Math 2412 Review 4(answers)

1. Expand $(x+2)^5$, using the Binomial Theorem, or using Pascal's Triangle.

$$\begin{array}{ccccccccccc} &&&&&1&&&&& \\ &&&&1&&1&&&& \\ &&&1&&2&&1&&& \\ &&1&&3&&3&&1&& \\ &1&&4&&6&&4&&1& \\ 1&&5&&10&&10&&5&&1 \end{array}$$

$$(x+2)^5=\sum_{j=0}^{5}\binom{5}{j}x^{5-j}\cdot 2^j = x^5+5x^4\cdot 2+10x^3\cdot 2^2+10x^2\cdot 2^3+5x\cdot 2^4+2^5$$

$$\boxed{x^5+10x^4+40x^3+80x^2+80x+32}$$

2. Expand $(x-1)^6$, using the Binomial Theorem, or using Pascal's Triangle.

$$\begin{array}{ccccccccccccc} &&&&&&1&&&&&& \\ &&&&&1&&1&&&&& \\ &&&&1&&2&&1&&&& \\ &&&1&&3&&3&&1&&& \\ &&1&&4&&6&&4&&1&& \\ &1&&5&&10&&10&&5&&1& \\ 1&&6&&15&&20&&15&&6&&1 \end{array}$$

$$(x-1)^6=\sum_{j=0}^{6}\binom{6}{j}x^{6-j}(-1)^j = x^6+6x^5(-1)+15x^4(-1)^2+20x^3(-1)^3+15x^2(-1)^4+6x(-1)^5+(-1)^6$$

$$\boxed{x^6-6x^5+15x^4-20x^3+15x^2-6x+1}$$

3. Find the coefficient of x^3 in the expansion of $(x+3)^{10}$.

$$\binom{10}{j}x^{10-j}\cdot 3^j \Rightarrow j=7 \Rightarrow \binom{10}{7}3^7=\boxed{262,440}$$

4. Find the coefficient of x^0 in the expansion of $\left(x-\frac{1}{x^2}\right)^9$.

$$\binom{9}{j}x^{9-j}x^{-2j}(-1)^j=\binom{9}{j}x^{9-3j}(-1)^j \Rightarrow j=3 \Rightarrow \binom{9}{3}(-1)^3=\boxed{-84}$$

Give the ***exact value*** *of the following.*

5. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\boxed{-\frac{\pi}{3}}$

6. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ $\boxed{\frac{5\pi}{6}}$

7. $\sec^{-1}(2)$ $\boxed{\frac{\pi}{3}}$

8. $\cos\left(\cos^{-1}(0)\right)$

$\boxed{0}$

9. $\csc^{-1}\left(\csc\left(\frac{3\pi}{4}\right)\right)$

$\boxed{\frac{\pi}{4}}$

10. $\sin\left(\cos^{-1}\left(-\frac{2}{3}\right)\right)$

$\boxed{\frac{\sqrt{5}}{3}}$

11. $\sin\left(2\cos^{-1}\left(\frac{4}{5}\right)\right)$ *{Hint:* $\sin 2A = 2\sin A\cos A$.*}*

$$2\sin\left(\cos^{-1}\left(\frac{4}{5}\right)\right)\cos\left(\cos^{-1}\left(\frac{4}{5}\right)\right)$$

$$= 2\cdot\frac{3}{5}\cdot\frac{4}{5} = \boxed{\frac{24}{25}}$$

12. $\sin\left[\sin^{-1}\left(-\frac{1}{4}\right) - \sin^{-1}\left(\frac{2}{3}\right)\right]$ *{Hint:* $\sin(A-B) = \sin A\cos B - \cos A\sin B$.*}*

$$\sin\left(\sin^{-1}\left(-\frac{1}{4}\right)\right)\cos\left(\sin^{-1}\left(\frac{2}{3}\right)\right) - \cos\left(\sin^{-1}\left(-\frac{1}{4}\right)\right)\sin\left(\sin^{-1}\left(\frac{2}{3}\right)\right)$$

$$= -\frac{1}{4}\cdot\frac{\sqrt{5}}{3} - \frac{\sqrt{15}}{4}\cdot\frac{2}{3} = \boxed{\frac{-\sqrt{5}-2\sqrt{15}}{12}}$$

13. $\tan\left(\frac{1}{2}\sin^{-1}\left(-\frac{1}{3}\right)\right)$ *{Hint:* $\tan\frac{A}{2} = \frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$.*}*

$$\frac{\sin\left(\sin^{-1}\left(-\frac{1}{3}\right)\right)}{1+\cos\left(\sin^{-1}\left(-\frac{1}{3}\right)\right)} \quad \textit{or} \quad \frac{1-\cos\left(\sin^{-1}\left(-\frac{1}{3}\right)\right)}{\sin\left(\sin^{-1}\left(-\frac{1}{3}\right)\right)}$$

$$= \frac{-\frac{1}{3}}{1+\frac{\sqrt{8}}{3}} = \boxed{-\frac{1}{3+\sqrt{8}}} \qquad = \frac{1-\frac{\sqrt{8}}{3}}{-\frac{1}{3}} = \boxed{\sqrt{8}-3}$$

Exactly solve the following trigonometric equations on the interval $[0, 2\pi)$.

14. $\cos^2 x = 1$

$\cos x = \pm 1$

$x = \boxed{0, \pi}$

15. $3\sin^2 x + 2\sin x - 1 = 0$

$(3\sin x - 1)(\sin x + 1) = 0$

$\sin x = \frac{1}{3}, \sin x = -1$

$x = \boxed{\sin^{-1}\left(\frac{1}{3}\right), \pi - \sin^{-1}\left(\frac{1}{3}\right), \frac{3\pi}{2}}$

16. $\csc^4(2x) = 4$

$\sin 2x = \pm\frac{1}{\sqrt{2}}$

$2x = \frac{\pi}{4}, \frac{\pi}{4} + 2\pi$

$2x = \frac{3\pi}{4}, \frac{3\pi}{4} + 2\pi$

$2x = \frac{5\pi}{4}, \frac{5\pi}{4} + 2\pi$

$2x = \frac{7\pi}{4}, \frac{7\pi}{4} + 2\pi$

$x = \boxed{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}}$

17. $\sec\left(\frac{x}{3}\right) = \cos\left(\frac{x}{3}\right)$

$\cos\left(\frac{x}{3}\right) = \pm 1$

$\frac{x}{3} = 0, \pi$

$x = \boxed{0}$

18. $\sin x = \sin 2x$ {*Hint:* $\sin 2A = 2\sin A\cos A.$}

$\sin x = 2\sin x\cos x$

$2\sin x\left(\cos x - \frac{1}{2}\right) = 0$

$\sin x = 0, \cos x = \frac{1}{2}$

$x = \boxed{0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}}$

19. $\cos 2x - \cos x = 0$ {*Hint:* $\cos 2A = 2\cos^2 A - 1.$}

$2\cos^2 x - \cos x - 1 = 0$

$(2\cos x + 1)(\cos x - 1) = 0$

$\cos x = -\frac{1}{2}, \cos x = 1$

$x = \boxed{\frac{2\pi}{3}, \frac{4\pi}{3}, 0}$

20. $\sin 2x = 2\cos^2 x$ {*Hint:* $\sin 2A = 2\sin A\cos A.$}

$2\sin x\cos x = 2\cos^2 x$

$2\cos x(\sin x - \cos x) = 0$

$\cos x = 0, \sin x = \cos x$

$x = \boxed{\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{4}}$

21. $\sqrt{2}\sin 3x - 1 = 0$

$$\sin 3x = \frac{1}{\sqrt{2}}$$

$$3x = \frac{\pi}{4}, \frac{\pi}{4} + 2\pi, \frac{\pi}{4} + 4\pi$$

$$3x = \frac{3\pi}{4}, \frac{3\pi}{4} + 2\pi, \frac{3\pi}{4} + 4\pi$$

$$x = \boxed{\frac{\pi}{12}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{11\pi}{12}, \frac{17\pi}{12}, \frac{19\pi}{12}}$$

22. $\cos\frac{x}{2} = 1$

$$\frac{x}{2} = 0$$

$$x = \boxed{0}$$

23. $\sqrt{3}\sin x + \cos x = 1$

$$\left(\sqrt{3}\sin x + \cos x\right)^2 = 1$$

$$3\sin^2 x + 2\sqrt{3}\sin x\cos x + \cos^2 x = 1$$

$$2\sin^2 x + 2\sqrt{3}\sin x\cos x = 0$$

$$2\sin x\cos x\left(\tan x + \sqrt{3}\right) = 0$$

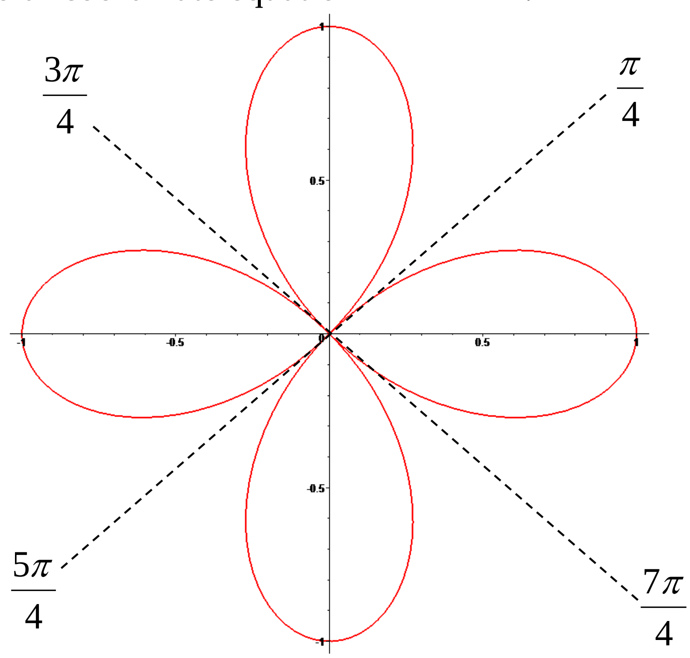
$$\sin 2x = 0, \tan x = -\sqrt{3}$$

$$2x = 0, \pi, 2\pi, 3\pi \Rightarrow x = 0, \tfrac{\pi}{2}, \pi, \tfrac{3\pi}{2}$$

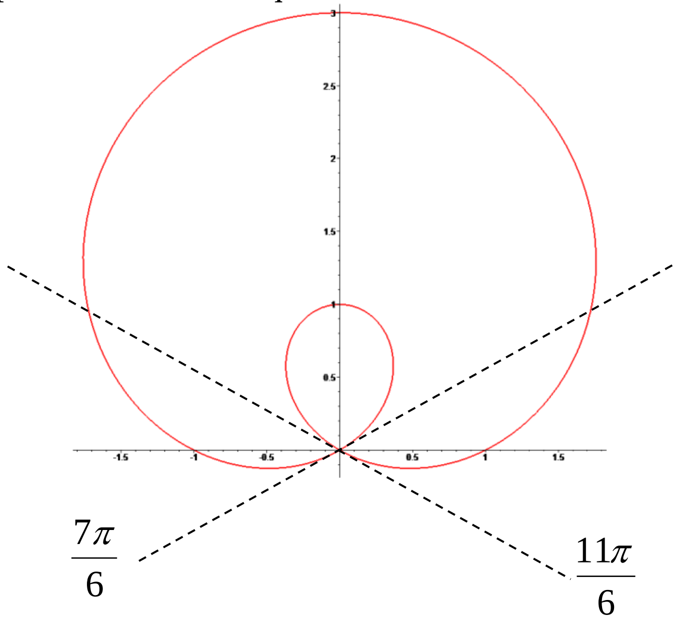
$$x = \tfrac{2\pi}{3}, \tfrac{5\pi}{3}$$

The ones that check out are $\boxed{0 \text{ and } \tfrac{2\pi}{3}}$.

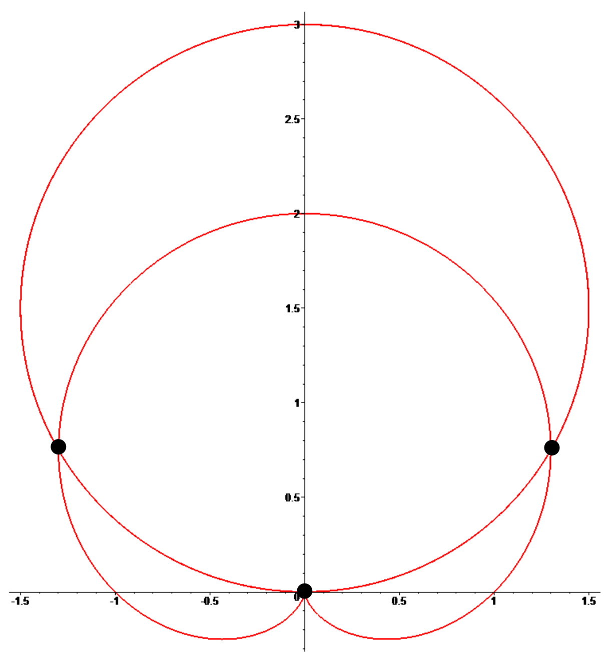
24. Sketch the graph of the polar coordinate equation $r = \cos 2\theta$.



25. Sketch the graph of the polar coordinate equation $r = 1 + 2\sin\theta$.

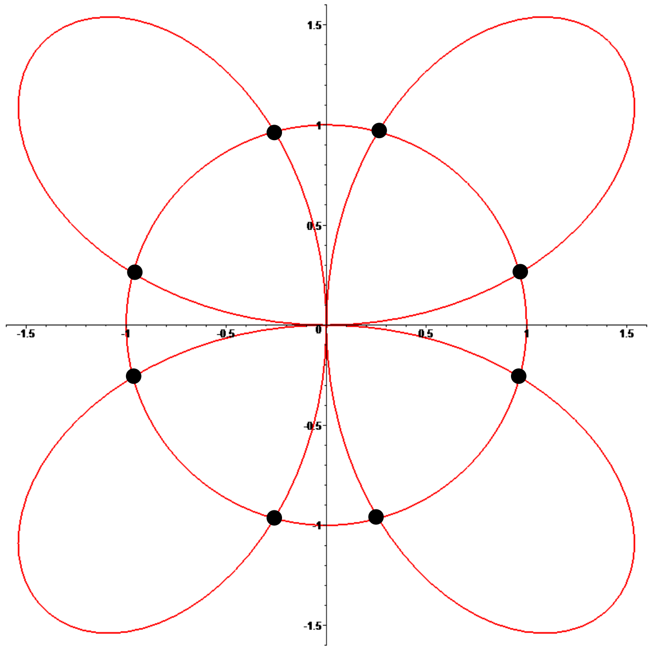


26. Find the points of intersection of the solution curves of the polar coordinate equations $r = 1 + \sin\theta$ and $r = 3\sin\theta$.



$$1 + \sin\theta = 3\sin\theta \Rightarrow \sin\theta = \tfrac{1}{2} \Rightarrow \theta = \tfrac{\pi}{6}, \tfrac{5\pi}{6} \Rightarrow \boxed{(0,0), \left(\tfrac{3}{2}, \tfrac{\pi}{6}\right), \left(\tfrac{3}{2}, \tfrac{5\pi}{6}\right)}$$

27. Find the points of intersection of the solution curves of the polar coordinate equations $r = 2\sin 2\theta$ and $r = 1$.



$$2\sin 2\theta = 1 \Rightarrow \sin 2\theta = \tfrac{1}{2} \Rightarrow 2\theta = \tfrac{\pi}{6}, \tfrac{\pi}{6} + 2\pi, \ldots \text{ or } \tfrac{5\pi}{6}, \tfrac{5\pi}{6} + 2\pi, \ldots \Rightarrow \theta = \tfrac{\pi}{12}, \tfrac{\pi}{12} + \pi, \ldots \text{ or } \tfrac{5\pi}{12}, \tfrac{5\pi}{12} + \pi, \ldots$$

$$\boxed{\left(1, \frac{\pi}{12}\right), \left(1, \frac{5\pi}{12}\right), \left(1, \frac{7\pi}{12}\right), \left(1, \frac{11\pi}{12}\right), \left(1, \frac{13\pi}{12}\right), \left(1, \frac{17\pi}{12}\right), \left(1, \frac{19\pi}{12}\right), \left(1, \frac{23\pi}{12}\right)}$$

28. Find zw and $\dfrac{z}{w}$ for $z = 4\left(\cos\frac{3\pi}{8} + i\sin\frac{3\pi}{8}\right)$ and $w = 2\left(\cos\frac{9\pi}{16} + i\sin\frac{9\pi}{16}\right)$. Leave your answers in polar form.

$$zw = 8\left[\cos\left(\tfrac{3\pi}{8} + \tfrac{9\pi}{16}\right) + i\sin\left(\tfrac{3\pi}{8} + \tfrac{9\pi}{16}\right)\right]$$

$$= \boxed{8\left(\cos\tfrac{15\pi}{16} + i\sin\tfrac{15\pi}{16}\right)}$$

$$\frac{z}{w} = 2\left[\cos\left(\tfrac{3\pi}{8} - \tfrac{9\pi}{16}\right) + i\sin\left(\tfrac{3\pi}{8} - \tfrac{9\pi}{16}\right)\right]$$

$$= 2\left(\cos\tfrac{-3\pi}{16} + i\sin\tfrac{-3\pi}{16}\right)$$

$$= \boxed{2\left(\cos\tfrac{29\pi}{16} + i\sin\tfrac{29\pi}{16}\right)}$$

29. Write the equivalent standard form of $\left[\sqrt{3}\left(\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}\right)\right]^6$.

$$\left[\sqrt{3}\left(\cos\tfrac{5\pi}{18}+i\sin\tfrac{5\pi}{18}\right)\right]^6$$

$$\left(\sqrt{3}\right)^6\left[\cos\left(6\cdot\tfrac{5\pi}{18}\right)+i\sin\left(6\cdot\tfrac{5\pi}{18}\right)\right]$$

$$27\left(\cos\tfrac{5\pi}{3}+i\sin\tfrac{5\pi}{3}\right)$$

$$27\left(\tfrac{1}{2}-i\tfrac{\sqrt{3}}{2}\right)$$

$$\boxed{\tfrac{27}{2}-i\tfrac{27\sqrt{3}}{2}}$$

30. Write the equivalent standard form of $\left(\sqrt{3}+i\right)^9$.

$$\left(\sqrt{3}+i\right)^9$$

$$\left[2\left(\tfrac{\sqrt{3}}{2}+i\tfrac{1}{2}\right)\right]^9$$

$$\left[2\left(\cos\tfrac{\pi}{6}+i\sin\tfrac{\pi}{6}\right)\right]^9$$

$$2^9\left(\cos\tfrac{3\pi}{2}+i\sin\tfrac{3\pi}{2}\right)$$

$$512\left(0-i\right)$$

$$\boxed{-512i}$$