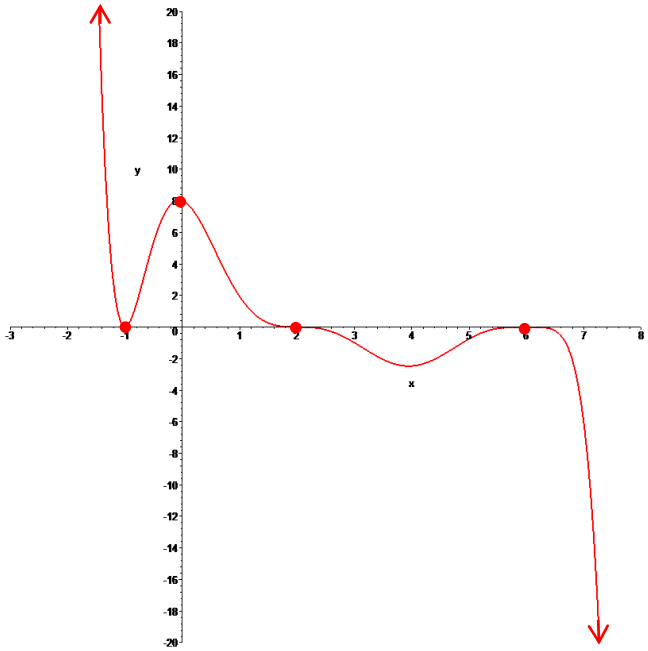
Math 2412 Review 2(answers)

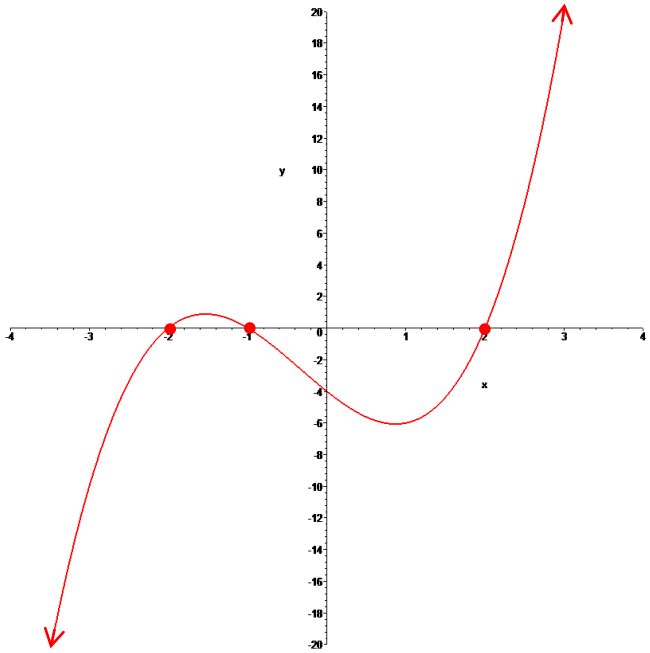
Sketch the graphs of the following polynomial functions.(1-4) ***Label the zeros and y-intercept.***

1. $f(x)=-\frac{1}{1296}(x+1)^2(x-2)^3(x-6)^4$



$-\frac{1}{1296}(0+1)^2(0-2)^3(0-6)^4=8$

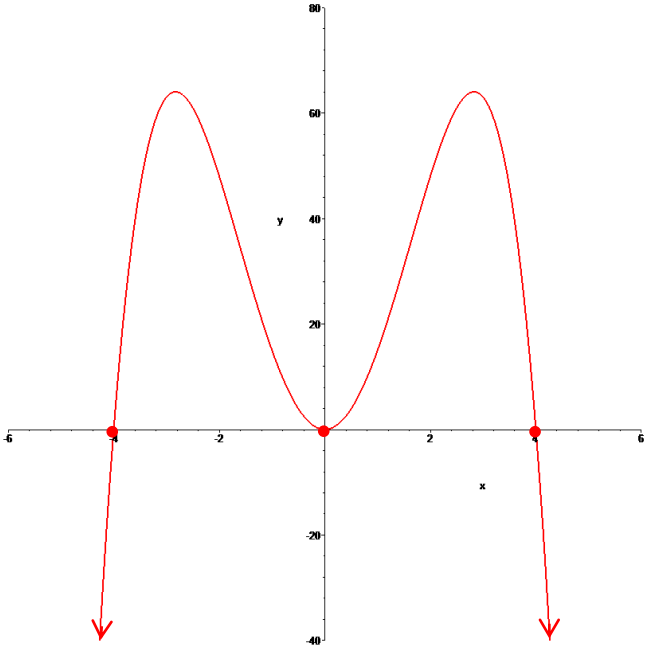
2. $f(x)=x^3+x^2-4x-4$



$x^2(x+1)-4(x+1)=(x+1)(x-2)(x+2)$

$0^3+0^2-4\cdot 0-4=4$

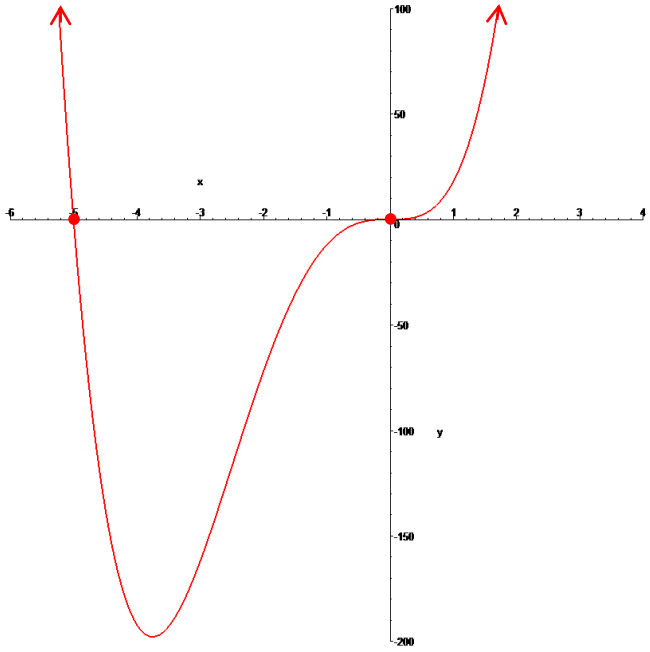
3. $f(x)=-x^4+16x^2$



$-x^2(x^2-16)=-x^2(x-4)(x+4)$

$-0^4+16\cdot 0^2=0$

4. $f(x)=3x^4+15x^3$



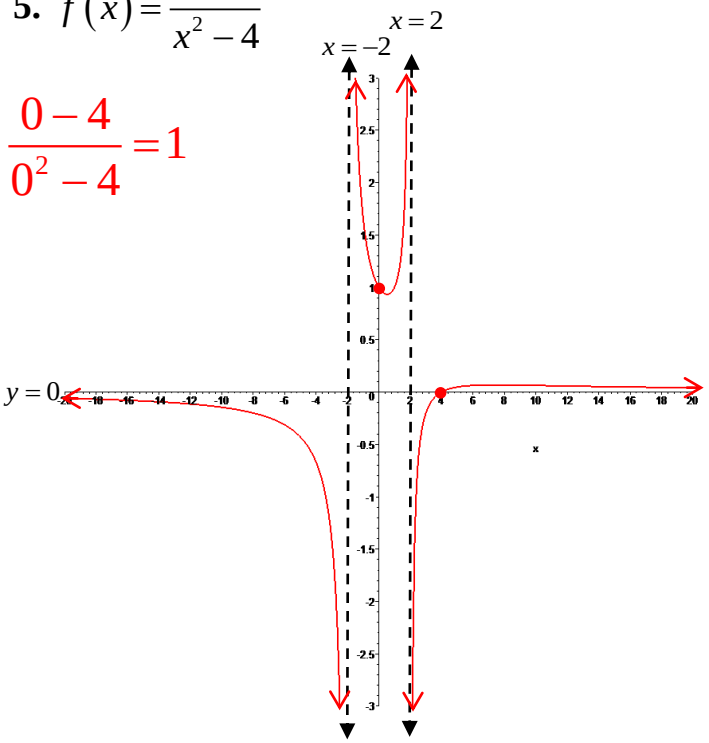
$3x^3(x+5)$

$3\cdot 0^4+15\cdot 0^3=0$

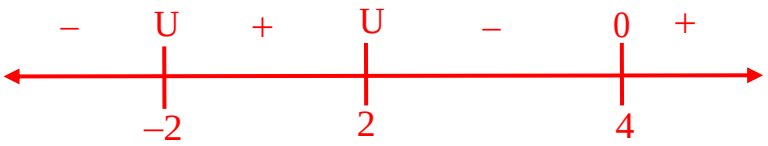
Complete the graphs of the following rational functions.(5-8) ***Label asymptotes and intercepts.***

5. $f(x)=\dfrac{x-4}{x^2-4}$

$\dfrac{0-4}{0^2-4}=1$

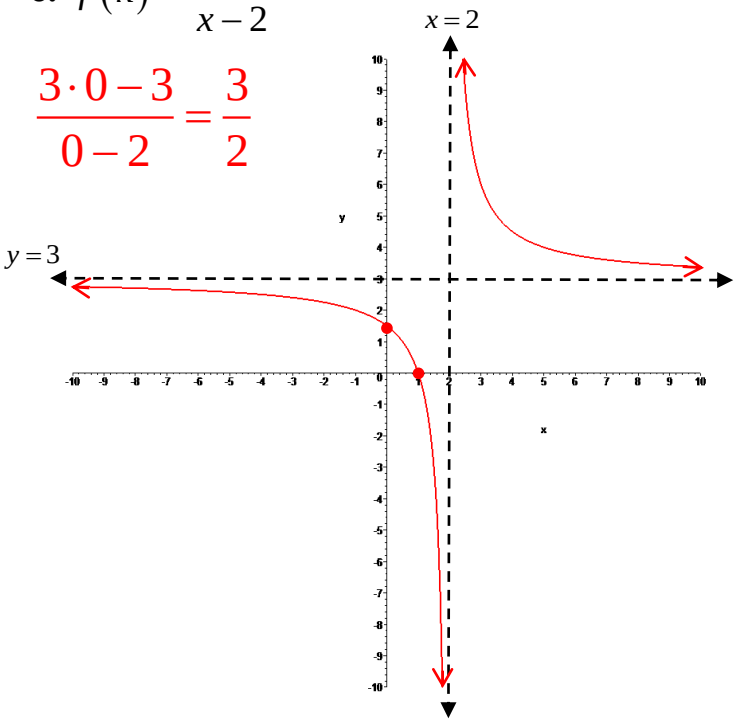


$\dfrac{x-4}{(x-2)(x+2)}$

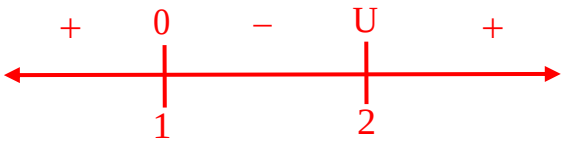


6. $f(x)=\dfrac{3x-3}{x-2}$

$\dfrac{3\cdot 0-3}{0-2}=\dfrac{3}{2}$

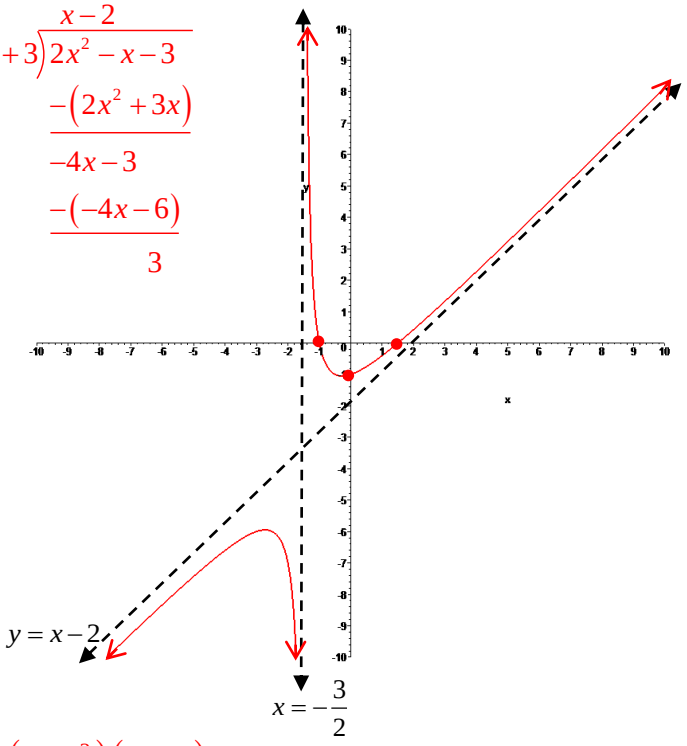


$\dfrac{3(x-1)}{x-2}$

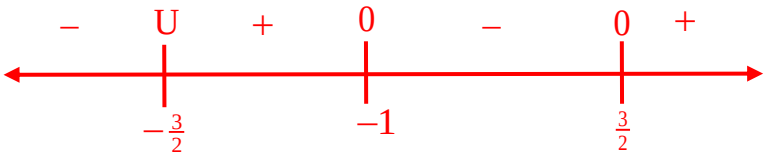


7. $f(x)=\dfrac{2x^2-x-3}{2x+3}$

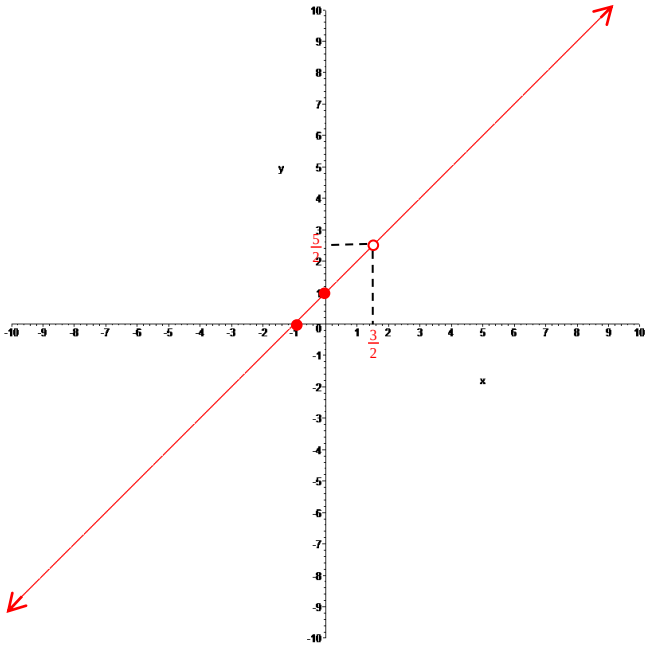
$$\begin{array}{r} x-2 \\ 2x+3\overline{)\,2x^2-x-3} \\ -(2x^2+3x) \\ \hline -4x-3 \\ -(-4x-6) \\ \hline 3 \end{array}$$



$$\frac{\left(x-\frac{3}{2}\right)(x+1)}{\left(x+\frac{3}{2}\right)}$$

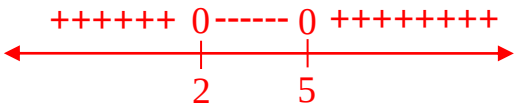


8. $f(x)=\dfrac{2x^2-x-3}{2x-3}=\dfrac{(2x-3)(x+1)}{2x-3}=x+1;x\neq\dfrac{3}{2}$



Solve the following inequalities by completing the sign charts, and expressing the solution in interval notation.(9-12)

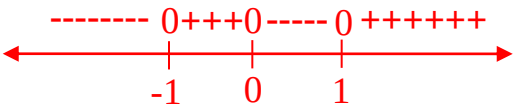
9. $(x-2)(x-5)>0$



$$\boxed{(-\infty,2),(5,\infty)}$$

10. $x^3\le x$

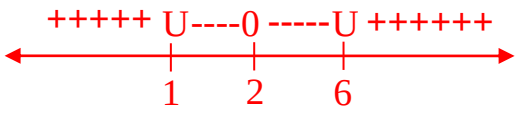
$$x(x-1)(x+1)\le 0$$



$$\boxed{(-\infty,-1],[0,1]}$$

11. $\dfrac{x^2-4x+4}{x^2-7x+6}<0$

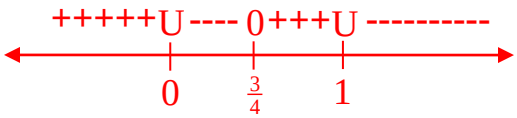
$$\frac{(x-2)^2}{(x-1)(x-6)}<0$$



$\boxed{(1,2),(2,6)}$

12. $\dfrac{1}{1-x}\geq\dfrac{3}{x}\Rightarrow\dfrac{x-3(1-x)}{x(1-x)}\geq 0\Rightarrow\dfrac{4x-3}{x(1-x)}\geq 0$

$$\frac{4\left(x-\frac{3}{4}\right)}{x(1-x)}\geq 0$$



$\boxed{(-\infty,0),\left[\frac{3}{4},1\right)}$

13. Approximate a solution of the polynomial equation $\underbrace{x^4-3x^2-1}_{f(x)}=0$ using the Bisection Method by completing the following table:

Left Endpoint(sign)	Midpoint(sign)	Right Endpoint(sign)	Error Bound
1(-)	$\frac{3}{2}$(-)	2(+)	$\frac{1}{2}$
$\frac{3}{2}$(-)	$\frac{7}{4}$(-)	2(+)	$\frac{1}{4}$
$\boxed{\frac{7}{4}(-)}$	$\boxed{\frac{15}{8}(+)}$	$\boxed{2(+)}$	$\boxed{\frac{1}{8}}$
$\boxed{\frac{7}{4}(-)}$	$\boxed{\frac{29}{16}(-)}$	$\boxed{\frac{15}{8}(+)}$	$\boxed{\frac{1}{16}}$

Find partial fraction decompositions for the following.(14-16)

14.

$$\frac{3x-4}{(x-1)(x-2)}=\frac{A}{x-1}+\frac{B}{x-2}$$

$$\Rightarrow 3x-4=A(x-2)+B(x-1)$$

$$x=1\Rightarrow -1=-A\Rightarrow A=1$$

$$x=2\Rightarrow 2=B$$

$$\boxed{\frac{1}{x-1}+\frac{2}{x-2}}$$

15.

$$\frac{4x^2-11x+9}{(x-1)^2(x-2)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x-2}$$

$$\Rightarrow 4x^2-11x+9=A(x-1)(x-2)+B(x-2)+C(x-1)^2$$

$$x=1\Rightarrow 2=-B\Rightarrow B=-2$$

$$x=2\Rightarrow 3=C$$

$$x=0\Rightarrow 9=2A+4+3\Rightarrow A=1$$

$$\boxed{\frac{1}{x-1}-\frac{2}{(x-1)^2}+\frac{3}{x-2}}$$

16. $\dfrac{3x^2-x+1}{x\left(x^2+1\right)}=\dfrac{A}{x}+\dfrac{Bx+C}{x^2+1}$

$\Rightarrow 3x^2-x+1=A\left(x^2+1\right)+x\left(Bx+C\right)=\left(A+B\right)x^2+Cx+A$

$\Rightarrow A+B=3, C=-1, A=1\Rightarrow B=2$

$\boxed{\dfrac{1}{x}+\dfrac{2x-1}{x^2+1}}$

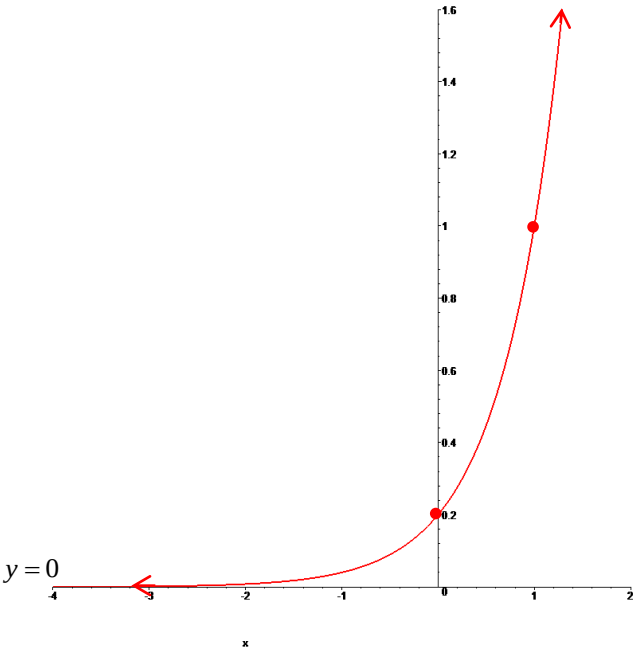
17. The first step in finding the partial fraction decomposition of $\frac{5x^2+5x+1}{x^2+x}$ would be to divide the numerator by the denominator. Perform this division.

$$x^2+x\overline{)5x^2+5x+1}\quad 5$$
$$-(5x^2+5x)$$
$$1$$

$$\frac{5x^2+5x+1}{x^2+x}=\boxed{5+\frac{1}{x^2+x}}$$

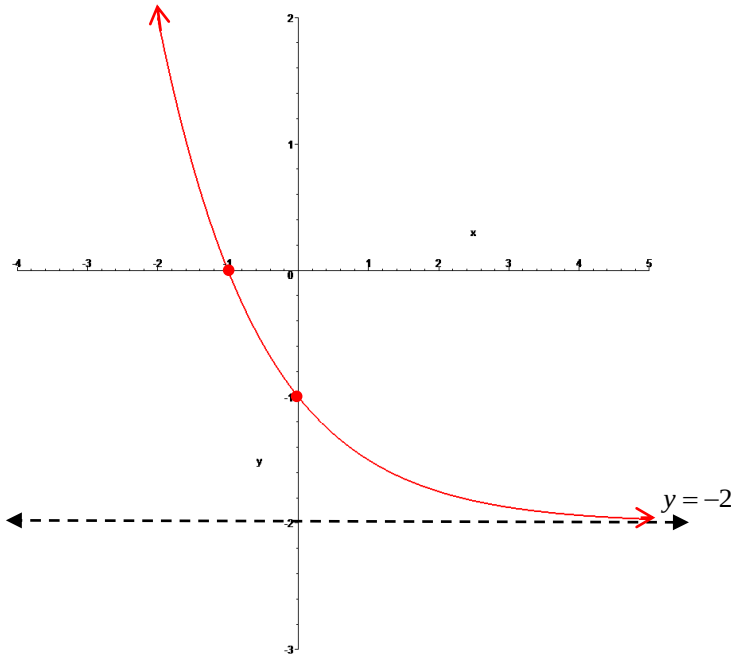
Sketch the graphs of the following exponential and logarithmic functions. Indicate the asymptotes and intercepts.(18-21)

18. $g(x)=5^{x-1}$



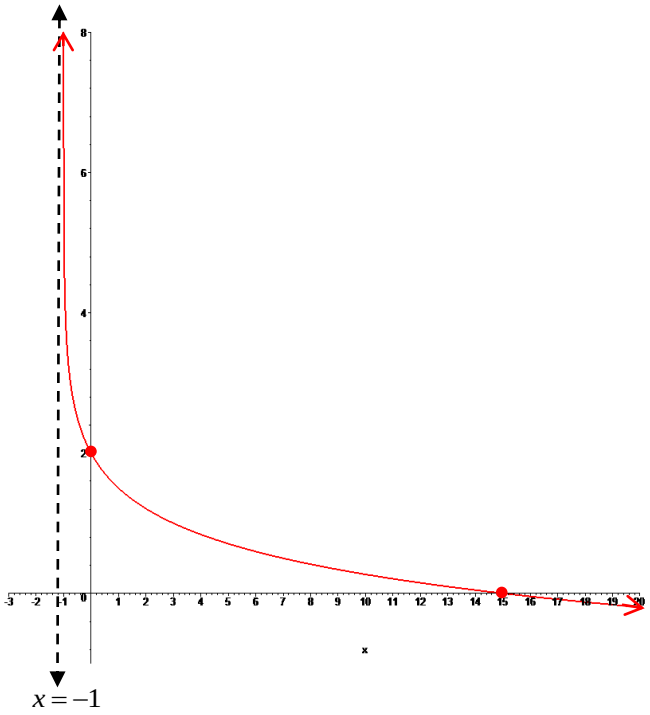
$5^{0-1}=\frac{1}{5}=.2$

19. $g(x)=\left(\frac{1}{2}\right)^x-2$



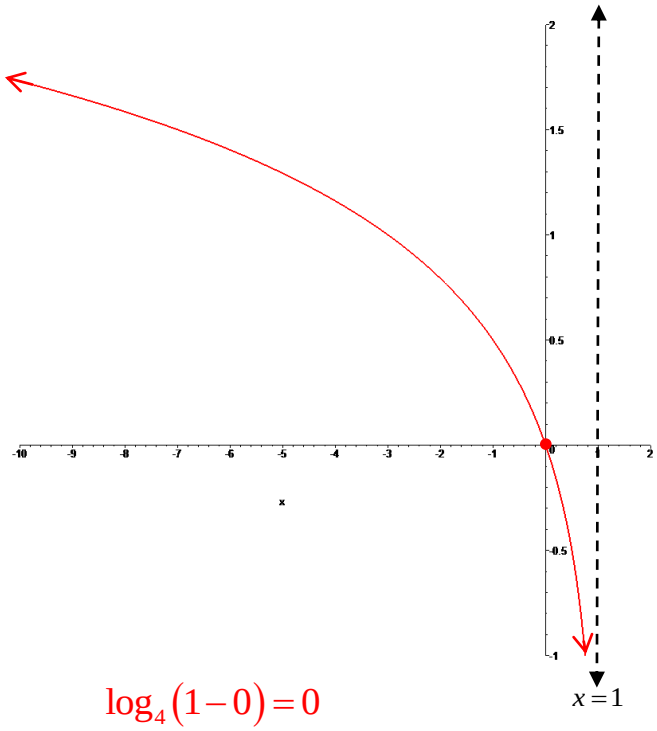
$\left(\frac{1}{2}\right)^0-2=-1,\left(\frac{1}{2}\right)^x-2=0\Rightarrow x=-1$

20. $g(x)=-\log_4(x+1)+2$



$-\log_4(0+1)+2=2$

21. $g(x)=\log_4(1-x)$



$\log_4(1-0)=0$

*Find the **<u>exact simplified value</u>** of the following expressions.(22-29)*

22. $\log_8 1$

$\boxed{0}$

23. $\log_2\left(16^{99}\right)$

$\log_2\left(\left(2^4\right)^{99}\right)=\boxed{396}$

24. $\log_{10}\left(\dfrac{10^{100}}{10^{106}}\right)$

$\log_{10}\left(10^{-6}\right)=\boxed{-6}$

25. $\log_3\left(\sqrt[70]{3}\right)$

$\boxed{\dfrac{1}{70}}$

26. $\log_{10}\left(\log_{10}\left(10^{100}\right)\right)$

$\log_{10}100=\boxed{2}$

27. $2^{\log_2\pi}$

$\boxed{\pi}$

28. $\log_8 3-\log_{8\frac{3}{16}}$

$\log_8\left(\dfrac{3}{\frac{3}{16}}\right)=\log_{2^3}\left(2^4\right)=\boxed{\dfrac{4}{3}}$

29. $\log_2\left(\dfrac{1}{2}\right)+\log_2\left(\dfrac{2}{3}\right)+\log_2\left(\dfrac{3}{4}\right)+\log_2\left(\dfrac{4}{5}\right)+\cdots+\log_2\left(\dfrac{126}{127}\right)+\log_2\left(\dfrac{127}{128}\right)$

$=\left(\log_2 1-\log_2 2\right)+\left(\log_2 2-\log_2 3\right)+\left(\log_2 3-\log_2 4\right)+\cdots+\left(\log_2 127-\log_2 128\right)$

$=\log_2 1-\log_2 128=\boxed{-7}$

Expand the following logarithmic expressions as much as possible. Simplify if possible.

30. $\log_2\left(xy^{10}\right)$

$\boxed{\log_2 x + 10\log_2 y}$

31. $\log_2\left(\dfrac{x^2}{8y^3}\right)$

$\boxed{2\log_2 x - 3 - 3\log_2 y}$

Compress the following logarithmic expressions into a single term. Simplify if possible.

32. $\log_{10} 6 + 4\log_{10} x$

$\boxed{\log_{10}\left(6x^4\right)}$

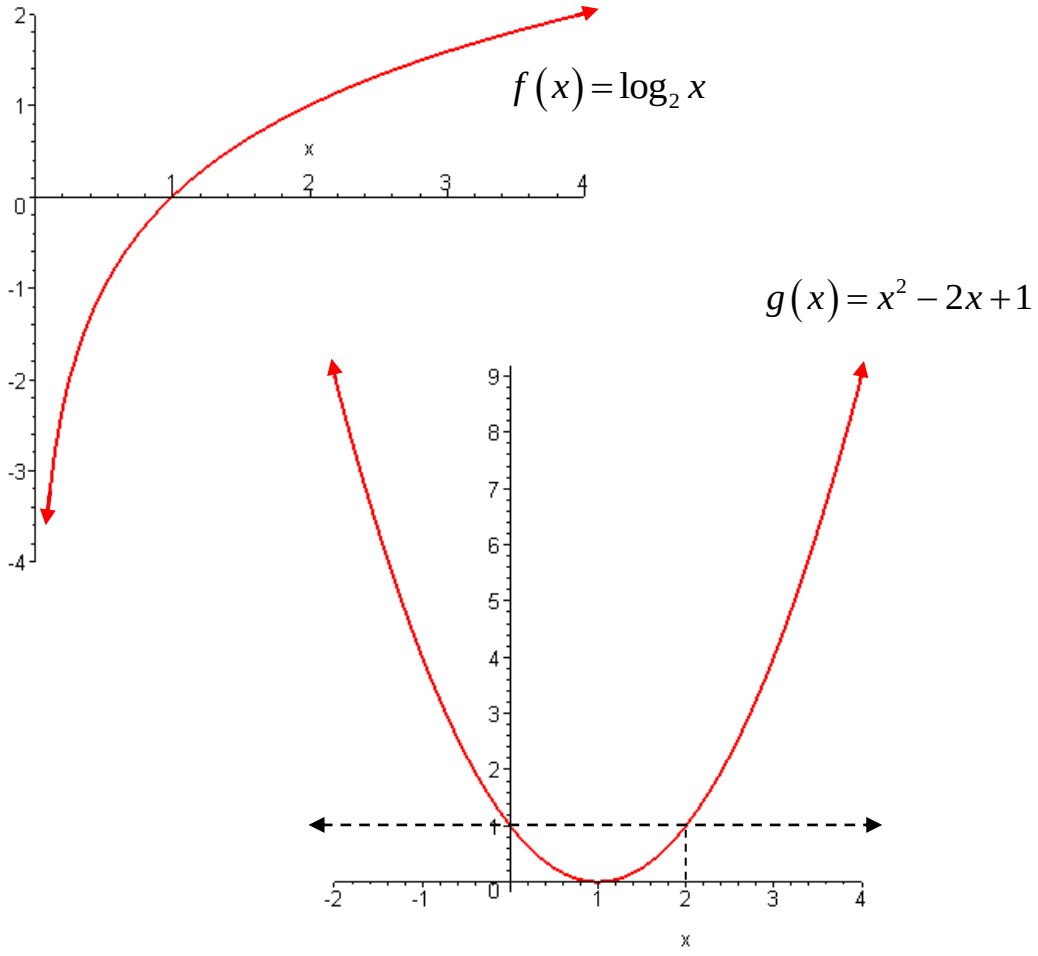
33. $\log_{10}(x-2) + \log_{10}(x+2) - \frac{1}{2}\log_{10}\left(x^2+4\right)$

$\boxed{\log_{10}\left(\dfrac{x^2-4}{\sqrt{x^2+4}}\right)}$

34. Evaluate $\log_4 15$ using a calculator to four decimal places.

$$\log_4 15 = \frac{\log 15}{\log 4} \approx \boxed{1.9534}$$

35. Solve the inequality $\log_2\left(x^2 - 2x + 1\right) > 0$. You may use the given graphs of $f(x) = \log_2 x$ and $g(x) = x^2 - 2x + 1$, if you like.



$\boxed{(-\infty, 0), (2, \infty)}$

Solve the following equations.(36-48)

36. $4^{1-2x}=2$

$\left(2^2\right)^{1-2x}=2$

$2-4x=1$

$\boxed{\frac{1}{4}}$

37. $3^{x^2-\frac{1}{2}x}=\sqrt{3}$

$x^2-\frac{1}{2}x=\frac{1}{2}$

$2x^2-x-1=0$

$(2x+1)(x-1)=0$

$\boxed{-\frac{1}{2},1}$

38. $9^{2x}=27^{3x-4}$

$\left(3^2\right)^{2x}=\left(3^3\right)^{3x-4}$

$4x=9x-12$

$\boxed{\frac{12}{5}}$

39. $25^{2x}=5^{x^2-12}$

$\left(5^2\right)^{2x}=5^{x^2-12}$

$4x=x^2-12$

$x^2-4x-12=0$

$(x-6)(x+2)=0$

$\boxed{6,-2}$

40. $4^{2x}+5\cdot 4^x-14=0$

$\left(4^x+7\right)\left(4^x-2\right)=0$

$\boxed{\frac{1}{2}}$

41. $\log_3\sqrt{x-2}=2$

$\sqrt{x-2}=9$

$x-2=81$

$\boxed{83}$

42. $\log_x 64=-3$

$x^{-3}=64$

$x=64^{-\frac{1}{3}}$

$\boxed{\frac{1}{4}}$

43. $\log_{\sqrt{2}} x=-6$

$x=\left(\sqrt{2}\right)^{-6}$

$\boxed{\frac{1}{8}}$

44. $\log_6(x+3)+\log_6(x+4)=1$

$(x+3)(x+4)=6$

$x^2+7x+12=6$

$x^2+7x+6=0$

$(x+1)(x+6)=0$

$\boxed{-1}$

45. $\log_{10}(7x-12)=2\log_{10} x$

$x^2=7x-12$

$x^2-7x+12=0$

$(x-3)(x-4)=0$

$\boxed{3,4}$

46. $8=4^{x^2}\cdot 2^{5x}$

$2^3=\left(2^2\right)^{x^2}\cdot 2^{5x}$

$3=2x^2+5x$

$2x^2+5x-3=0$

$(2x-1)(x+3)=0$

$\boxed{\frac{1}{2},-3}$

47. $\log_2 x+\log_2(x-2)=\log_2(x+4)$

$x(x-2)=x+4$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$\boxed{4}$

48. $\log_{10}(2x)-\log_{10}(x-3)=1$

$\frac{2x}{x-3}=10$

$2x=10x-30$

$\boxed{\frac{15}{4}}$