Math 1314 Review 4(answers)

Sketch the graphs of the following rational functions. Indicate the asymptotes and intercepts

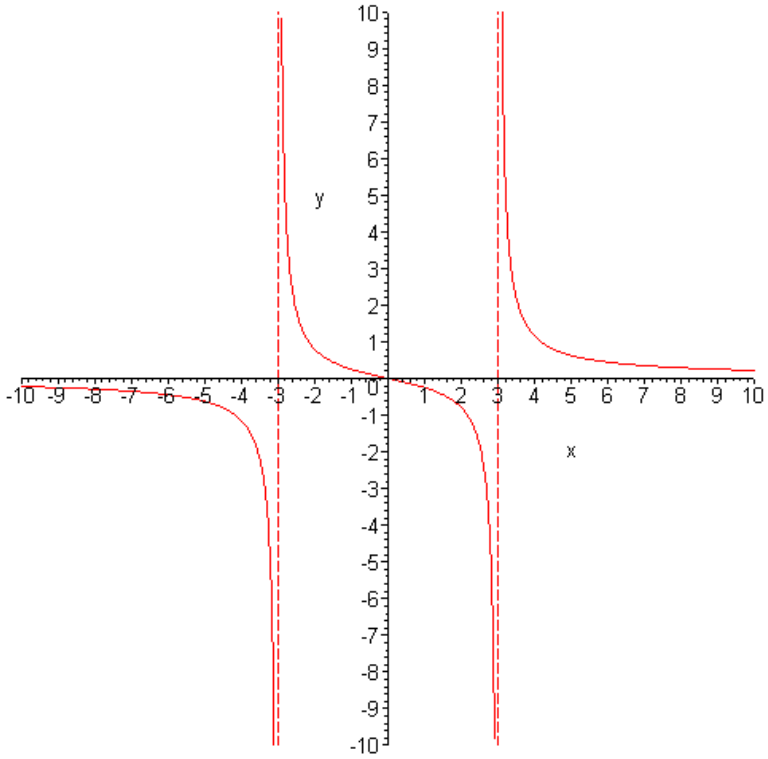
1. $f(x)=\dfrac{2x}{x^2-9}$

$V.A.:\boxed{x=3, x=-3}$

$H.A.:\boxed{y=0}$

$x-\text{int}.:\boxed{0}$

$y-\text{int}:\boxed{0}$



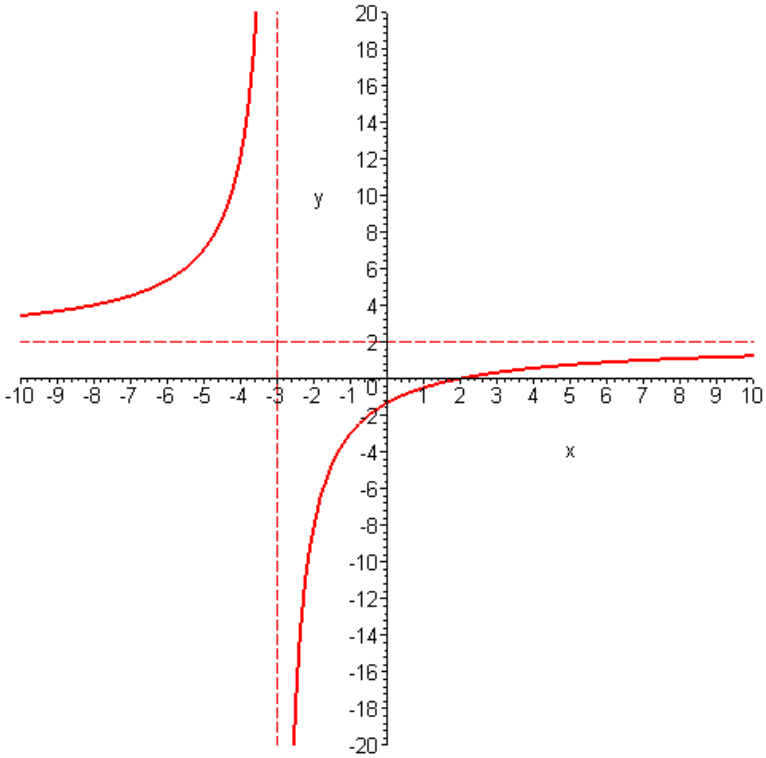
2. $f(x)=\dfrac{2x-4}{x+3}$

$V.A.:\boxed{x=-3}$

$H.A.:\boxed{y=2}$

$x-\text{int}:\boxed{2}$

$y-\text{int}:\boxed{-\dfrac{4}{3}}$



3. $f(x)=\dfrac{4x^2-16x+16}{2x-3}$

$V.A.:\ \boxed{x=\dfrac{3}{2}}$

$S.A.:$

$$2x-3\overline{)\,4x^2-16x+16}$$

$$2x-5+\frac{1}{2x-3}$$

$$-\left(4x^2-6x\right)$$

$$-10x+16$$

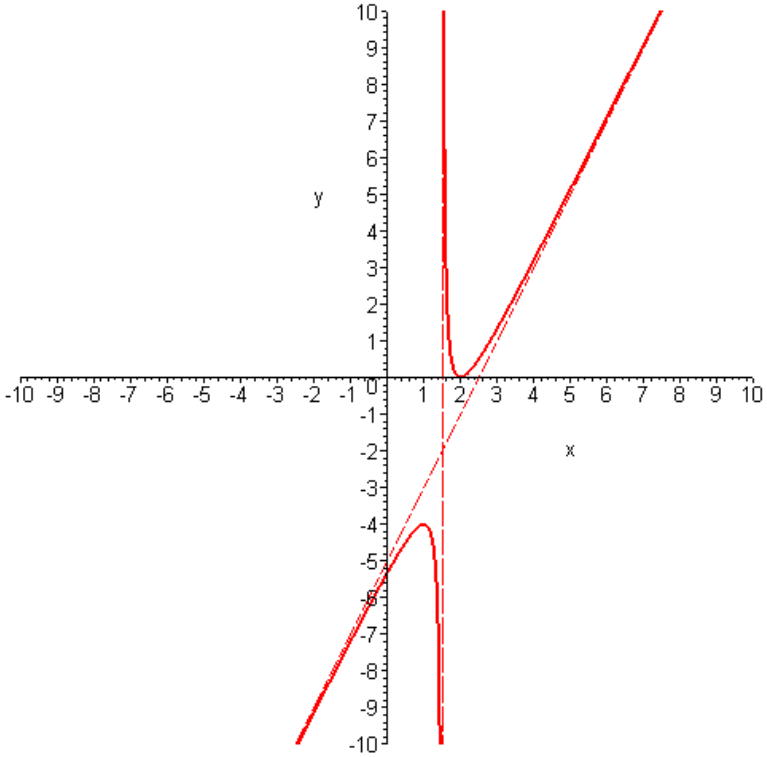
$$-\left(-10x+15\right)$$

$$1$$

$\boxed{y=2x-5}$

$x-\text{int}:\ \boxed{2}$

$y-\text{int}:\ \boxed{-\dfrac{16}{3}}$

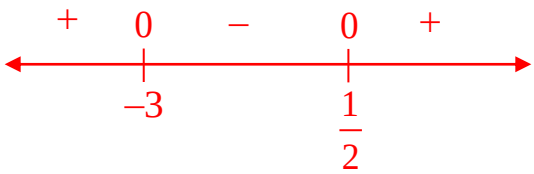


Solve the following inequalities, and express the solution in interval notation.

4. $2x^2+5x-3<0$

$(2x-1)(x+3)<0$

$2\left(x-\frac{1}{2}\right)(x+3)<0$



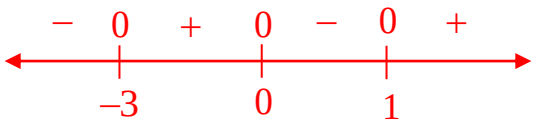
$\boxed{\left(-3,\tfrac{1}{2}\right)}$

5. $x^3+2x^2\geq 3x$

$x^3+2x^2-3x\geq 0$

$x\left(x^2+2x-3\right)\geq 0$

$x(x+3)(x-1)\geq 0$



$\boxed{[-3,0],[1,\infty)}$

6. $\dfrac{x-6}{x+2} \le 0$

$\boxed{(-2,6]}$

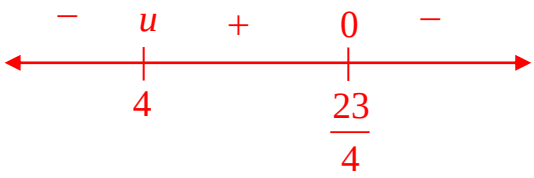
7. $\dfrac{x+3}{x-4} \le 5$

$$\frac{x+3}{x-4} - 5 \le 0$$

$$\frac{x+3}{x-4} - \frac{5(x-4)}{x-4} \le 0$$

$$\frac{-4x+23}{x-4} \le 0$$

$$\frac{-4\left(x-\frac{23}{4}\right)}{x-4} \le 0$$



$\boxed{(-\infty,4),\left[\frac{23}{4},\infty\right)}$

Sketch the graphs of the following exponential and logarithmic functions. Indicate the asymptotes, intercepts, domain, and range.

8. $g(x) = 5^x - 1$

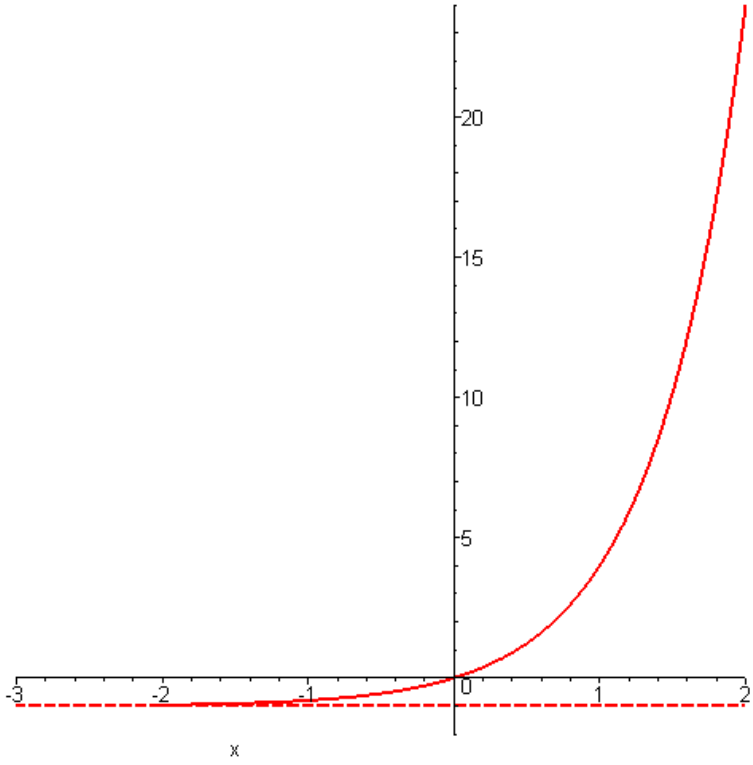
H.A.: $\boxed{y=-1}$

$x-\text{int.}:\ \boxed{0}$

$y-\text{int.}:\ \boxed{0}$

Domain: $\boxed{(-\infty,\infty)}$

Range: $\boxed{(-1,\infty)}$



9. $g(x)=-\left(\frac{1}{2}\right)^x+2$

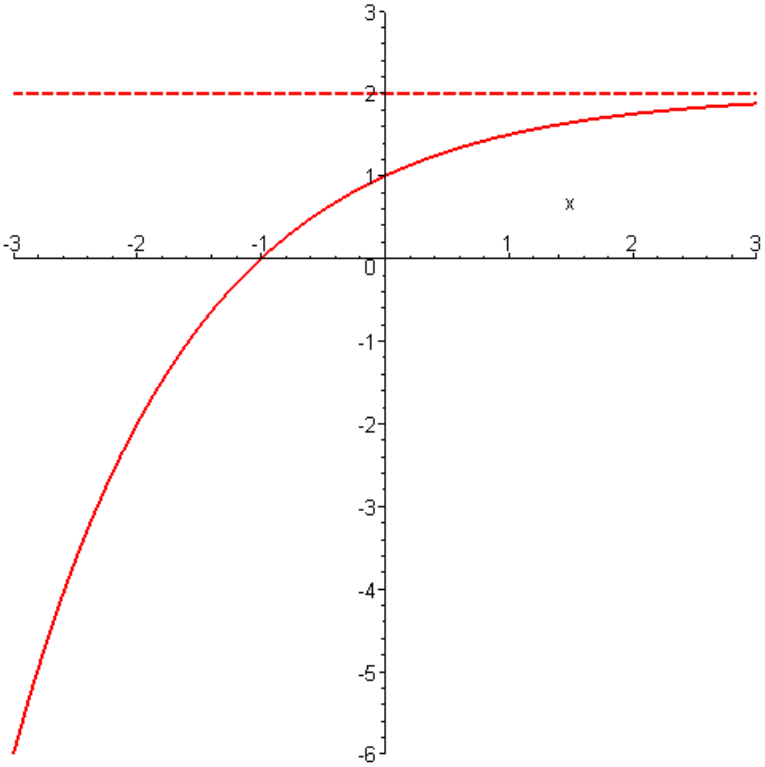
$H.A.:\boxed{y=2}$

$x-\text{int.}:\boxed{-1}$

$y-\text{int.}:\boxed{1}$

$Domain:\boxed{(-\infty,\infty)}$

$Range:\boxed{(-\infty,2)}$



10. $g(x)=\log_4(x-1)+2$

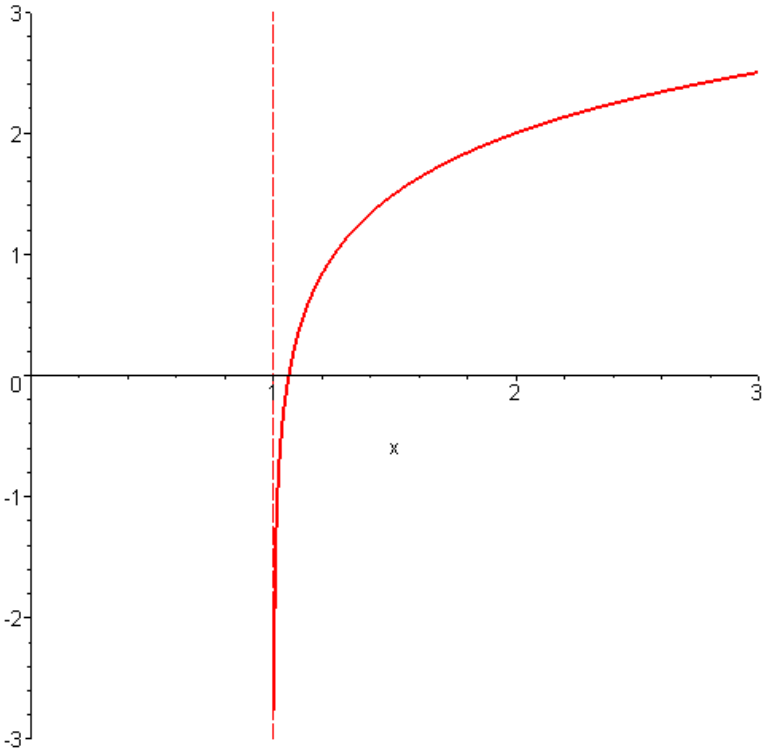
$V.A.:\boxed{x=1}$

$x-\text{int.}:\boxed{\frac{17}{16}}$

$y-\text{int.}:\boxed{none}$

$Domain:\boxed{(1,\infty)}$

$Range:\boxed{(-\infty,\infty)}$



11. $g(x)=\log_4(2-x)$

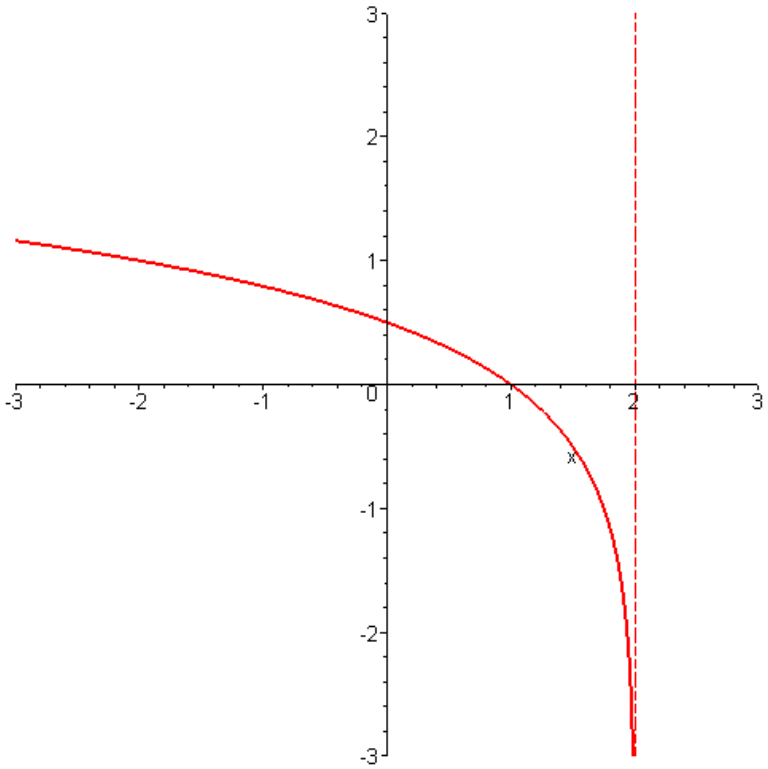
$V.A.:\boxed{x=2}$

$x-\text{int.}:\boxed{1}$

$y-\text{int.}:\boxed{\frac{1}{2}}$

$Domain:\boxed{(-\infty,2)}$

$Range:\boxed{(-\infty,\infty)}$



Evaluate the following expressions.

12. $\log_4 64$

$\boxed{3}$

13. $\log_5\left(\frac{1}{25}\right)$

$\boxed{-2}$

14. $\log_{16} 4$

$\boxed{\frac{1}{2}}$

15. $\log_3\left(\frac{1}{\sqrt{3}}\right)$

$\boxed{-\frac{1}{2}}$

16. $\log_5\left(5^{10,000}\right)$

$\boxed{10,000}$

17. $9^{\log_9\left(3^{10,000}\right)}$

$\boxed{3^{10,000}}$

18. $(\log_2 3)(\log_3 4)(\log_4 5)\cdots(\log_{63} 64)$

$=\log_2 3\cdot\frac{\log_2 4}{\log_2 3}\cdot\frac{\log_2 5}{\log_2 4}\cdot\frac{\log_2 6}{\log_2 5}\cdots\frac{\log_2 63}{\log_2 62}\cdot\frac{\log_2 64}{\log_2 63}$

$=\log_2 64=\boxed{6}$

19. $\log_3(\log_8 8)$

$\boxed{0}$

20. $\log\left(\frac{1}{1000}\right)$

$\boxed{-3}$

Expand the following logarithmic expressions as much as possible. Simplify if possible.

21. $\log_6\left(36x^3\right)$

$\log_6 36+\log_6\left(x^3\right)$

$\boxed{2+3\log_6 x}$

22. $\log_2\left(\dfrac{xy^2}{64}\right)$

$\log_2\left(xy^2\right)-\log_2 64$

$\boxed{\log_2 x+2\log_2 y-6}$

23. $\log_4\left(\dfrac{\sqrt{x}}{64}\right)$

$\log_4\left(\sqrt{x}\right)-\log_4 64$

$\boxed{\dfrac{1}{2}\log_4 x-3}$

Compress the following logarithmic expressions into a single term.

24. $\log_{10}3-3\log_{10}x$

$\boxed{\log_{10}\left(\dfrac{3}{x^3}\right)}$

25. $\frac{1}{2}\log_{10}x+\frac{1}{3}\log_{10}y$

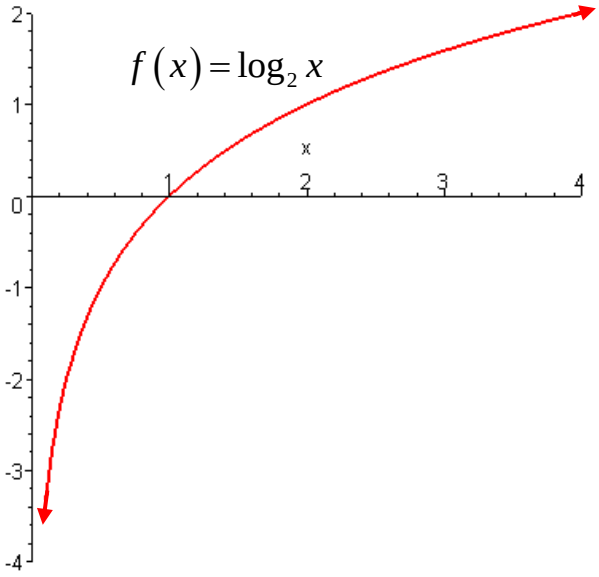
$\boxed{\log_{10}\left(x^{\frac{1}{2}}y^{\frac{1}{3}}\right)}$

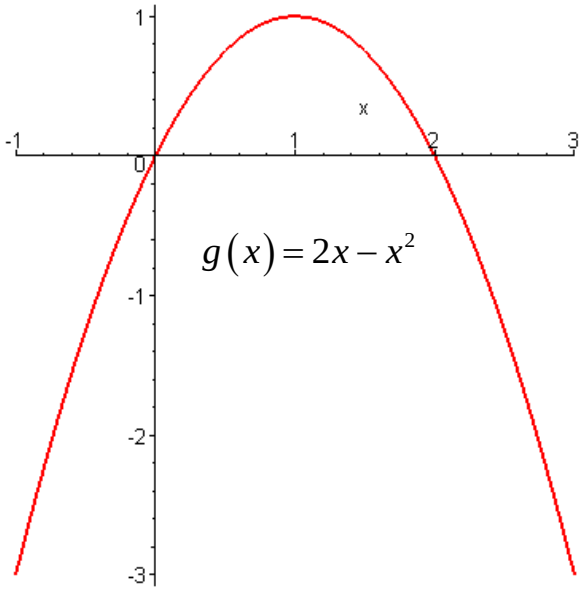
26. Evaluate $\log_4 1000$ using a calculator to four decimal places.

$$\log_4 1000=\frac{\log 1000}{\log 4}=4.9828921\ldots$$

$\approx \boxed{4.9829}$

27. Use the graphs of $f(x)=\log_2 x$ and $g(x)=2x-x^2$ to solve the inequality $\log_2\left(2x-x^2\right)\le 0$





$\boxed{(0,2)}$