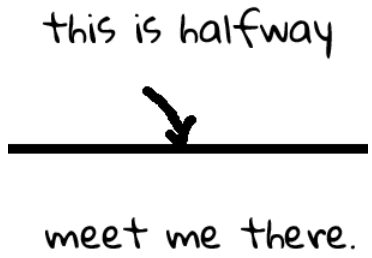
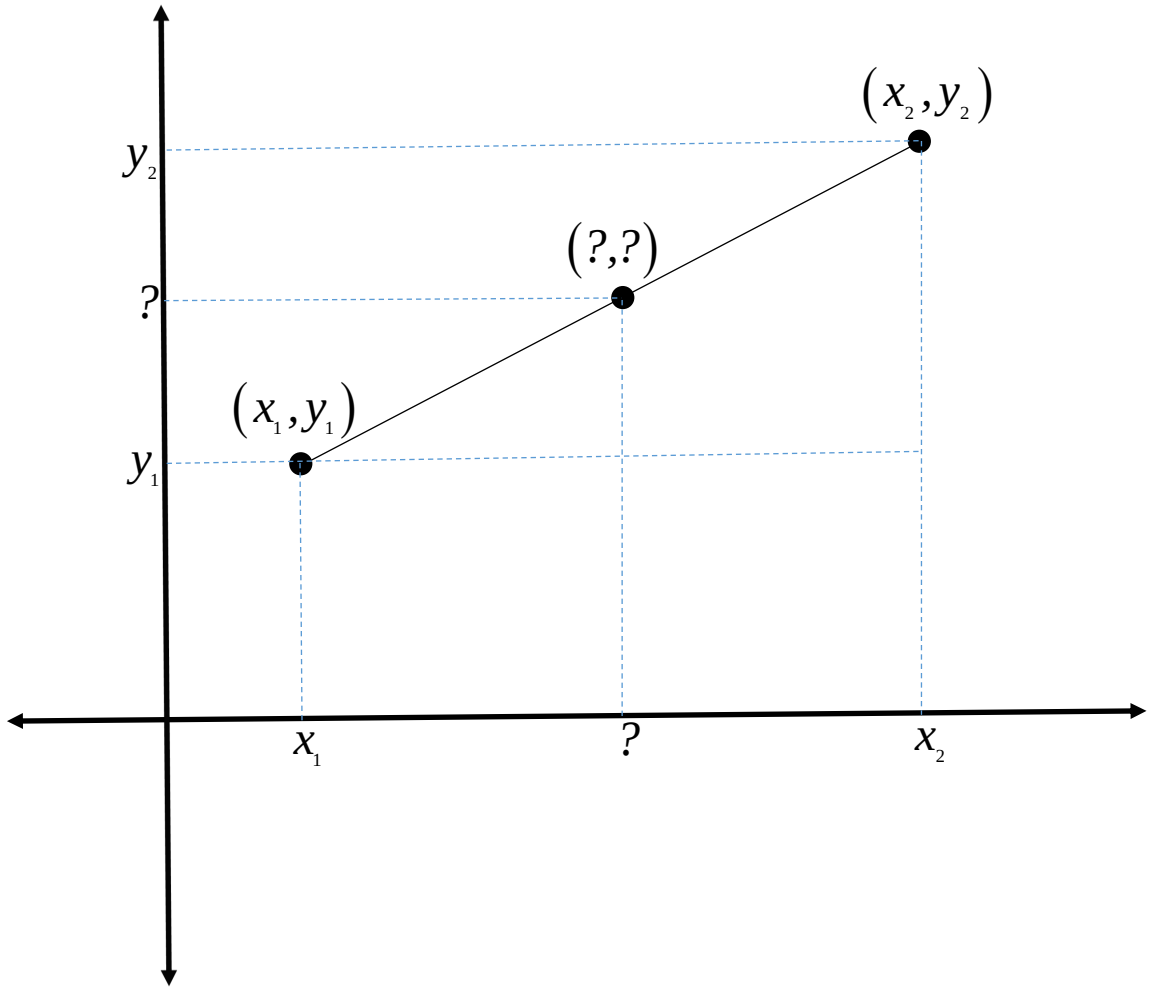
Midpoint Formula:

Given two points (x_1, y_1) and (x_2, y_2), the point on the line segment connecting them that is halfway between them is called their midpoint.





So $\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$**.**

Examples: Find the midpoints of the following pairs of points.

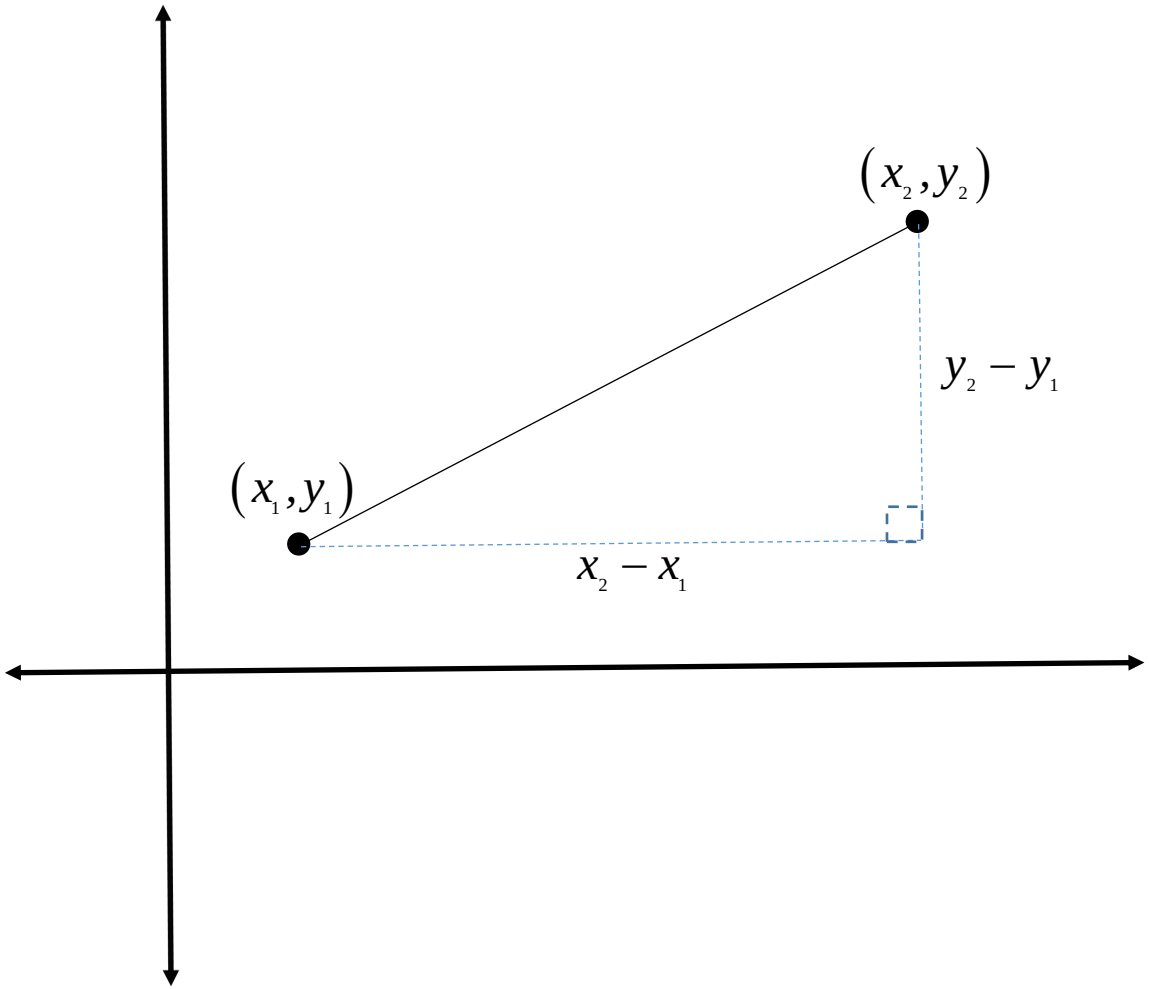
1. $(1,2)$ and $(3,4)$

2. $(1,-2)$ and $(-3,4)$

3. $(1,-2)$ and $(4,-6)$

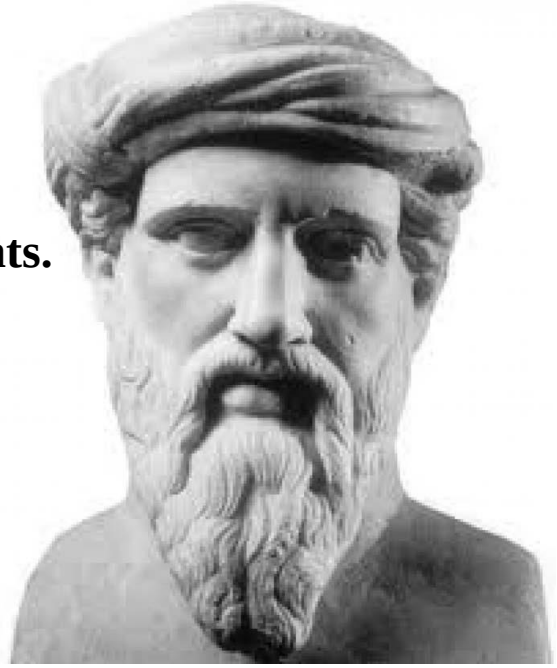
Distance Formula:

Given two points (x_1, y_1) and (x_2, y_2), the length of the line segment connecting them is the distance between the two points.



"I thought you measured the distance to the ground. I hope Bob's okay."

From the Pythagorean Theorem, we get that $\left(\text{Distance}\right)^2 = \left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2$**, so we get that** $\text{Distance} = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$**.**



Examples: Find the distance between the following pairs of points.

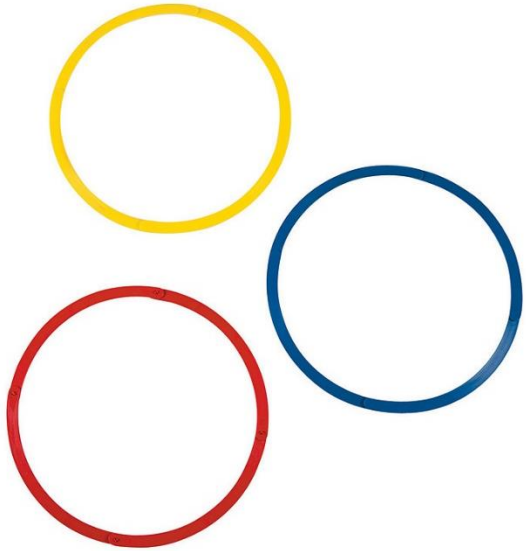
1. $\left(1,2\right)$ and $\left(4,6\right)$

2. $\left(1,-2\right)$ and $\left(-4,10\right)$

3. $\left(1,-2\right)$ and $\left(4,-5\right)$

Equations of Circles:

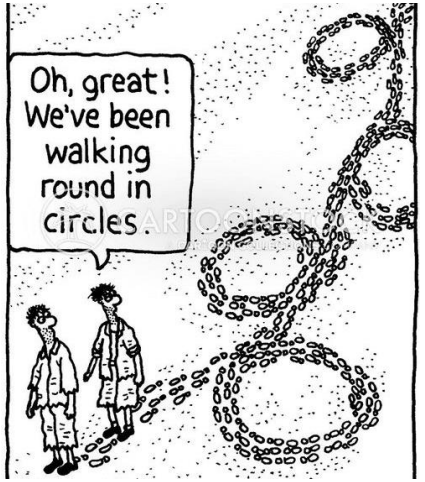
Let's find an equation for a circle with center of (h,k) and radius of *r*. It would be the set of points (x,y) whose distance to (h,k) is equal to *r*.



$$\sqrt{(x-h)^2+(y-k)^2}=r$$

$$(x-h)^2+(y-k)^2=r^2$$

This is called the standard form of the equation of a circle.



Examples: Find equations for the circles described.

1. center of $(1,2)$ and radius of 3

2. center of $(-1,2)$ and radius of $\sqrt{3}$

3. center of $(0,-2)$ and radius of 5

4. $(1,2)$ and $(3,4)$ are the endpoints of a diameter

There is another form of the equation of a circle called the general form.

$$x^2 + y^2 + cx + dy + e = 0$$

The graph of the solutions of an equation of this form can be a circle, a point, or nothing at all.

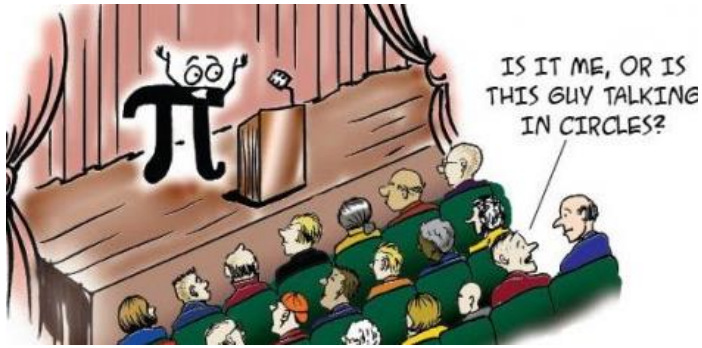
To determine the nature of the graph, you complete the square in both *x* and *y* to convert the equation to standard form.

$$(x-h)^2 + (y-k)^2 = c$$

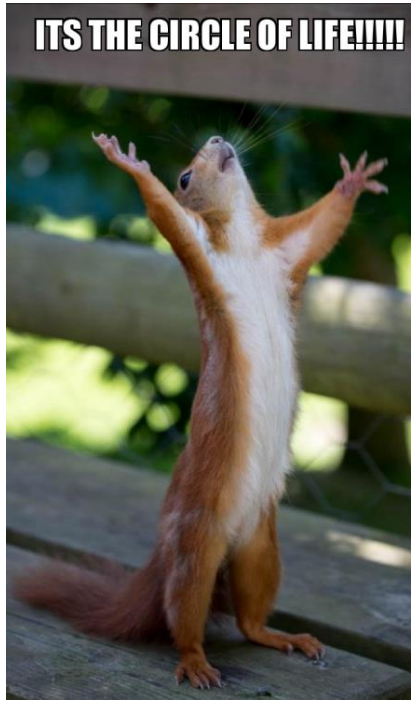
If $c > 0$, then the graph is a circle.

If $c = 0$, then the graph is a point.

If $c < 0$, then there is no graph.



Examples: **Determine if the graph of the given equation is a circle, a point, or nothing at all. If it's a circle, give its center and radius. If it's a point give the coordinates.**



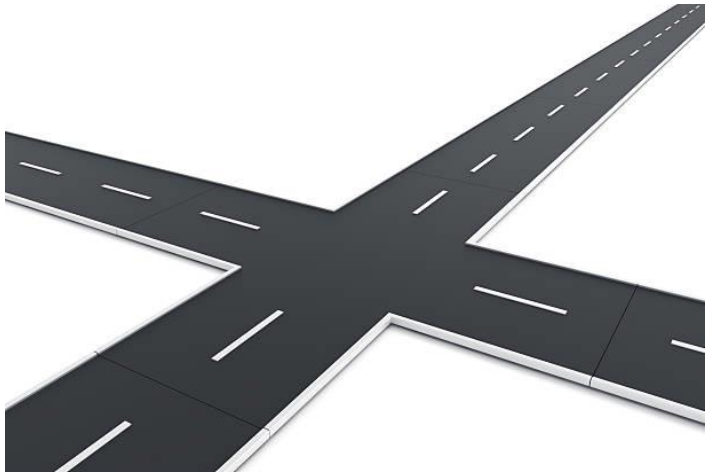
1. $x^2 + y^2 - 8x + 2y - 19 = 0$

2. $x^2 + y^2 + 2x - 6y + 10 = 0$

3. $x^2 + y^2 - 4x - 8y + 21 = 0$

Points of Intersection:

Find the points of intersection of the graphs of the solutions of the given pair of equations.



1. $x^2 + y^2 = 1$ and $x + y = 1$

2. $x^2 + y^2 = 1$ and $(x-1)^2 + y^2 = 1$