Math 1314 Review 3(Solutions)

1. For the two points $(-2,3)$ and $(3,-9)$, find the distance between them, and the midpoint of the segment that joins them.

$$d\left[(-2,3),(3,-9)\right]=\sqrt{(-2-3)^2+(3-(-9))^2}=\sqrt{25+144}=\sqrt{169}=\boxed{13}$$

$$\text{midpoint}=\left(\frac{-2+3}{2},\frac{3+-9}{2}\right)=\boxed{\left(\frac{1}{2},-3\right)}$$

2. Find an equation for the circle centered at $(-2,4)$ with a radius of $\sqrt{8}$.

$$(x-(-2))^2+(y-4)^2=\left(\sqrt{8}\right)^2$$

$$\boxed{(x+2)^2+(y-4)^2=8}$$

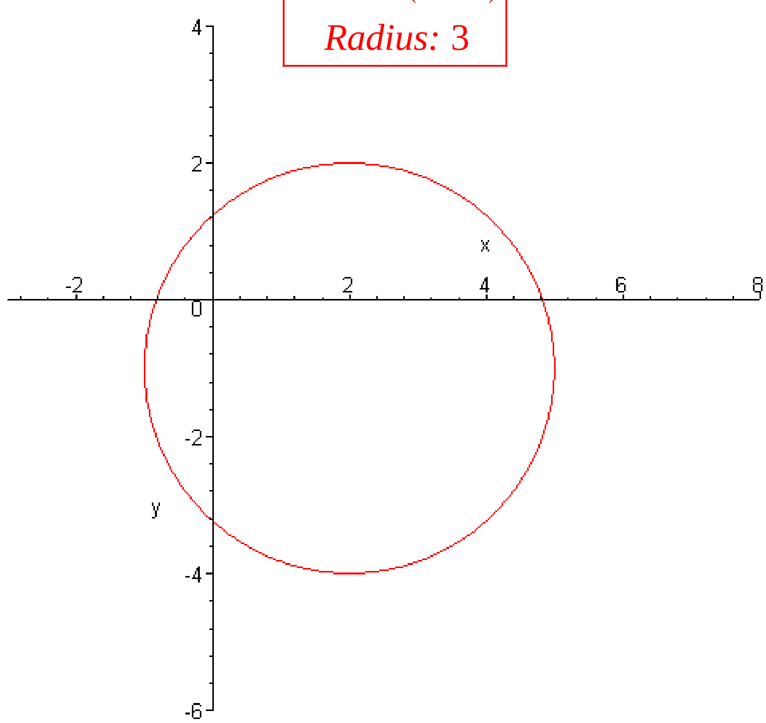
3. Find the center and radius of the circle with equation $x^2+y^2-4x+2y-4=0$. Use the graph to determine the domain and range of the relation.

$$x^2-4x+y^2+2y=4$$

$$\left(x^2-4x+4\right)+\left(y^2+2y+1\right)=4+4+1$$

$$(x-2)^2+(y+1)^2=9$$

$$\boxed{\begin{array}{c}\textit{Center:}(2,-1)\\ \textit{Radius: } 3\end{array}}$$



$$\boxed{\begin{array}{c}\text{Domain: } [-1,5]\\ \text{Range: } [-4,2]\end{array}}$$

4. Find an equation for the circle which has $(3,6)$ and $(5,4)$ as endpoints of a diameter.

$$\text{Center of the circle: } \left(\frac{3+5}{2},\frac{6+4}{2}\right)=(4,5)$$

$$(x-4)^2+(y-5)^2=r^2$$

Plug in $(3,6)$:

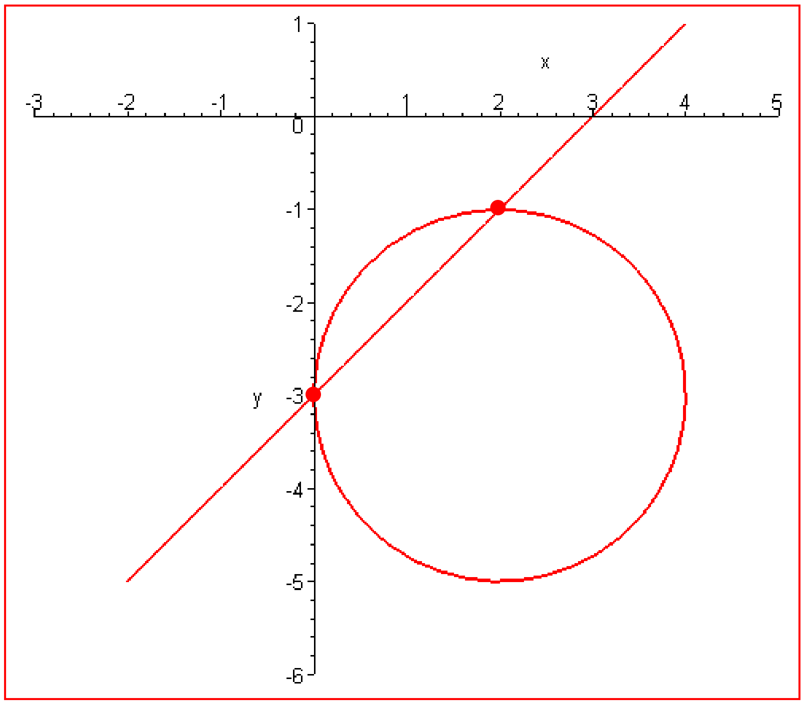
$$(3-4)^2+(6-5)^2=r^2$$

so $r^2=1+1=2$

$$\boxed{(x-4)^2+(y-5)^2=2}$$

5. Graph the equations $\begin{matrix}(x-2)^2+(y+3)^2=4\\ y=x-3\end{matrix}$, and find their points of intersection.

$$(x-2)^2+(y+3)^2=4$$
$$y=x-3$$
$$(x-2)^2+(x-3+3)^2=4$$
$$(x-2)^2+x^2=4$$
$$x^2-4x+4+x^2=4$$
$$2x^2-4x+4=4$$
$$2x^2-4x=0$$
$$2x(x-2)=0$$
$$x=0,2$$

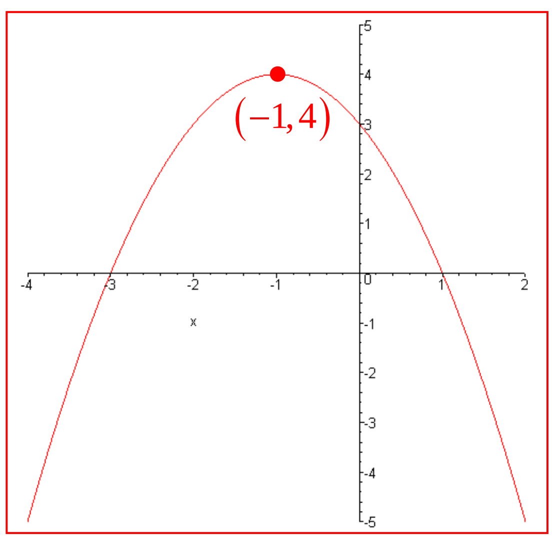


So the solutions are $(0,-3)$ *and* $(2,-1)$.

Sketch the graphs of the following quadratic functions. Indicate the vertex, intercepts, and range of the function.

6. $f(x)=-(x+1)^2+4$

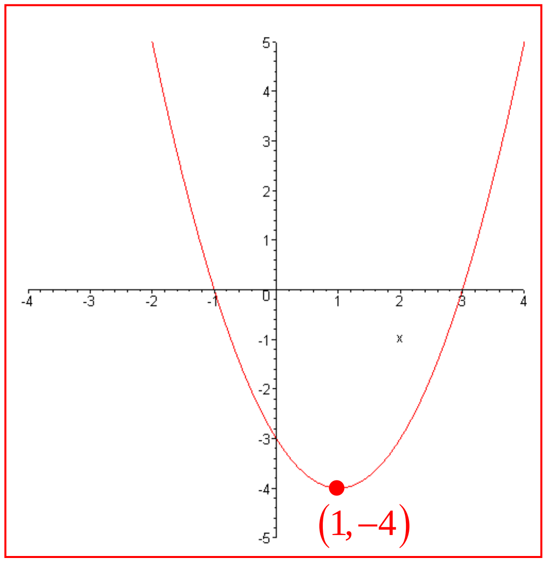
Range: $(-\infty,4]$



7. $f(x) = x^2 - 2x - 3$

$$f(x) = (x^2 - 2x) - 3$$
$$= (x^2 - 2x + 1) - 3 - 1$$
$$= (x-1)^2 - 4$$

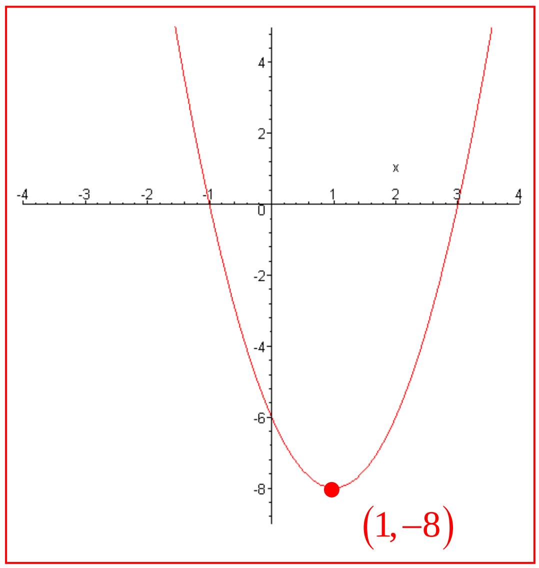
Range: $[-4, \infty)$



8. $f(x) = 2x^2 - 4x - 6$

$$f(x) = (2x^2 - 4x) - 6$$
$$= 2(x^2 - 2x) - 6$$
$$= 2(x^2 - 2x + 1) - 6 - 2$$
$$= 2(x-1)^2 - 8$$

Range: $[-8, \infty)$



Find the maximum or minimum value of the following quadratic functions.

9. $f(x) = -x^2 + 14x - 106$

$$f(x) = (-x^2 + 14x) - 106$$
$$= -(x^2 - 14x) - 106$$
$$= -(x^2 - 14x + 49) - 106 + 49$$
$$= -(x-7)^2 - 57$$

Maximum is -57, and there is no minimum.

10. $f(x) = 2x^2 + 12x + 703$

$$\begin{aligned} f(x) &= 2x^2 + 12x + 703 \\ &= \left(2x^2 + 12x\right) + 703 \\ &= 2\left(x^2 + 6x\right) + 703 \\ &= 2\left(x^2 + 6x + 9\right) + 703 - 18 \\ &= 2(x+3)^2 + 685 \end{aligned}$$

$$\boxed{\textit{Minimum is 685, and there is no maximum.}}$$

Find a formula for the quadratic function whose graph satisfies the given conditions.

11. The vertex is $(-3,-4)$ and the graph passes through the point $(1,4)$.

$f(x) = a(x-h)^2 + k$

$f(x) = a(x-(-3))^2 + (-4)$ now plug in $(1,4)$ to get $\begin{aligned} 4 &= a(1+3)^2 - 4 \\ 4 &= 16a - 4 \\ 8 &= 16a \\ a &= \frac{1}{2} \end{aligned}$ so the quadratic function

$f(x) = a(x+3)^2 - 4$

is $\boxed{f(x) = \frac{1}{2}(x+3)^2 - 4}$.

12. The graph passes through the points $(1,4)$ and $(3,4)$ and the maximum value is 6.

$f(x) = a(x-h)^2 + k$ and since the maximum is 6, you know that $k = 6$.

$f(x) = a(x-h)^2 + 6$, so plug in $(1,4)$ and $(3,4)$ to get $\begin{aligned} 4 &= a(1-h)^2 + 6 \\ 4 &= a(3-h)^2 + 6 \end{aligned}$, which implies that

$(1-h)^2 = (3-h)^2$, so $1-h = 3-h$ or $1-h = h-3$. This leads to $h = 2$. So now you know that

$f(x) = a(x-2)^2 + 6$. If you plug in $(1,4)$, you get $\begin{aligned} 4 &= a(1-2)^2 + 6 \\ 4 &= a + 6 \\ a &= -2 \end{aligned}$. So the quadratic function

is $\boxed{f(x) = -2(x-2)^2 + 6}$.

OR

You could notice that the *y*-coordinates of both points are 4, which means that the *x*-coordinate of the vertex is halfway between the two *x*-coordinates of the points: 1 and 3.

So now you know that $f(x)=a(x-2)^2+6$, and if you plug in one of the points, you can determine the value of *a*.

13. The graph passes through the points $(0,4)$, $(1,3)$, and $(2,6)$.

$f(x)=ax^2+bx+c$, so plug in the points to determine the values of *a*, *b*, and *c*.

$4=c$

$3=a+b+c$, subbing the first equation into the next two leads to $\begin{matrix}-1=a+b\\1=2a+b\end{matrix}$, and subtracting

$6=4a+2b+c$

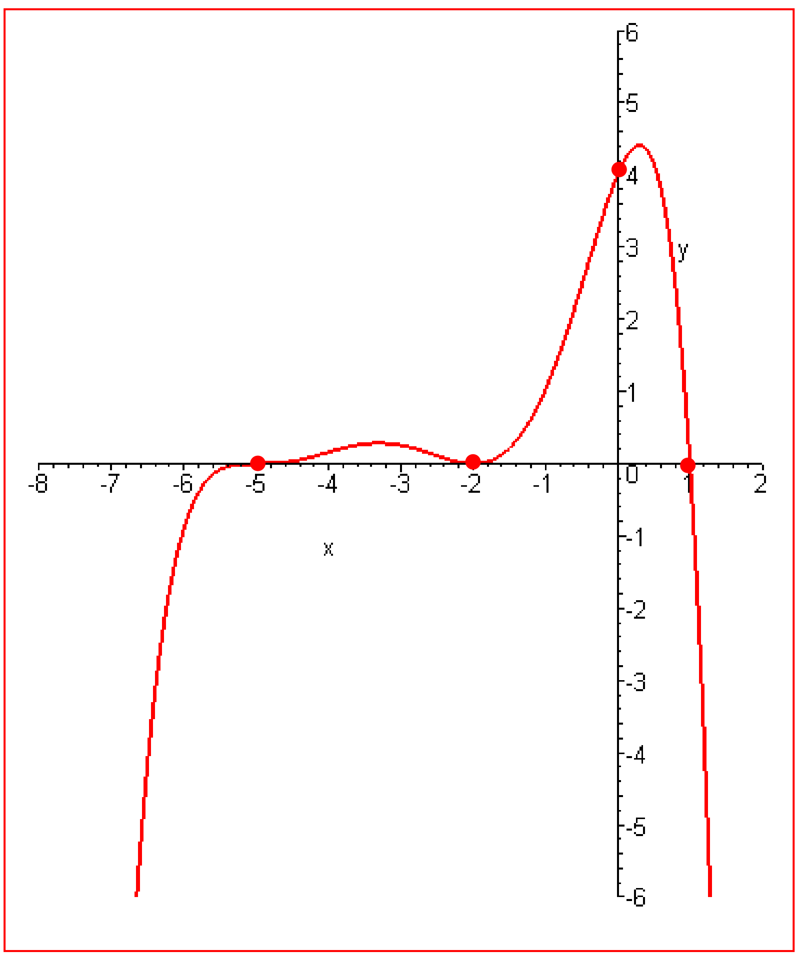
the second equation from the first leads to $-2=-a$. So you get $a=2, b=-3, c=4$, which leads to $\boxed{f(x)=2x^2-3x+4}$.

Sketch the graph of the following polynomial functions. Label the zeros and y-intercept.

14. $f(x)=-\frac{1}{125}(x-1)(x+2)^2(x+5)^3$

x-intercepts: $-5,-2,1$

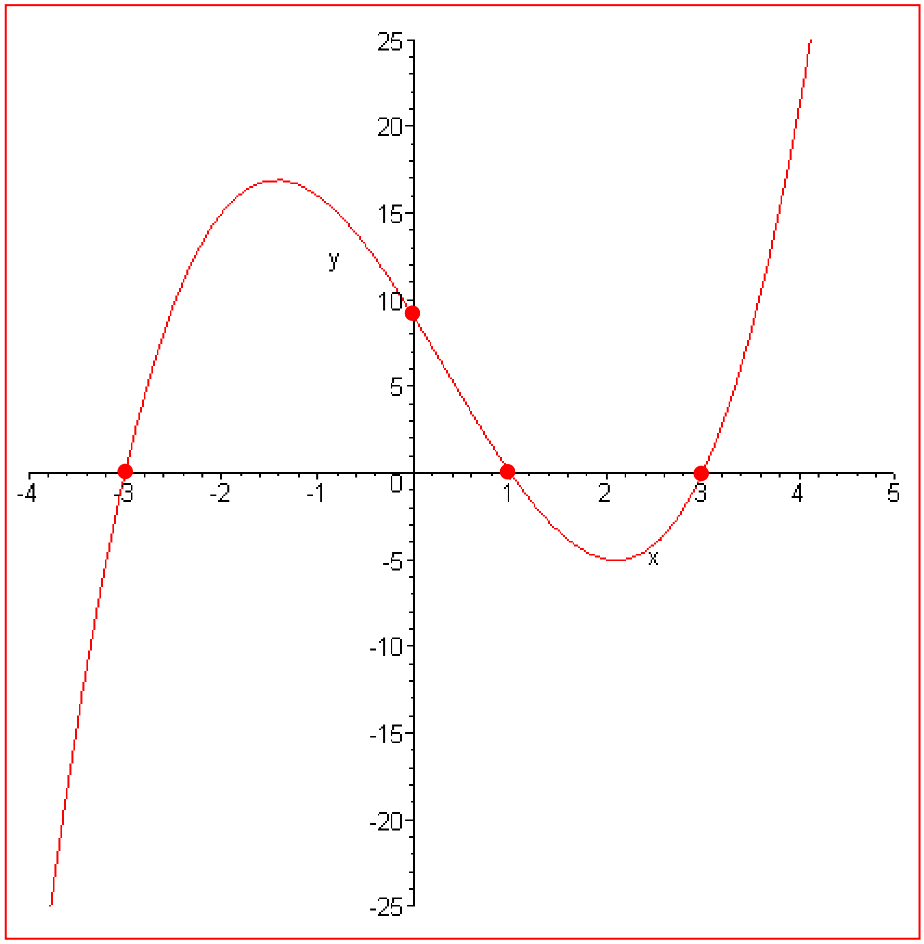
y-intercept: 4



15. $f(x) = x^3 - x^2 - 9x + 9$

$$x^3 - x^2 - 9x + 9$$
$$(x^3 - x^2) - (9x - 9)$$
$$x^2(x-1) - 9(x-1)$$
$$(x-1)(x^2-9)$$
$$(x-1)(x-3)(x+3)$$

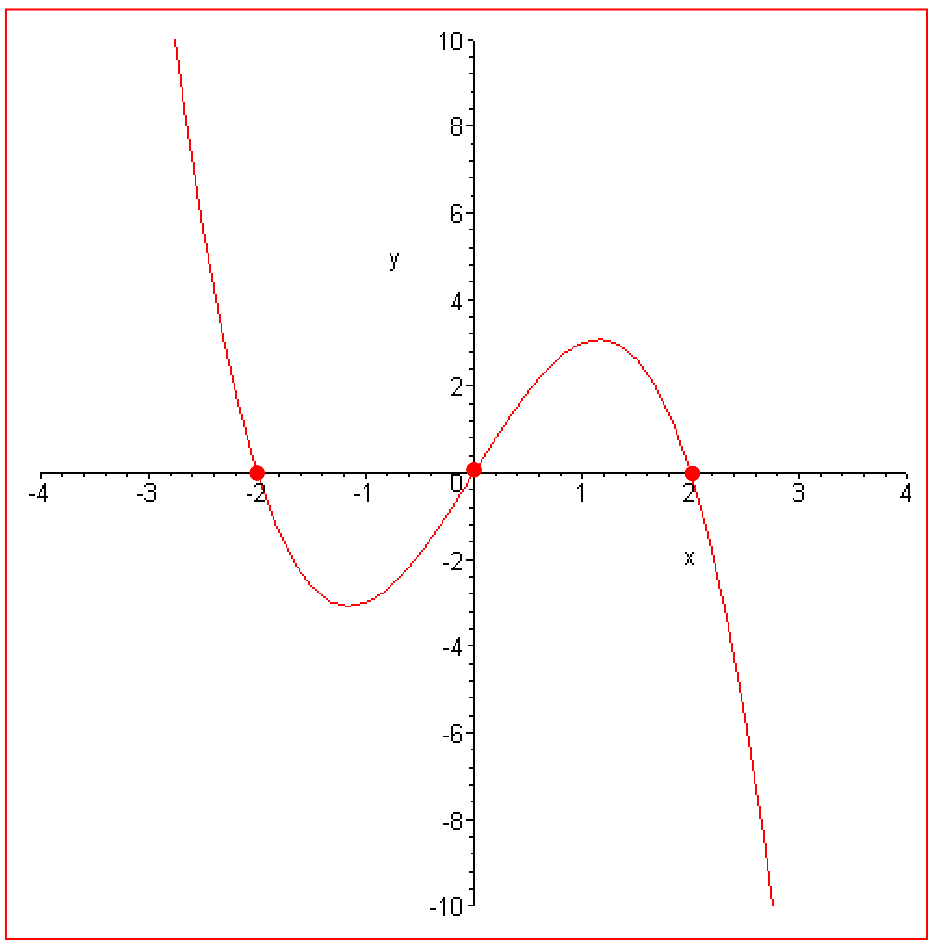
x-intercepts: $1, 3, -3$
y-intercept: 9



16. $f(x) = 4x - x^3$

$$4x - x^3$$
$$x(4 - x^2)$$
$$x(2-x)(2+x)$$
$$-x(x-2)(x+2)$$

x-intercepts: $-2, 0, 2$
y-intercept: 0



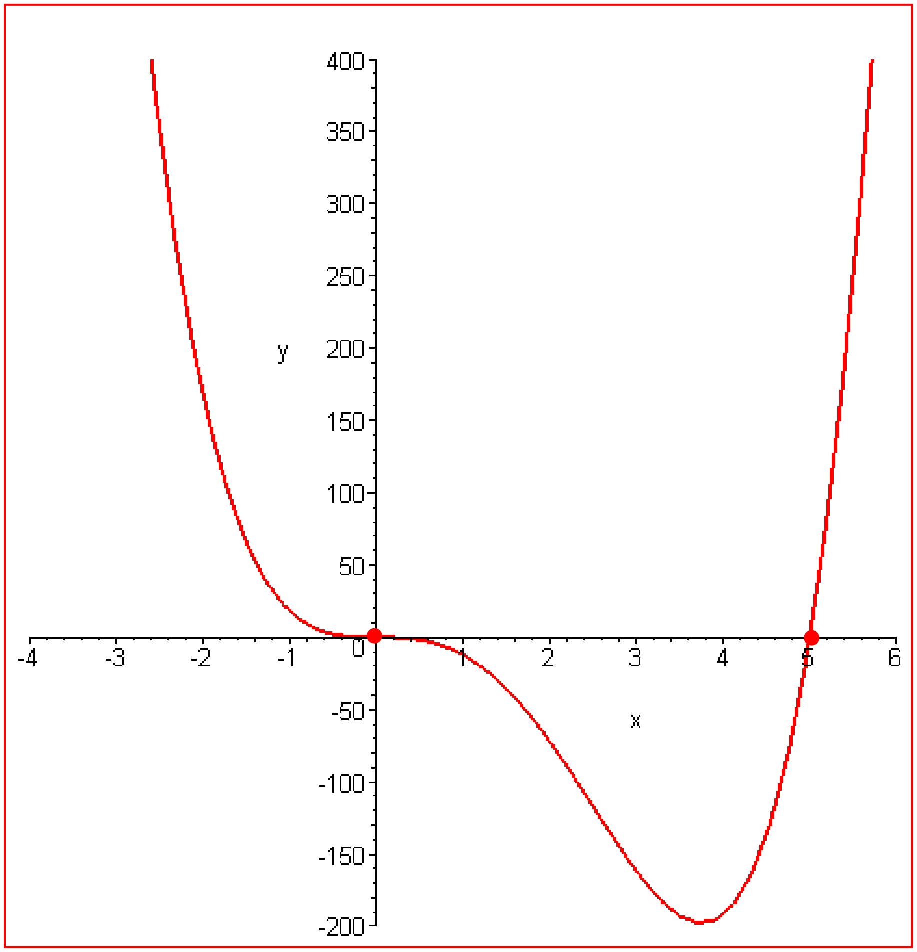
17. $f(x)=3x^4-15x^3$

$3x^4-15x^3$

$3x^3(x-5)$

x-intercepts: 0,5

y-intercept: 0



18. Use long division to find $(4x^4+6x^3+3x-1)\div(2x^2+1)$.

$$
\begin{array}{r l}
 & 2x^2+3x-1 \\
2x^2+1 \overline{)} & 4x^4+6x^3+3x-1 \\
 & -(4x^4+2x^2) \\
\hline
 & 6x^3-2x^2+3x-1 \\
 & -(6x^3+3x) \\
\hline
 & -2x^2-1 \\
 & -(-2x^2-1) \\
\hline
 & 0
\end{array}
$$

So $(4x^4+6x^3+3x-1)\div(2x^2+1)=\boxed{2x^2+3x-1}$.

19. Use synthetic division to find $(3x^4+11x^3-20x^2+7x+35)\div(x+5)$.

$$\begin{array}{r|rrrrr} -5 & 3 & 11 & -20 & 7 & 35 \\ & & -15 & 20 & 0 & -35 \\ \hline & 3 & -4 & 0 & 7 & \boxed{0} \end{array}$$

So $(3x^4+11x^3-20x^2+7x+35)\div(x+5)=\boxed{3x^3-4x^2+7}$.

20. Given $f(x)=-2x^3+7x^2-9x+3$, use the remainder theorem to find $f(3)$.

$$\begin{array}{r|rrrr} 3 & -2 & 7 & -9 & 3 \\ & & -6 & 3 & -18 \\ \hline & -2 & 1 & -6 & \boxed{-15} \end{array}$$

So $f(3)=\boxed{-15}$.

21. Solve the equation $x^3-17x+4=0$, given that 4 is a solution.

$$\begin{array}{r|rrrr} 4 & 1 & 0 & -17 & 4 \\ & & 4 & 16 & -4 \\ \hline & 1 & 4 & -1 & \boxed{0} \end{array}$$

So $x^3-17x+4=(x-4)(x^2+4x-1)$, and the other two solutions come from the quadratic formula applied to $x^2+4x-1=0$. $\dfrac{-4\pm\sqrt{16+4}}{2}$ $\dfrac{-4\pm2\sqrt{5}}{2}$ $-2\pm\sqrt{5}$ So the three solutions of the equation are $\boxed{4,-2+\sqrt{5},-2-\sqrt{5}}$.

Use The Rational Zero Theorem to list all the possible rational zeros of the following polynomials.

22. $f(x)=x^4-6x^3+14x^2-14x+5$

$\{\pm5,\pm1\}$

23. $f(x)=3x^5-2x^4-15x^3+10x^2+12x-8$

$\left\{\pm8,\pm4,\pm2,\pm1,\pm\frac{8}{3},\pm\frac{4}{3},\pm\frac{2}{3},\pm\frac{1}{3}\right\}$

Use Descarte's Rule of Signs to determine the possible number of positive and negative zeros of the following polynomials.

24. $f(x) = 3x^4 - 2x^3 - 8x + 5$

Possible number of positive zeros	Possible number of negative zeros
2 or 0	**0**

25. $f(x) = 2x^5 - 3x^3 - 5x^2 + 3x - 1$

Possible number of positive zeros	Possible number of negative zeros
3 or 1	**2 or 0**

Find all the zeros of the following polynomials.

26. $f(x) = x^3 + 3x^2 - 4$

$$\begin{array}{r|rrrr} 1 & 1 & 3 & 0 & -4 \\ & & 1 & 4 & 4 \\ \hline & 1 & 4 & 4 & \lfloor 0 \end{array}$$

So $x^3 + 3x^2 - 4 = (x-1)(x^2+4x+4) = (x-1)(x+2)^2$. So the zeros are $\boxed{1, -2}$.

27. $f(x) = 2x^3 + 9x^2 - 7x + 1$

$$\begin{array}{r|rrrr} \frac{1}{2} & 2 & 9 & -7 & 1 \\ & & 1 & 5 & -1 \\ \hline & 2 & 10 & -2 & \lfloor 0 \end{array}$$

So $2x^3 + 9x^2 - 7x + 1 = (x - \tfrac{1}{2})(2x^2 + 10x - 2) = 2(x - \tfrac{1}{2})(x^2 + 5x - 1)$. The other two zeros come from applying the quadratic formula to the equation $x^2 + 5x - 1 = 0$. $\dfrac{-5 \pm \sqrt{25+4}}{2}$ $\dfrac{-5 \pm \sqrt{29}}{2}$ So the zeros are $\boxed{\dfrac{1}{2}, \dfrac{5+\sqrt{29}}{2}, \dfrac{5-\sqrt{29}}{2}}$.

28. $f(x)=4x^4+7x^2-2$

$4x^4+7x^2-2=(4x^2-1)(x^2+2)=(2x+1)(2x-1)(x^2+2)$. So the zeros are $\boxed{\frac{1}{2},-\frac{1}{2},\sqrt{2}i,-\sqrt{2}i}$.

Find an n^{th} degree polynomial function with real coefficients that satisfies the given conditions.

29. $n=3$, 2 and $2-3i$ are zeros; $f(1)=-10$

$$[x-(2-3i)][x-(2+3i)]=[(x-2)+3i][(x-2)-3i]=x^2-4x+13$$

So $f(x)=a(x-2)(x^2-4x+13)$. Plug in $x=1$ to get the value of a.

$$-10=a(1-2)((1)^2-4(1)+13)$$

$$-10=-10a$$

$$a=1$$

So the polynomial is $\boxed{f(x)=(x-2)(x^2-4x+13)}$.

30. $n=4$, i is a zero and -3 is a zero of multiplicity 2; $f(-1)=16$

$$(x-i)[x-(-i)]=(x-i)(x+i)=x^2+1$$

So $f(x)=a(x+3)^2(x^2+1)$. Plug in $x=-1$ to get the value of a.

$$16=a(-1+3)^2((-1)^2+1)$$

$$16=8a$$

$$a=2$$

So the polynomial is $\boxed{f(x)=2(x+3)^2(x^2+1)}$.

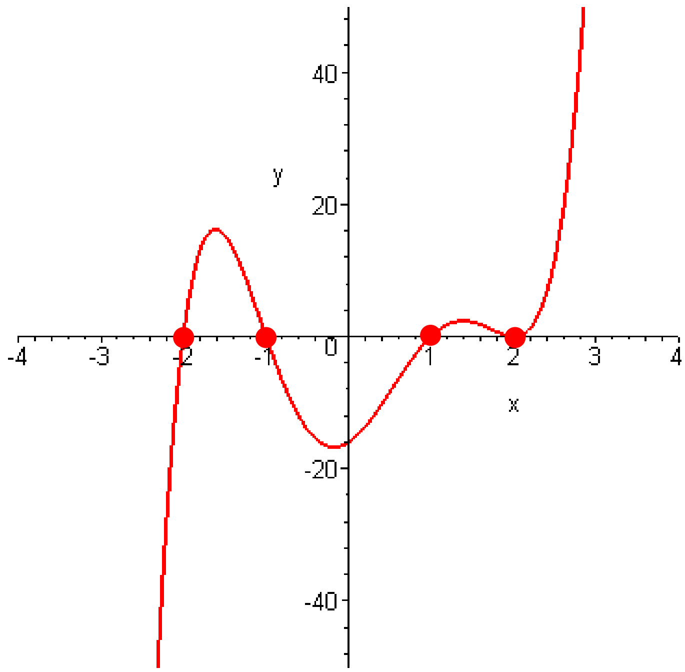
31. Find all the zeros of $f(x)=2x^4+3x^3+3x-2$ and write the polynomial as a product of linear factors.

-2	2	3	0	3	-2
		-4	2	-4	2
	2	-1	2	-1	0

So $f(x)=2x^4+3x^3+3x-2=(x+2)(2x^3-x^2+2x-1)=(x+2)(2x-1)(x^2+1)$. So as a product of linear factors, you get $f(x)=\boxed{2(x+2)\left(x-\frac{1}{2}\right)(x+i)(x-i)}$.

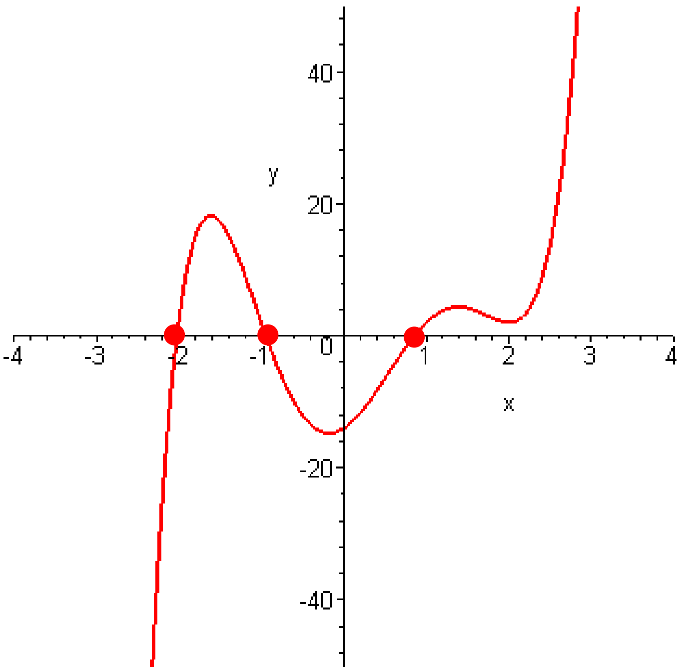
Graphs of fifth-degree polynomials are given. Tell the number of real zeros and the number of imaginary zeros.

32.



5 real zeros and no imaginary zeros.

33.



3 real zeros and 2 imaginary zeros.

34. How many real zeros does the polynomial $f(x) = 2x^5 + x + 8$ have?

It has no positive zeros, it has one negative zero, and 0 is not a zero.

This means that it has 1 real zero.

35. Find the value of k so that $x-3$ is a factor of $x^5 - 3x^4 + 6x^2 - k$.

$$\begin{array}{r|rrrrrr} 3 & 1 & -3 & 0 & 6 & 0 & -k \\ & & 3 & 0 & 0 & 18 & 54 \\ \hline & 1 & 0 & 0 & 6 & 18 & \boxed{54-k} \end{array}$$

So $k = \boxed{54}$.

36. What is the remainder when $x^{999} + 500x^{998} + x^3 + 500x^2 + x + 499$ is divided by $x + 500$?

$$(-500)^{999} + 500(-500)^{998} + (-500)^3 + 500(-500)^2 + (-500) + 499$$

$$-(500)^{999} + (500)^{999} - (500)^3 + (500)^3 - 500 + 499$$

$$0 + 0 - 1$$

$$\boxed{-1}$$

37. Is $x-1$ a factor of $x^{567} - 3x^{400} + x^9 + 2$?

$$(1)^{567} - 3(1)^{400} + (1)^9 + 2$$

$$1 - 3 + 1 + 2$$

$$1$$

So $x-1$ is not a factor.