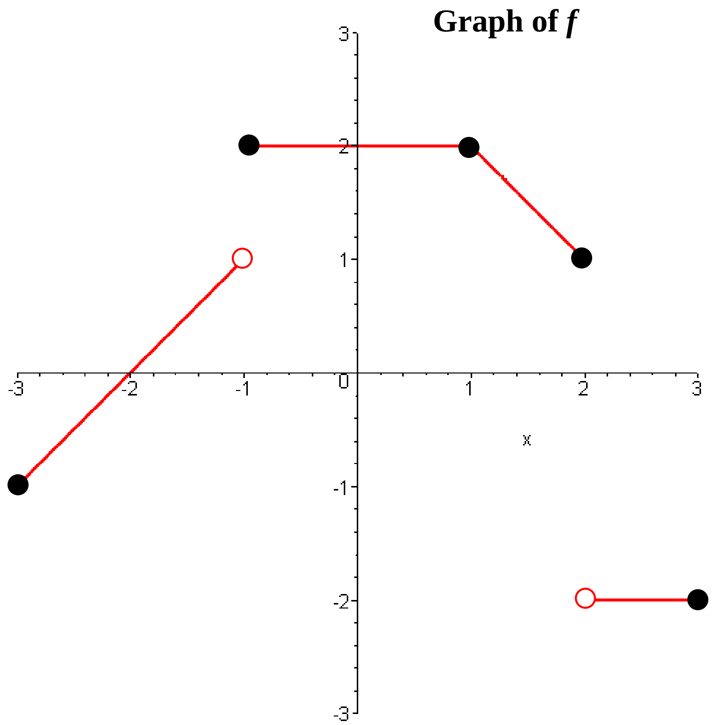
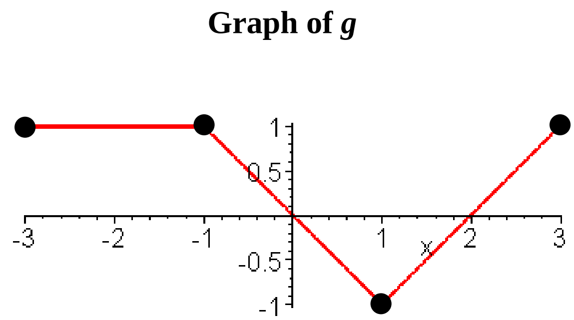
Math 2412 Review 1

1. Use the graphs of the functions *f* and *g* to determine the following:





a) $f(0)$ $\boxed{2}$

b) $g\left(\frac{1}{2}\right)$ $\boxed{-\frac{1}{2}}$

c) $f(-1)$ $\boxed{2}$

d) $(f+g)(-2)$ $\boxed{1}$

e) $(f-g)\left(\frac{3}{2}\right)$ $\boxed{2}$

f) $(f\cdot g)(-3)$ $\boxed{-1}$

g) $\left(\dfrac{f}{g}\right)(2)$ $\boxed{\text{undefined}}$

h) $(f\circ g)(1)$ $\boxed{2}$

i) $(f\circ f)(1)$ $\boxed{1}$

j) $(g\circ g)\left(\frac{1}{2}\right)$ $\boxed{\frac{1}{2}}$

k) $(f\circ g\circ f)(1)$ $\boxed{2}$

l) $(g\circ f\circ g)(1)$ $\boxed{0}$

m) Where is *f* increasing? $\boxed{[-3,-1]}$

n) Where is *f* decreasing? $\boxed{[1,2]}$

o) Where is *f* constant? $\boxed{[-1,1],(2,3]}$

p) Does *f* have a local maximum at $x=-1$? $\boxed{\text{yes}}$

q) Does *f* have a local minimum at $x=2$? $\boxed{\text{no}}$

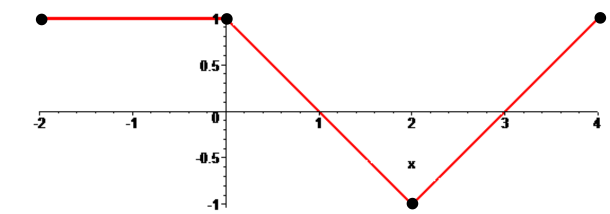
r) Solve $-1\le g(x)<0$. $\boxed{(0,2)}$

s) Solve $0<f(x)\le 1$. $\boxed{(-2,-1),\{2\}}$

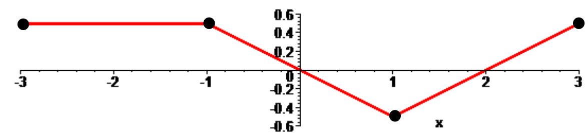
t) Solve $g(x)=\frac{1}{2}$. $\boxed{-\frac{1}{2},\frac{5}{2}}$

u) Solve $f(x)=-\frac{3}{2}$. $\boxed{\text{no solution}}$

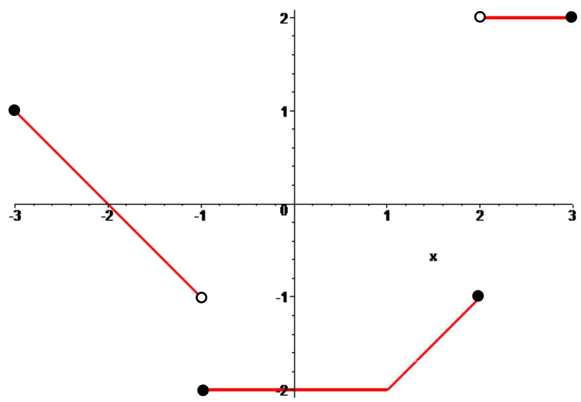
v) Graph $h(x) = g(x-1)$.



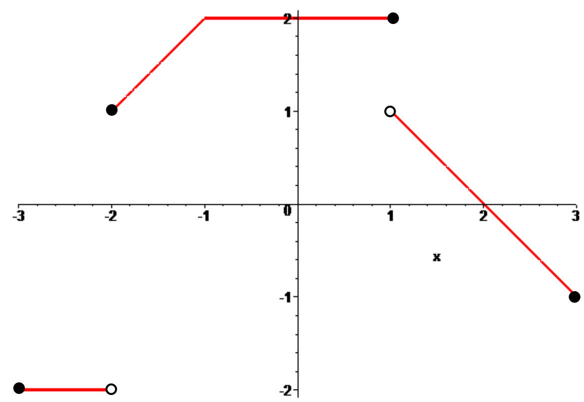
w) Graph $h(x) = \frac{1}{2}g(x)$.



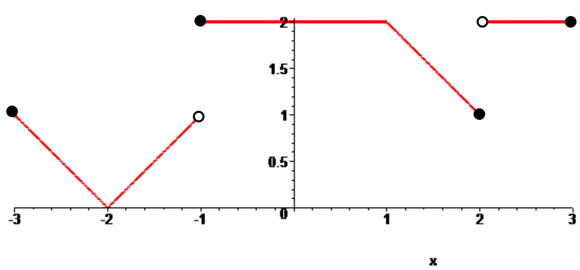
x) Graph $h(x) = -f(x)$.



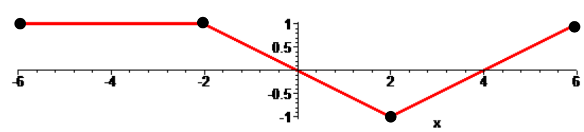
y) Graph $h(x) = f(-x)$.



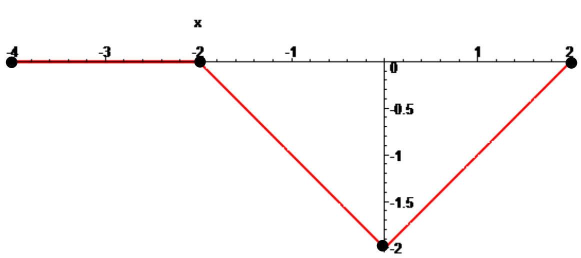
z) Graph $h(x) = |f(x)|$.



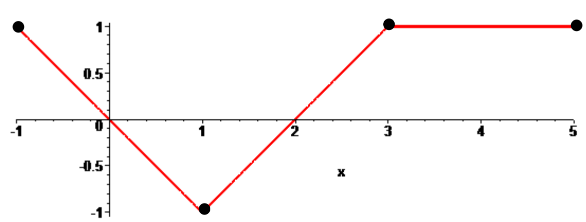
aa) Graph $h(x) = g\left(\frac{1}{2}x\right)$.



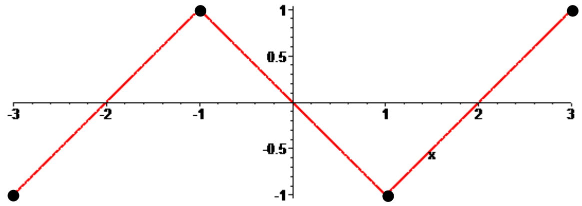
bb) Graph $h(x) = g(x+1) - 1$.



cc) Graph $h(x) = g(2-x)$.



dd) Graph $h(x)=\begin{cases} g(x); x \geq -1 \\ f(x); x < -1 \end{cases}$.



2. Determine the domains of the following functions.

a) $f(x)=\dfrac{1}{x^3+9x}$

$x^3+9x=0$

$x(x^2+9)=0 \Rightarrow x=0$

$\boxed{(-\infty,0),(0,\infty)}$

b) $g(x)=\dfrac{2x-9}{\sqrt{x-1}}+\dfrac{1}{x-3}$

$x-1 \geq 0 \Rightarrow x \geq 1$

$x \neq 1, x \neq 3$

$\boxed{(1,3),(3,\infty)}$

c) $h(x)=\dfrac{\sqrt[4]{5-x}}{\sqrt{x-1}}$

$5-x \geq 0 \Rightarrow x \leq 5$

$x-1 \geq 0 \Rightarrow x \geq 1, x \neq 1$

$\boxed{(1,5]}$

3. Determine if the following functions are even, odd, neither, or both. ***Justify your answers.***

a) $f(x)=x^5-x$

$f(-x)=(-x)^5-(-x)$

$=-x^5+x$

$=-(x^5-x)=-f(x)$

$\boxed{\text{odd}}$

b) $g(x)=\dfrac{x^5+x^3}{x^3-x}$

$g(-x)=\dfrac{(-x)^5+(-x)^3}{(-x)^3-(-x)}$

$=\dfrac{-x^5-x^3}{-x^3+x}$

$=\dfrac{x^5+x^3}{x^3-x}=g(x)$

$\boxed{\text{even}}$

c) $h(x) = x^2 - (-x)^2$

$h(x) = x^2 - (-x)^2$
$= x^2 - x^2 = 0$

$h(-x) = 0 = h(x)$
$= -0 = -h(x)$

both

d) $j(x) = \begin{cases} x-1 & ;1 \le x \le 2 \\ x+1 & ;-2 \le x \le -1 \end{cases}$

$$j(-x) = \begin{cases} (-x)-1 & ;1 \le (-x) \le 2 \\ (-x)+1 & ;-2 \le (-x) \le -1 \end{cases}$$
$$= \begin{cases} -(x+1) & ;-2 \le x \le -1 \\ -(x-1) & ;1 \le x \le 2 \end{cases}$$
$$= \begin{cases} -(x-1) & ;1 \le x \le 2 \\ -(x+1) & ;-2 \le x \le -1 \end{cases} = -j(x)$$

odd

4. Use the piecewise-defined function $f(x) = \begin{cases} -(x+3)^2 & ;-3 \le x < -1 \\ x-1 & ;-1 \le x < 1 \\ 3-x & ;1 \le x \le 3 \end{cases}$ to determine the following:

a) $f(-4)$

undefined

b) $f(-2)$

-1

c) $f(-1)$

-2

d) $f\left(\frac{3}{2}\right)$

$\frac{3}{2}$

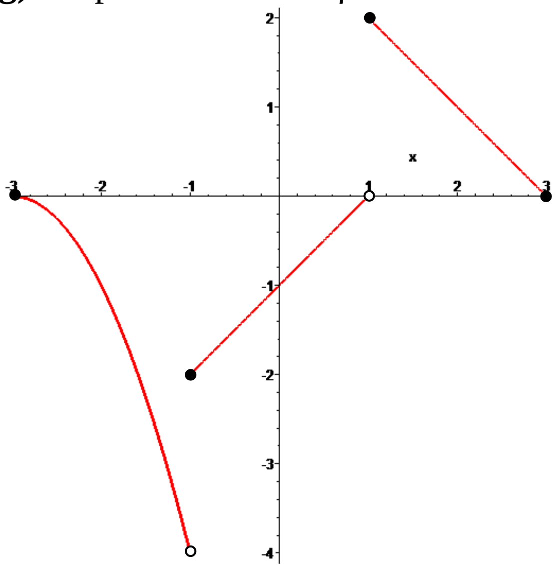
e) $f(4)$

undefined

f) $(f \circ f)\left(\frac{5}{2}\right)$

$-\frac{1}{2}$

g) Graph the function f.



h) Determine the range of f.

$(-4, 2]$

i) What is the absolute maximum value of f?

2

j) What is the absolute minimum value of f?

Does not exist.

k) Where is f increasing?

$[-1, 1]$

l) Where is f decreasing?

$[-3, -1), [1, 3]$

m) Where does f have local maxima?

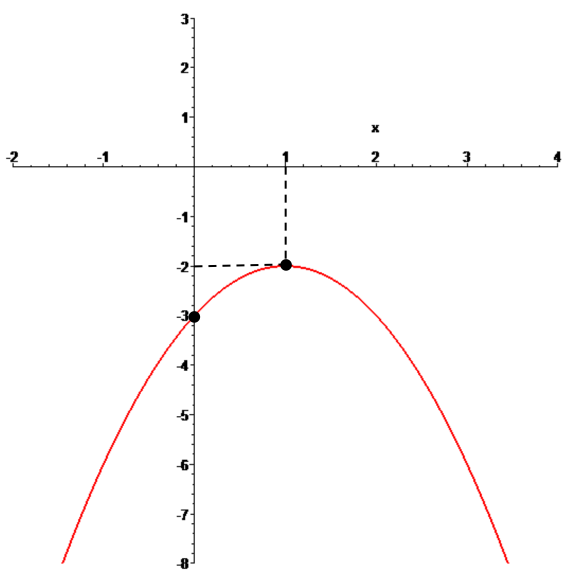
$x = 1$

n) Where does f have local minima?

nowhere

Sketch the graphs of the following quadratic functions. Indicate the vertex and intercepts.

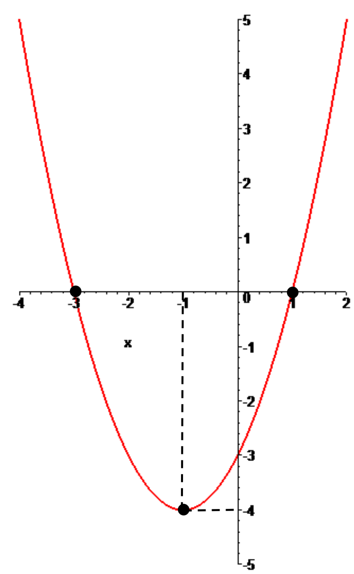
5. $f(x) = -(x-1)^2 - 2$



$-(0-1)^2 - 2 = -3$

$\Rightarrow$ y-int: -3

6. $f(x) = x^2 + 2x - 3$



$x^2 + 2x - 3 = (x+3)(x-1) \Rightarrow$ x-int: $-3, 1$

$\frac{-3+1}{2} = -1 \Rightarrow (-1, -4)$ is the vertex

$0^2 + 2 \cdot 0 - 3 = -3 \Rightarrow$ y-int: -3

Find a formula for the quadratic function whose graph satisfies the given conditions.

7. Its vertex is at $(-2,4)$ and the graph contains the point $(1,-5)$.

$$f(x)=a(x+2)^2+4$$

$$a(1+2)^2+4=-5 \Rightarrow 9a=-9 \Rightarrow a=-1$$

$$\boxed{f(x)=-(x+2)^2+4}$$

8. Its graph has x-intercepts of 1 and -2, and it contains the point $(4,9)$.

$$f(x)=a(x-1)(x+2)$$

$$a(4-1)(4+2)=9 \Rightarrow 18a=9 \Rightarrow a=\frac{1}{2}$$

$$\boxed{f(x)=\tfrac{1}{2}(x-1)(x+2)}$$

9. Its graph contains the points $(1,-1)$, $(0,1)$, and $(2,2)$.

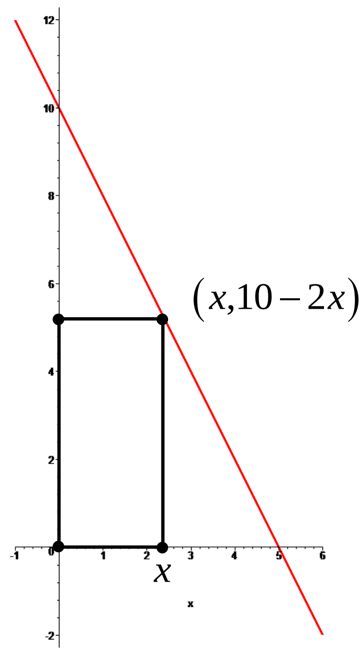
$$f(x)=ax^2+bx+c$$

$$a+b+c=-1, \underline{c=1}, 4a+2b+c=2$$

$$\Rightarrow \underline{a=\tfrac{5}{2}}, \underline{b=-\tfrac{9}{2}}$$

$$\boxed{f(x)=\tfrac{5}{2}x^2-\tfrac{9}{2}x+1}$$

10. A rectangle in the first quadrant has one vertex on the line $y=10-2x$, another at the origin, one on the positive x-axis, and one on the positive y-axis. *(See the figure.)*



a) Express the area A of the rectangle as a function of x.

$$\boxed{A(x)=x(10-2x)}$$

b) What's the domain of $A(x)$?

$$\boxed{(0,5)}$$

c) What value of x produces the maximum area?

$$A(x)=x(10-2x)=-2x^2+10x=-2(x^2-5x)$$

$$=-2\left(x^2-5x+\tfrac{25}{4}\right)+\tfrac{25}{2}=-2\left(x-\tfrac{5}{2}\right)^2+\tfrac{25}{2}$$

So the maximum area occurs when $x=\boxed{\frac{5}{2}\textit{units}}$.

d) What is the maximum area?

The maximum area is $\boxed{\frac{25}{2}\textit{square units}}$.