Math 2412 Review 3

1. The decay of radium is modeled by $A(t) = A_0 e^{kt}$. If the half-life of radium is 1690 years, and you have 10 grams now, how much will be present in 50 years (rounded to three decimal places)?

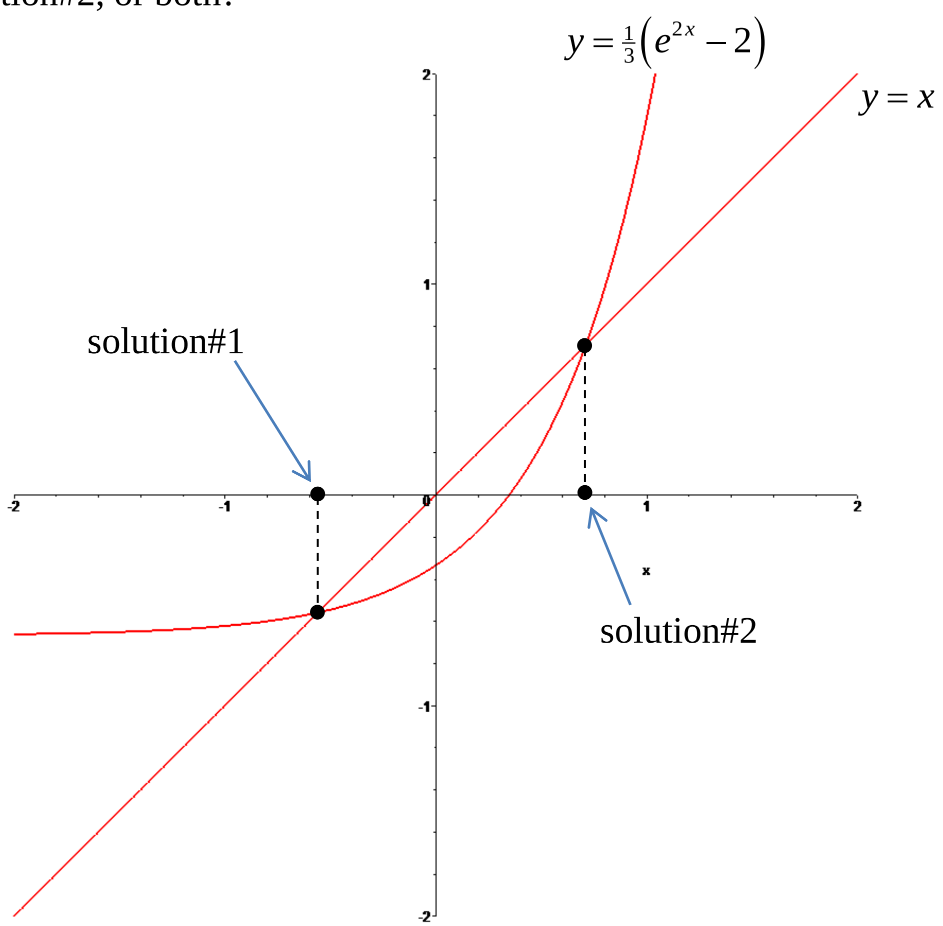
2. The logistic growth model $P(t) = \dfrac{1000}{1 + 32.33e^{-.439t}}$ represents the population (in grams) of bacteria after t hours.

a) What is the carrying capacity of the environment?

b) What is the initial population size (rounded to two decimal places)?

c) When will the population be 700 grams (rounded to two decimal places)?

3. The equation $x = \frac{1}{3}\left(e^{2x} - 2\right)$ has two solutions. In approximating them using the Method of Successive Approximations(see the graph), which solution is a repelling solution: solution#1, solution#2, or both?



4. Use the method of finite differences to find a formula for generating the terms of the sequence $1, 3, 7, 13, 21, 31, 43, 57, \ldots$.

5. Write out the first five terms of the following sequences:

a) $a_n = \dfrac{n}{n+2}$

b) $a_1 = 1, a_{n+1} = \dfrac{a_n}{n}; n \geq 1$

6. Use the following formulas:

$$\sum_{k=1}^{n} k = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1+4+9+16+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1+8+27+64+\cdots+n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

to find the exact values of the following series:

a) $\sum_{k=1}^{200} k$ **b)** $\sum_{k=1}^{40} (k^2+4)$ **c)** $\sum_{k=4}^{34} k^3$

7. Consider the arithmetic sequence $5, 2, -1, -4, -7, -10, \ldots$.

a) Write a formula that will generate the terms of the sequence.

b) Find the sum of the first 100 terms of this sequence.

8. Find the first term and the common difference for each of the following arithmetic sequences:

a) 15^{th} term is 0 and the 40^{th} term is -50

b) 12^{th} term is 4 and the 18^{th} term is 28

9. Find x so that $2x, 3x+2, 5x+3$ are consecutive terms of an arithmetic sequence.

10. How many terms must be added in an arithmetic sequence whose first term is 78 and whose common difference is -4 to get a sum of 702?

11. Express the sum of the series $\sum_{n=1}^{1000} \left(\frac{1}{n} - \frac{1}{n+1}\right)$ as a single fraction.

12. Consider the geometric sequence $2, -4, 8, -16, 32, -64, \ldots$.

a) Write a formula that will generate the terms of the sequence.

b) Find the sum of the first 12 terms of this sequence.

13. If $x, x+1, x+4$ are the first three terms of a geometric sequence, then what is the value of x?

14. Determine if the following geometric series converge or diverge. If a series converges, write what it converges to.

a) $1 + -1 + 1 + -1 + 1 + -1 + \cdots$

b) $1 + -\frac{1}{2} + \frac{1}{4} + -\frac{1}{8} + \cdots$

15. Use Mathematical Induction to prove that $3+5+7+\cdots+(2n+1)=n(n+2)$ for all natural numbers, *n*.

16. Use Mathematical Induction to prove that $\left(1+\frac{1}{1}\right)\left(1+\frac{1}{2}\right)\left(1+\frac{1}{3}\right)\cdots\left(1+\frac{1}{n}\right)=n+1$ for all natural numbers, *n*.

17. Use Mathematical Induction to prove that n^2+n is divisible by 2 for all natural numbers, *n*.