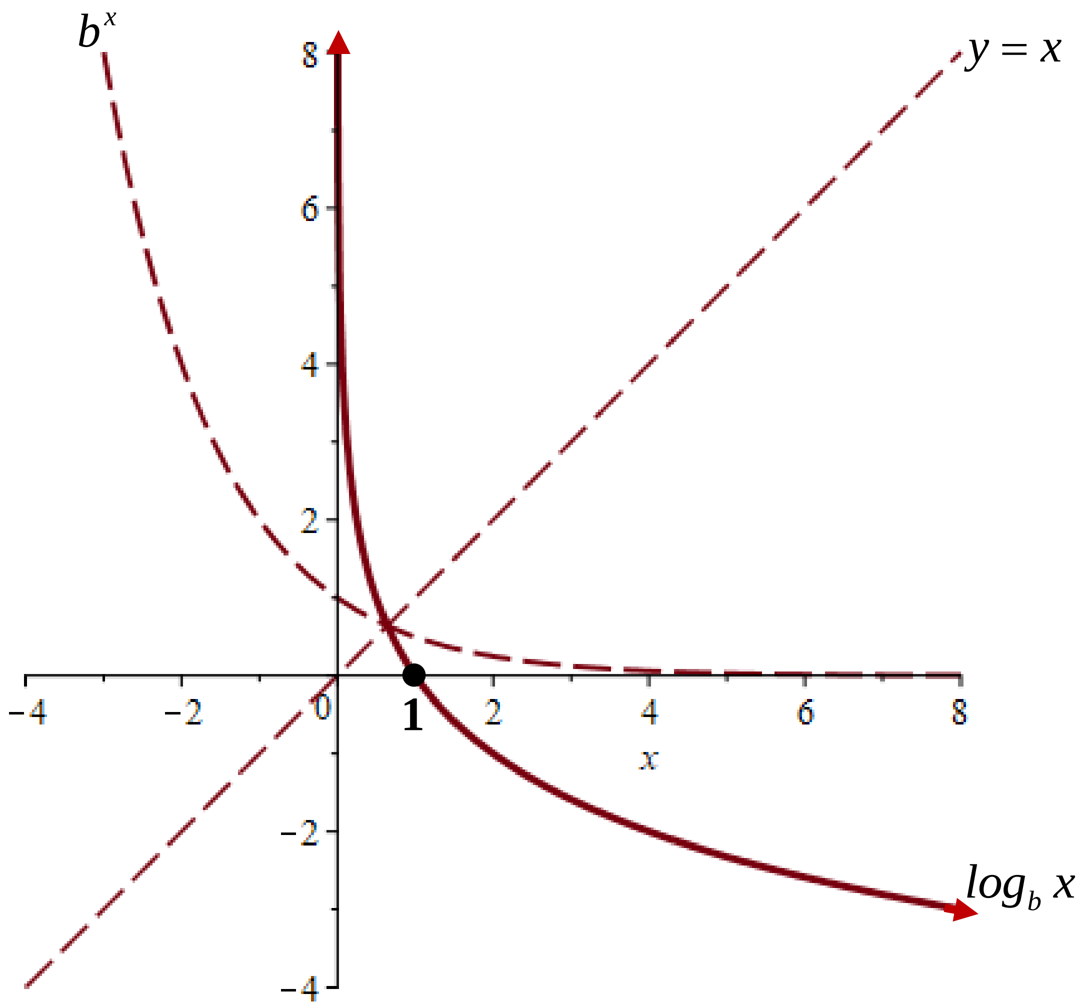
Domain: $(0,\infty)$

Range: $(-\infty,\infty)$

Vertical Asymptote: $x = 0$ **from the right**

Increasing: $(0,\infty)$

For $0 < b < 1$**,**



Domain: $(0,\infty)$

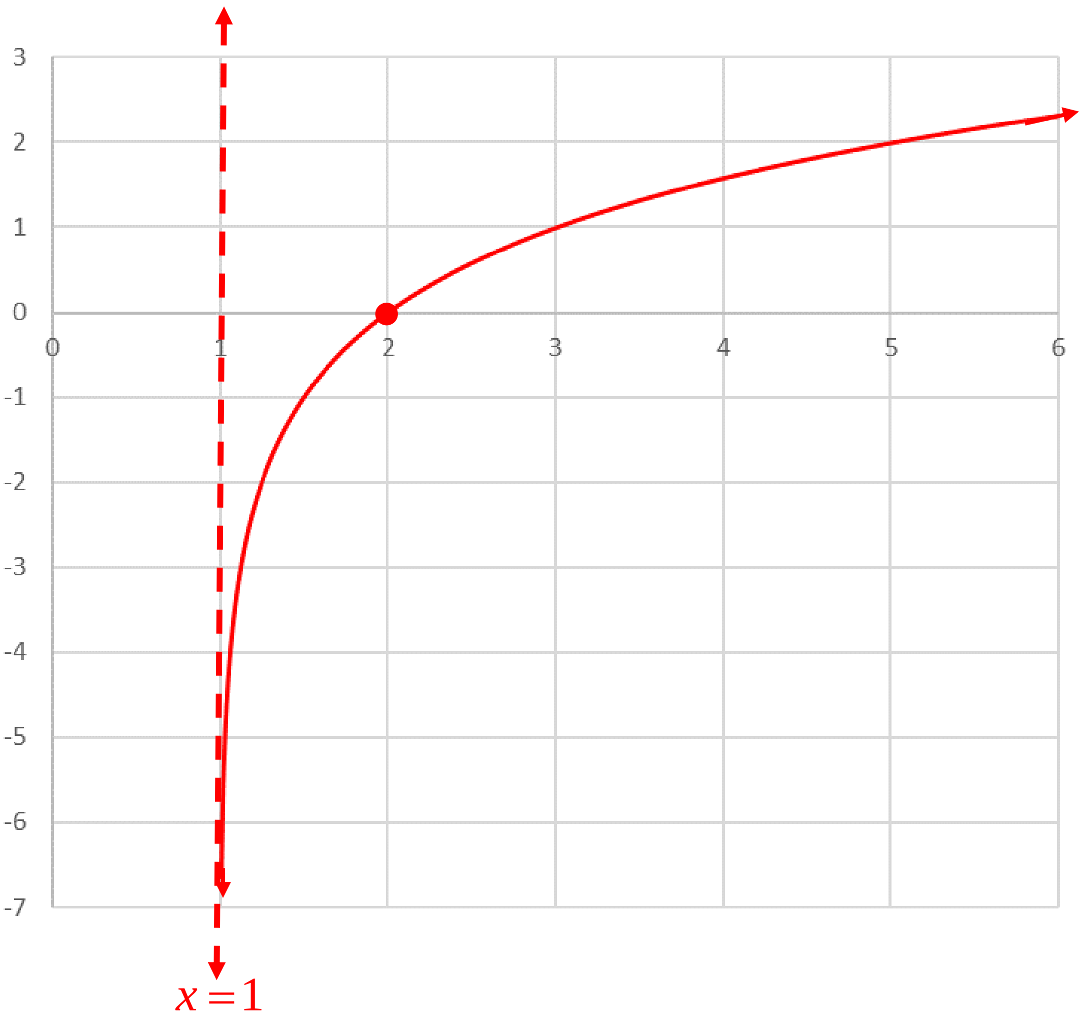
Range: $(-\infty,\infty)$

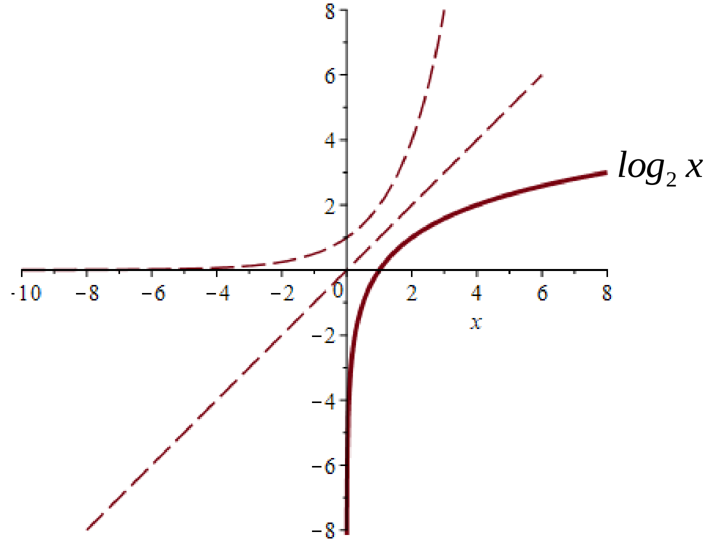
Vertical Asymptote: $x = 0$ **from the right**

Decreasing: $(0,\infty)$

Transformations of Logarithmic Functions:

1. $f\left(x\right)=log_2\left(x-1\right)$





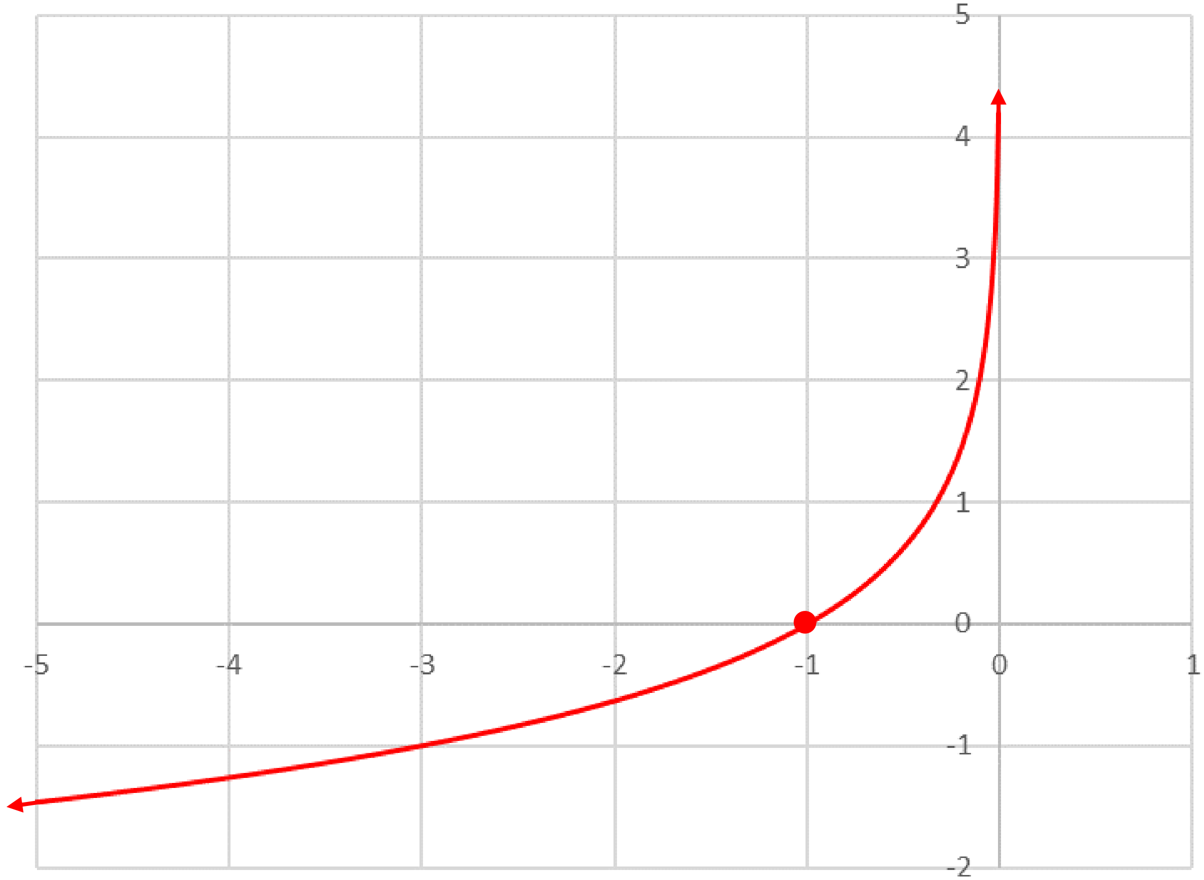
$\log_2\left(x-1\right)=0 \Rightarrow x-1=1 \Rightarrow x=2$

Domain: $\left(1,\infty\right)$

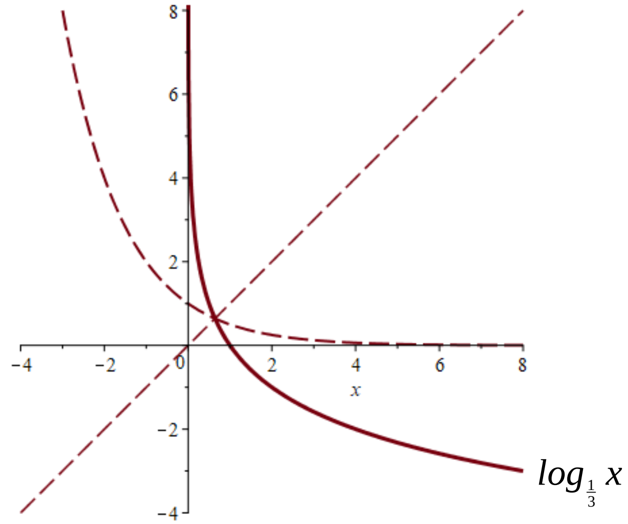
Range: $\left(-\infty,\infty\right)$

Shift the original graph 1 unit to the right.

2. $f(x) = log_{\frac{1}{3}}(-x)$



Reflect the original graph across the *y*-axis.

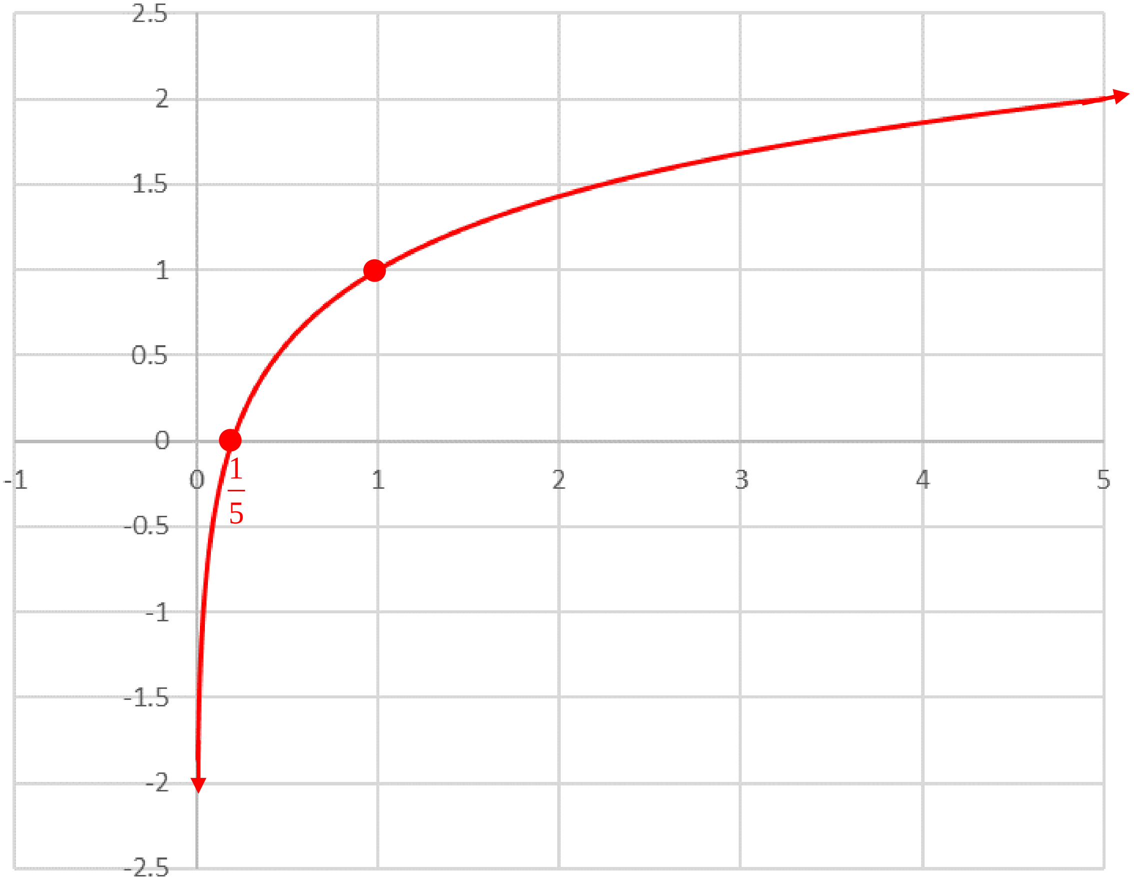


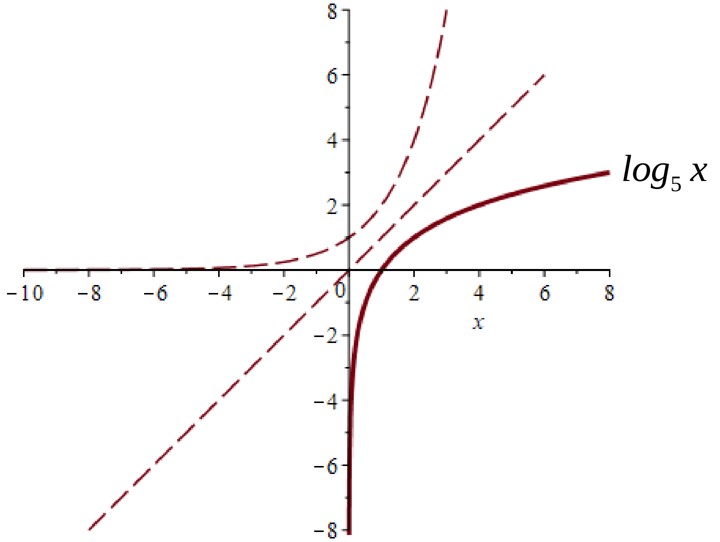
$\log_{\frac{1}{3}}(-x) = 0 \Rightarrow -x = 1 \Rightarrow x = -1$

Domain: $(-\infty, 0)$

Range: $(-\infty, \infty)$

3. $f(x) = log_5 x + 1$





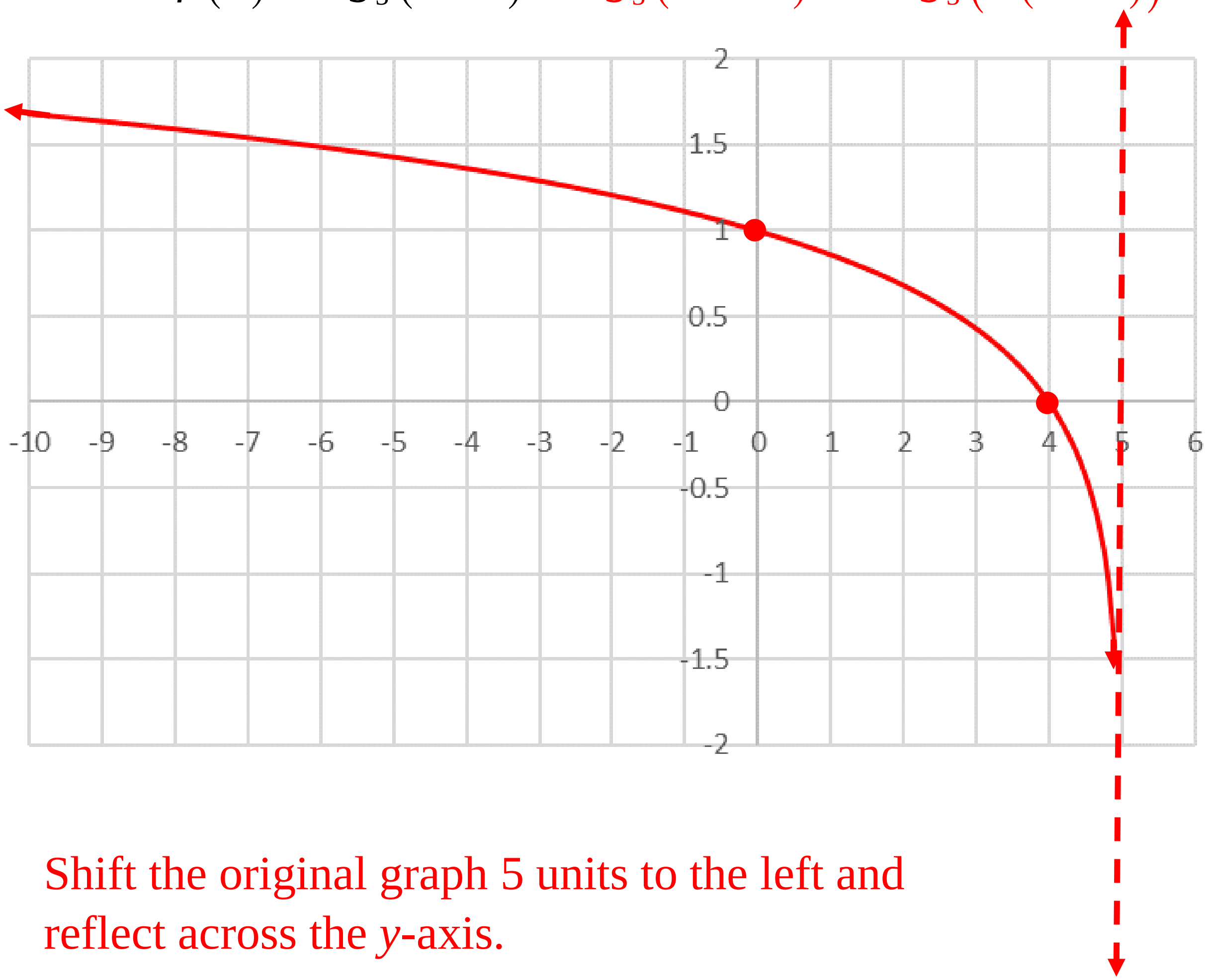
$\log_5 x + 1 = 0 \Rightarrow \log_5 x = -1 \Rightarrow x = -\frac{1}{5}$

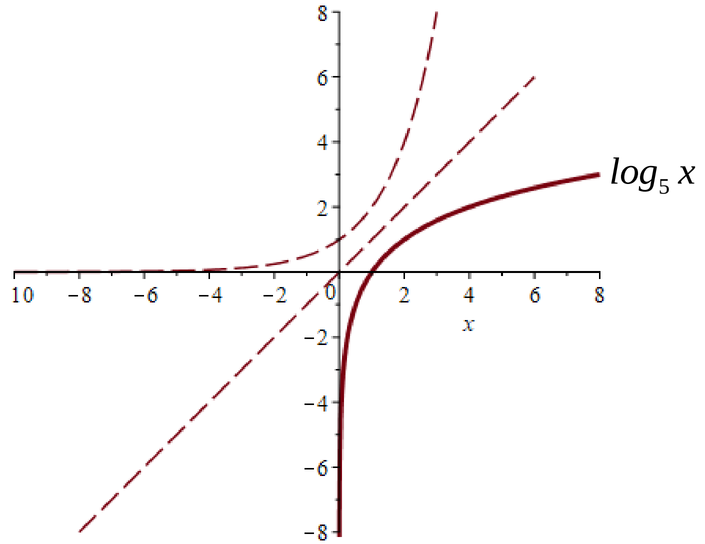
Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

Shift the original graph 1 unit up.

4. $f(x) = log_5(5-x) = log_5(-x+5)$ or $log_5(-(x-5))$





$f(0) = \log_5 5 = 1,$

$\log_5(5-x) = 0 \Rightarrow 5-x = 1 \Rightarrow x = 4$

Domain: $(-\infty, 5)$

Range: $(-\infty, \infty)$

Shift the original graph 5 units to the left and reflect across the *y*-axis.

Or

Reflect the original graph across the *y*-axis and shift 5 units to the right.

Logarithms are actually exponents. $log_b x$ **is the power or exponent that you raise *b* to in order to get *x*.**

1. $log_8 8$

1

2. $log_3 9$

2

3. $log_2 \frac{1}{4}$

-2

4. $log_{\frac{1}{2}} 8$

-3

5. $log_6 \sqrt{6}$

$\sqrt{6} = 6^{\frac{1}{2}} \Rightarrow \boxed{\frac{1}{2}}$

6. $log_5 \sqrt[3]{25}$

$\sqrt[3]{25} = 5^{\frac{2}{3}} \Rightarrow \boxed{\frac{2}{3}}$

7. $log_{\sqrt{3}} 9$

4

Important Logarithmic Properties:

$$\log_b\left(b^x\right) = x; \text{for all } x$$

$$b^{\log_b x} = x; \text{for } x > 0$$

Solve the following basic logarithmic equations:

1. $\log_5 x = 2$

$$\Rightarrow 5^{\log_5 x} = 5^2 \Rightarrow x = \boxed{25}$$

2. $\log_3\left(3x-2\right) = 3$

$$\Rightarrow 3^{\log_3(3x-2)} = 3^3 \Rightarrow 3x - 2 = 27$$

$$\Rightarrow 3x = 29 \Rightarrow x = \boxed{\frac{29}{3}}$$

3. $log_x 4 = 2$

$\Rightarrow x^{log_x 4} = x^2 \Rightarrow x^2 = 4$

$\Rightarrow x = \pm 2 \Rightarrow x = \boxed{2}$

The base must be positive.

4. $log_4 64 = x$

$log_4 64 = x \Rightarrow x = \boxed{3}$

5. $log_3\left(x^2+1\right) = 2$

$\Rightarrow 3^{log_3\left(x^2+1\right)} = 3^2 \Rightarrow x^2 + 1 = 9$

$\Rightarrow x^2 = 8 \Rightarrow x = \boxed{\pm\sqrt{8}}$

6. $log_5\left(x^2+4x+4\right) = 2$

$\Rightarrow 5^{log_5\left(x^2+4x+4\right)} = 5^2 \Rightarrow x^2 + 4x + 4 = 25$

$\Rightarrow x^2 + 4x - 21 = 0 \Rightarrow (x+7)(x-3) = 0$

$\Rightarrow x = \boxed{-7,3}$