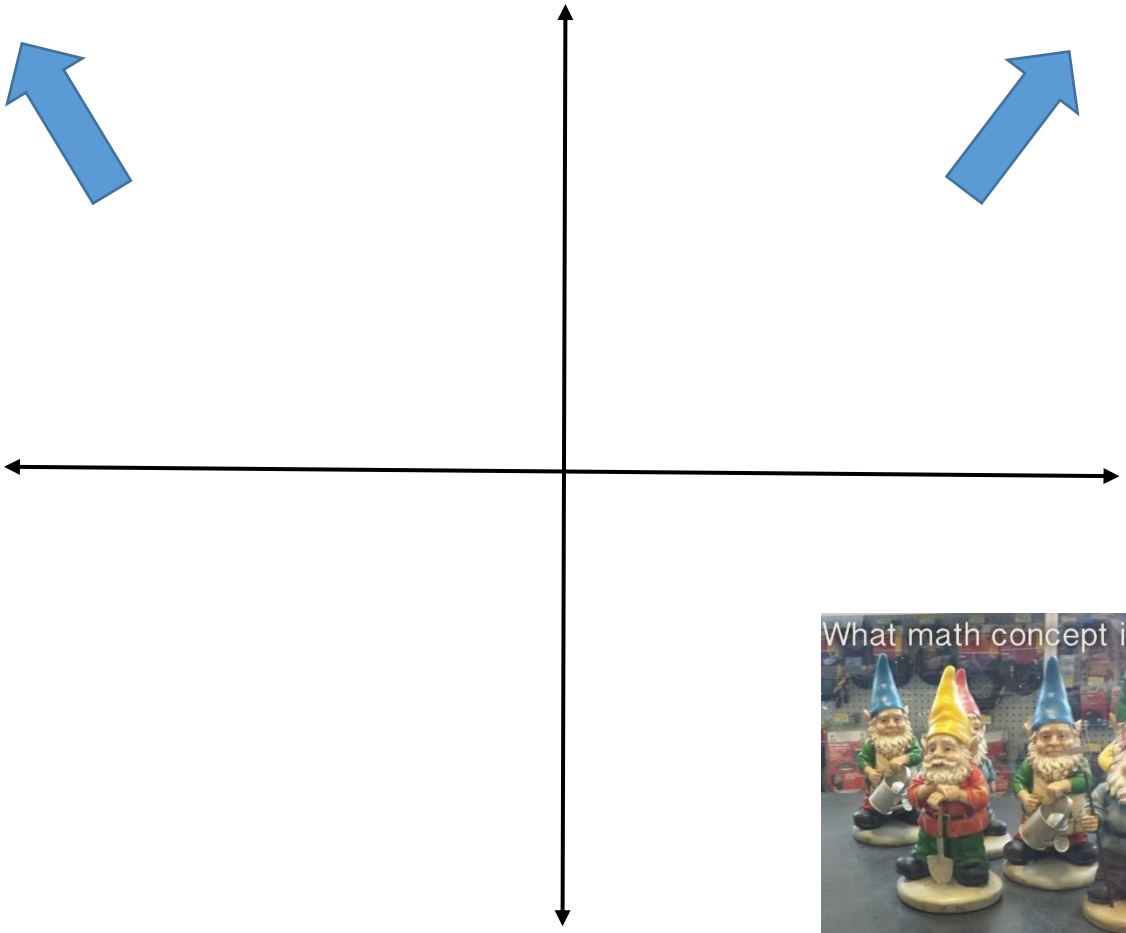
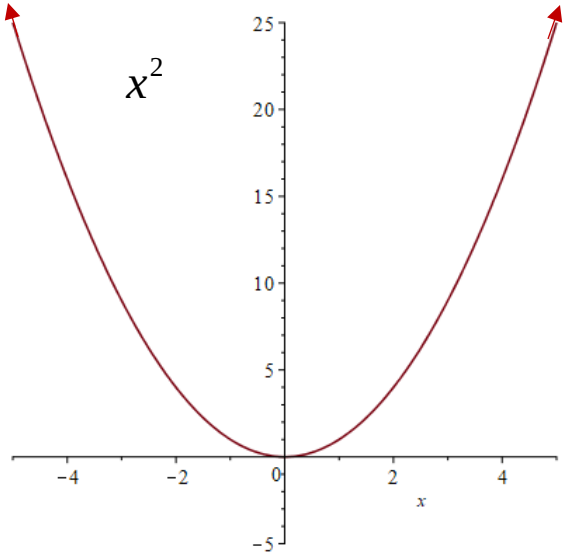
Graphing Polynomial Functions:

The Leading Coefficient Test and End Behavior:

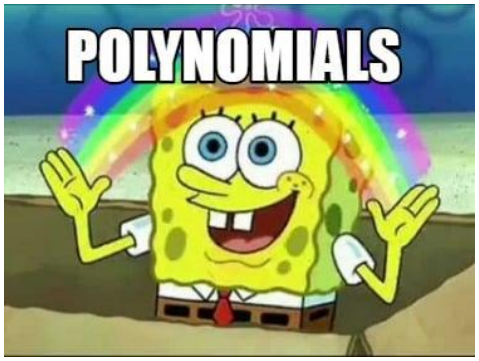
For an n^{th} –degree polynomial function $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ with $a_n \neq 0$,

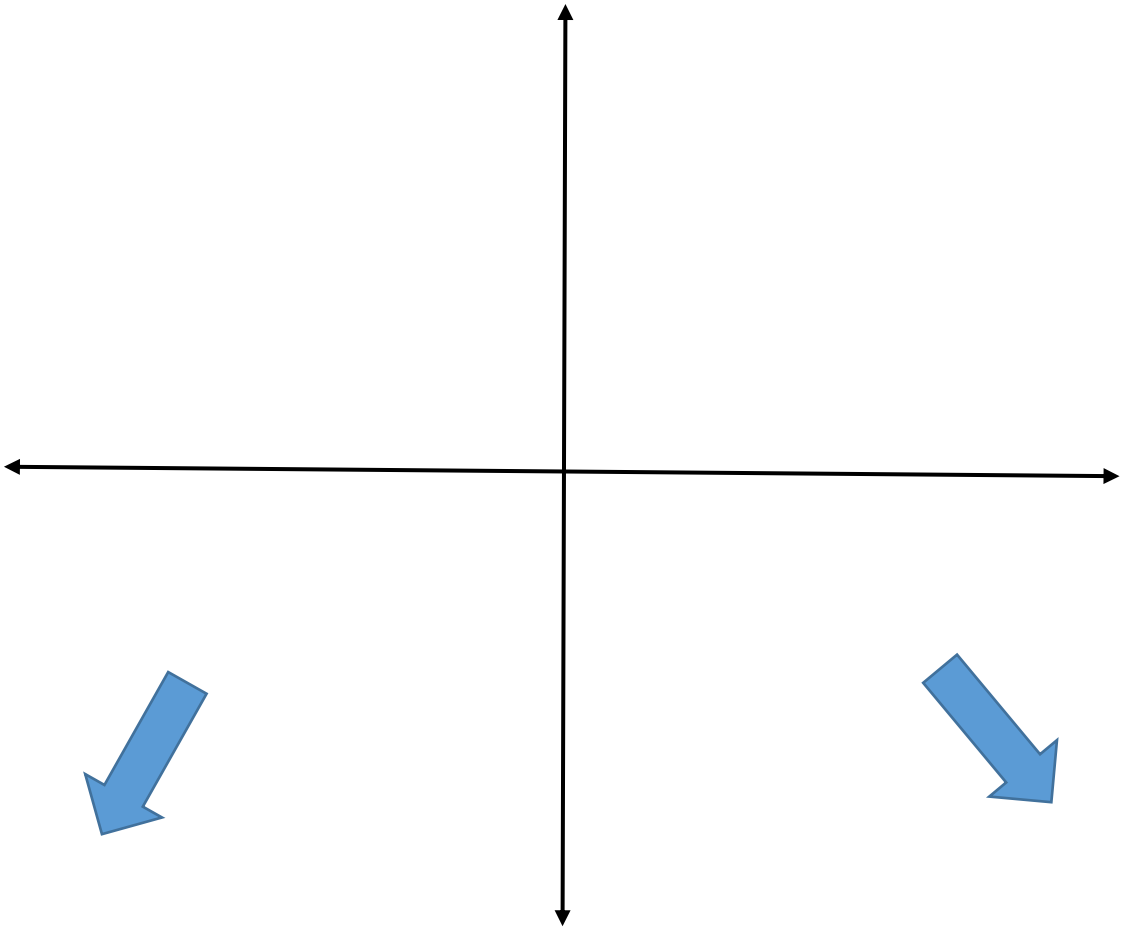
If *n* is even and $a_n > 0$, then

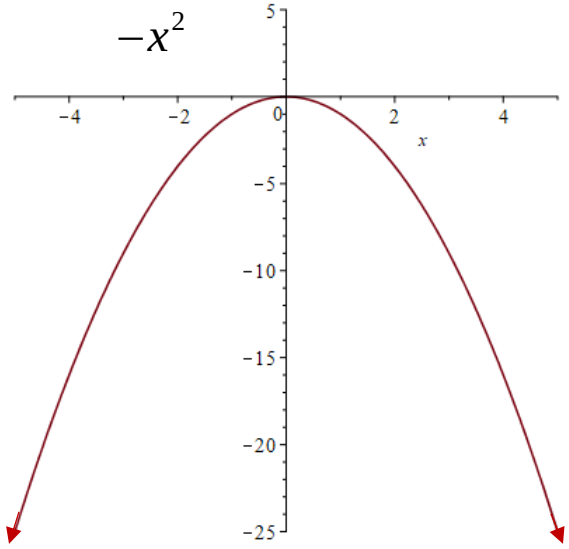




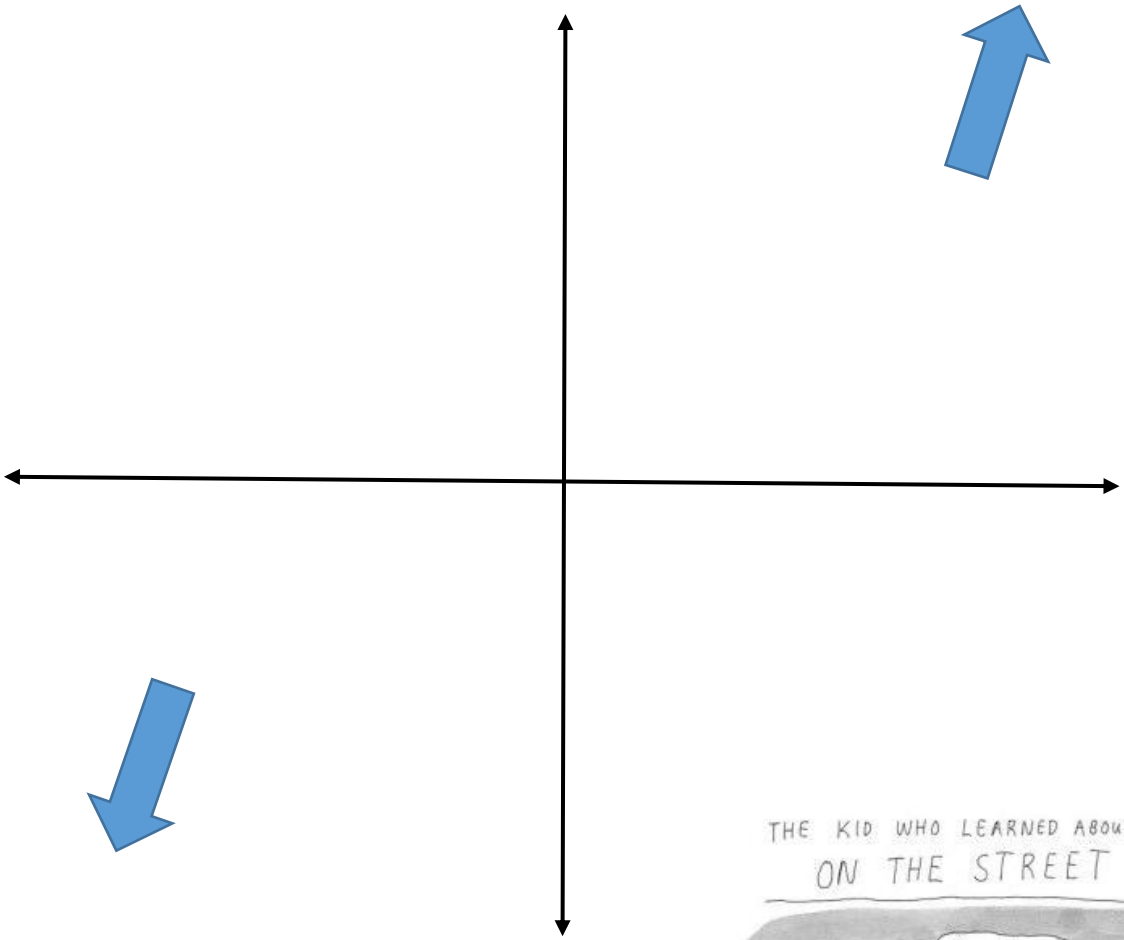
If *n* is even and $a_n < 0$, then

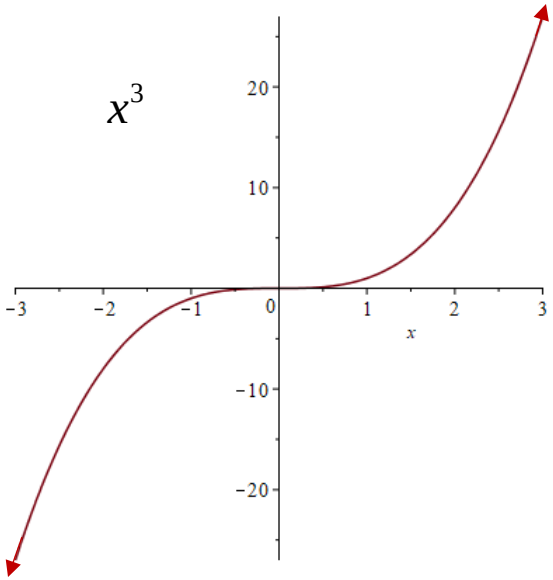




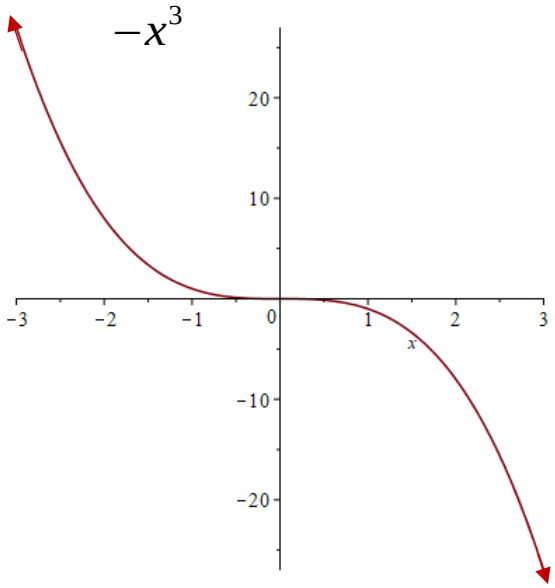


If *n* is odd and $a_n > 0$, then





If *n* is odd and $a_n < 0$, then



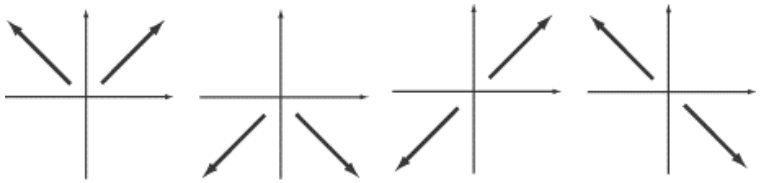
Determine the end behavior of the following polynomial functions.

1. $f(x) = 4x - x^3$ **Left:** **Right:**

2. $f(x) = 2x^4 + 12x - 4$ **Left:** **Right:**

3. $f(x) = x^3 + 2x^2 - 8x$ **Left:** **Right:**

4. $f(x) = 4x - x^6$ **Left:** **Right:**

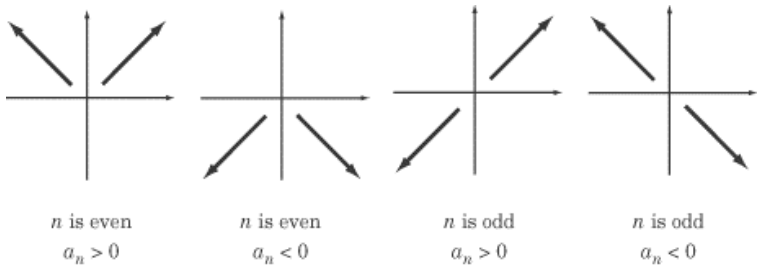


5. $f(x)=x^2(x-3)$ **Left:** **Right:**

6. $f(x)=-2(x+2)(x-2)^3$ **Left:** **Right:**

7. $f(x)=(x+1)^2(x-2)^2$ **Left:** **Right:**

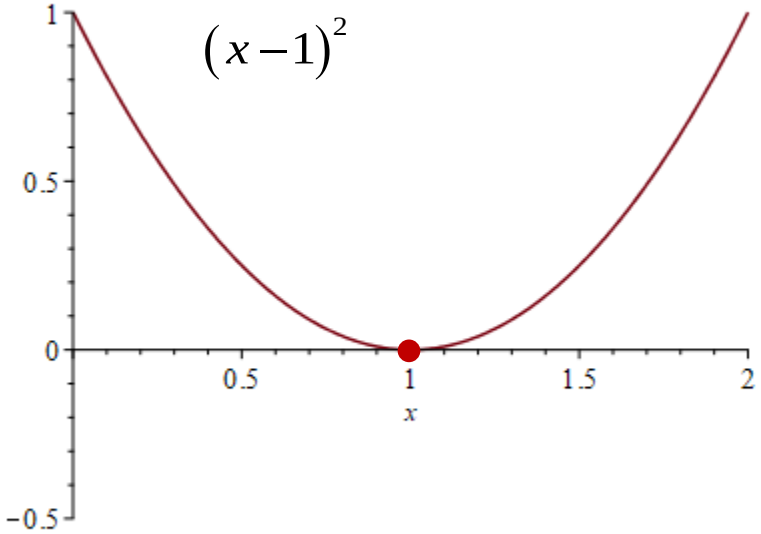
8. $f(x)=-2(x+2)^2(x-2)^3$ **Left:** **Right:**

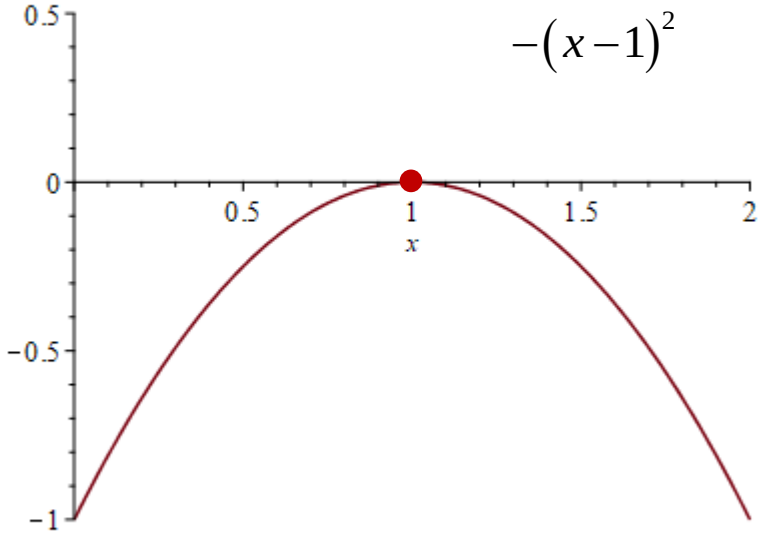


Behavior at the x-intercepts:

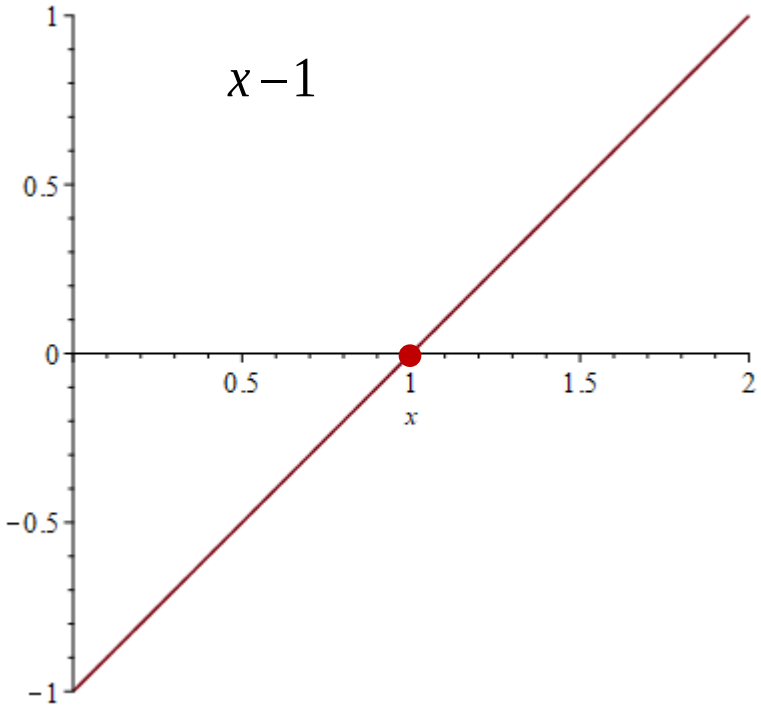
If $(x-c)^k$ is the highest power of $(x-c)$ that is a factor of $f(x)$, with *c* a real number, then

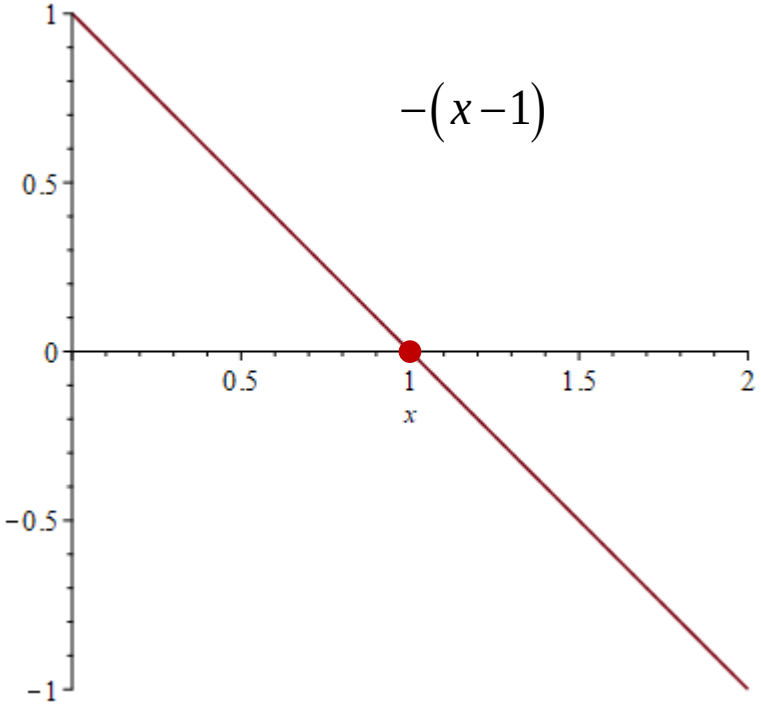
If *k* is even, then the graph touches the *x*-axis at *c* but doesn't cross the axis.



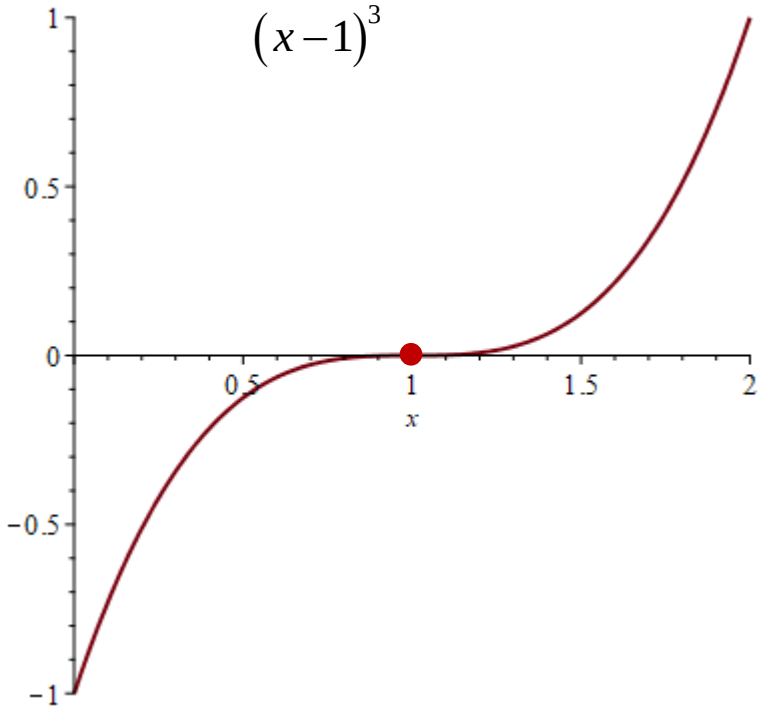


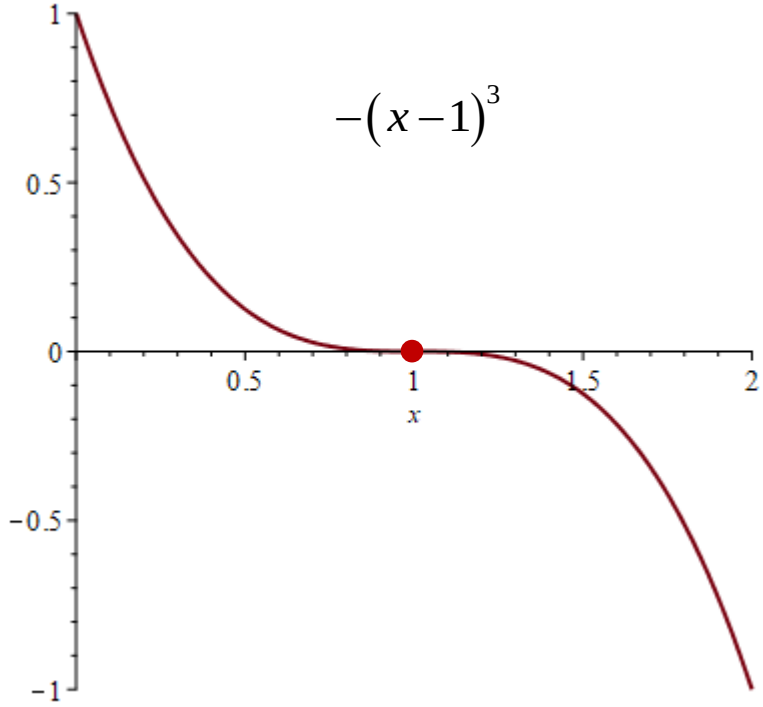
If *k* is 1, then the graph crosses the *x*-axis at *c* with a non-zero angle.





If *k* is odd and greater than 1, then the graph crosses the *x*-axis at *c* with a zero angle(flat).





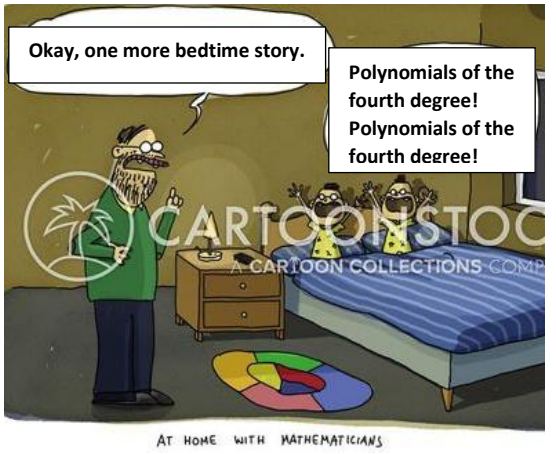
Steps for sketching graphs of polynomial functions:

1. Determine the end behavior, and indicate it on the graph with arrows.

2. Find all the real zeros(*x*-intercepts) of $f\left(x\right)$, and indicate them on the graph with points.

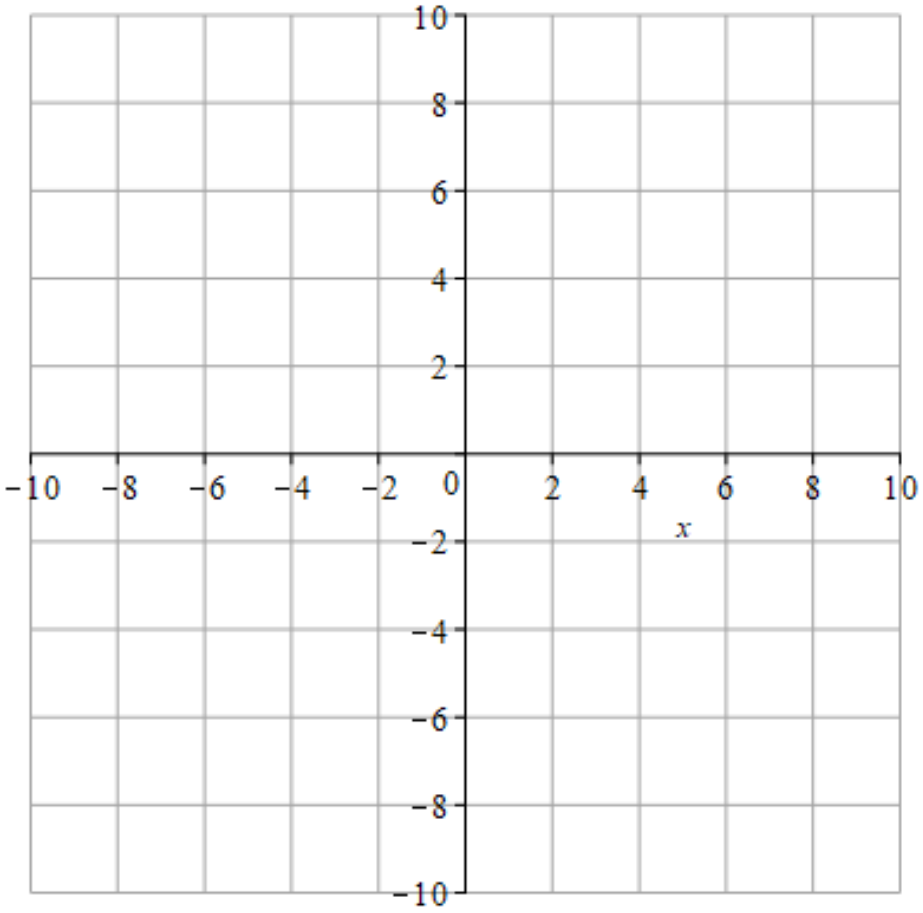
3. Find the *y*-intercept, and indicate it on the graph with a point.

4. Use the end behavior and *x*-intercept behavior to connect the previous points and arrows into a reasonable graph.

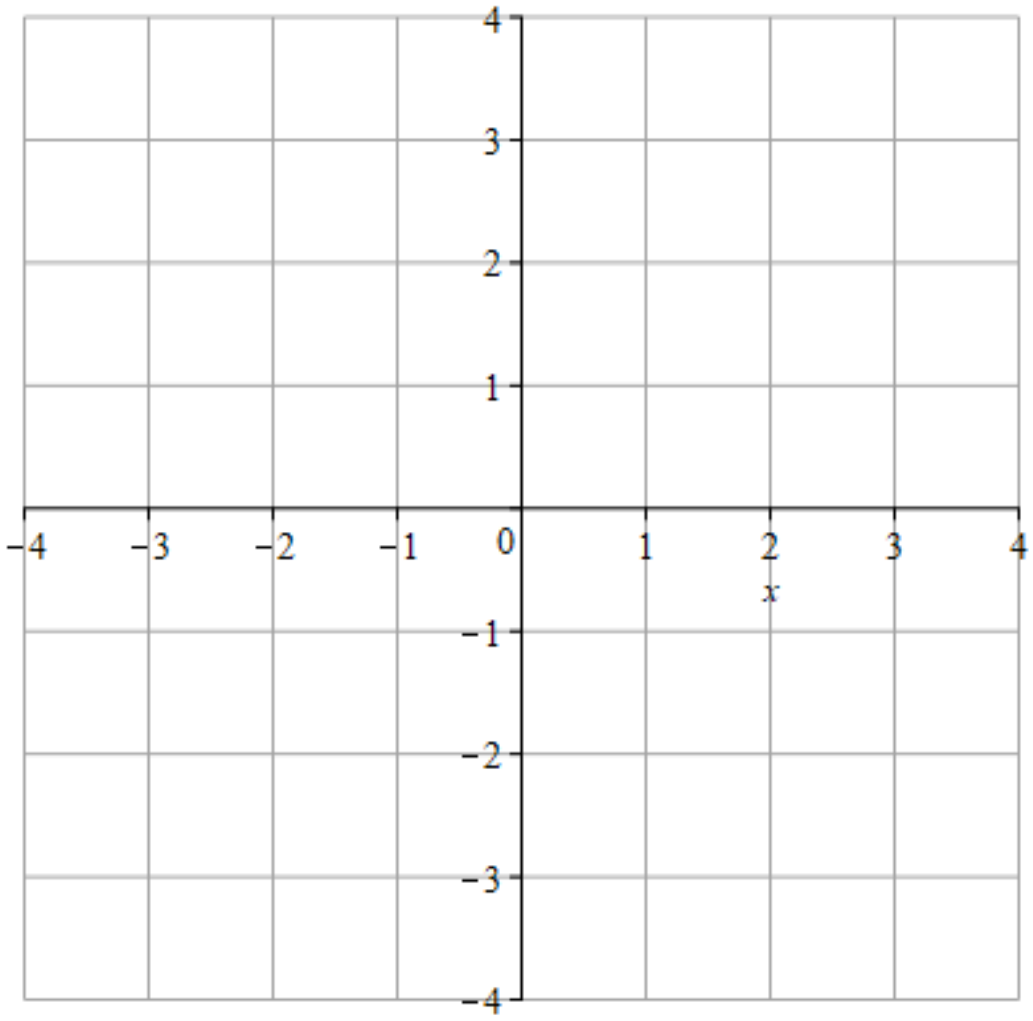


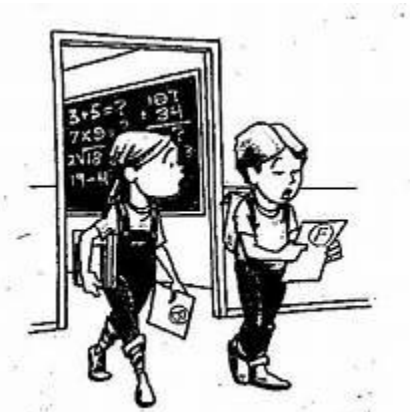
Sketch the graphs of the following polynomial functions.

1. $f(x)=\frac{1}{27}(x+4)(x-3)^3$

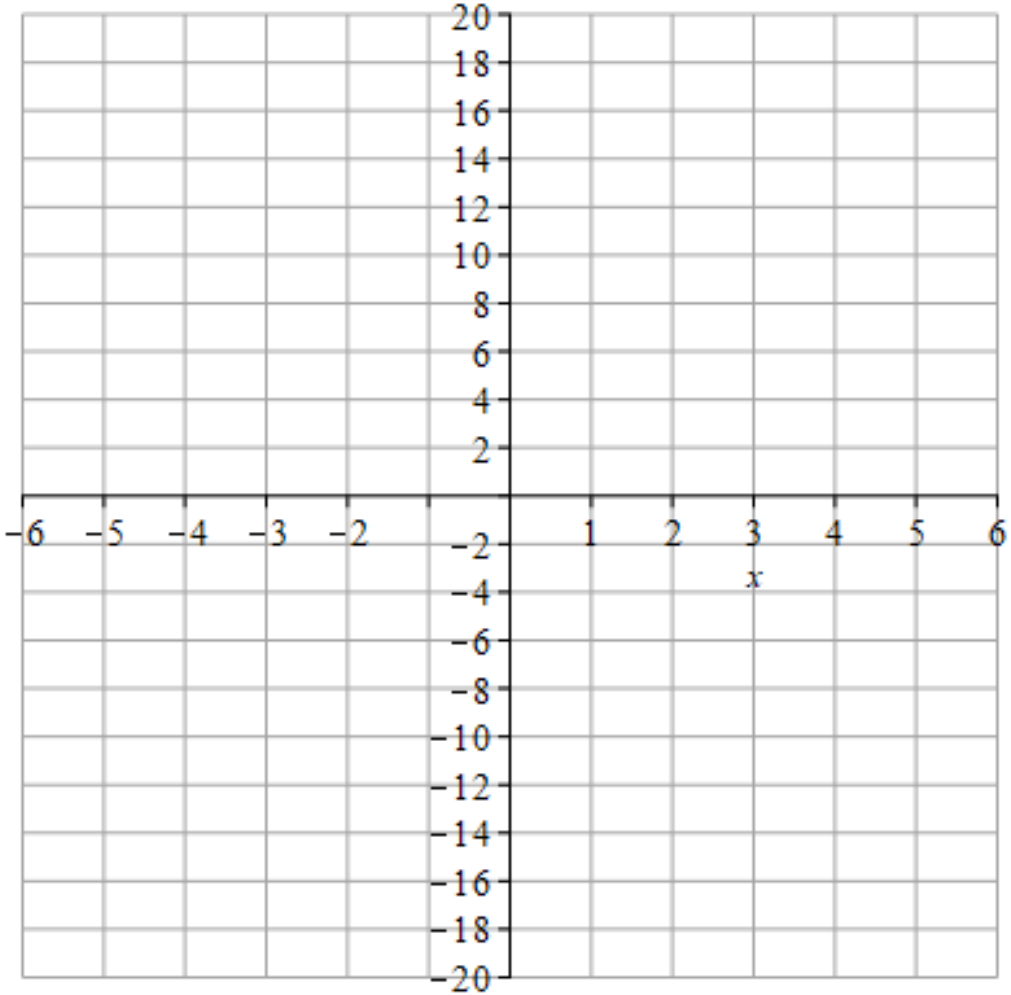


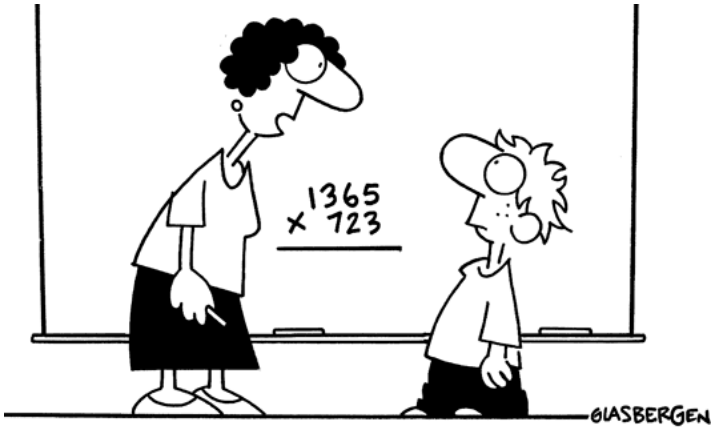
2. $f(x) = x^2(x-2)$


4
3
2
1
−4
−3
−2
−1
0
1
2
3
4
x
−1
−2
−3
−4


3+5=?
7×9
2√18
19-4
"It's not the math I hate...it's the *aftermath*."

3. $f(x) = -(x+2)(x-2)^3$

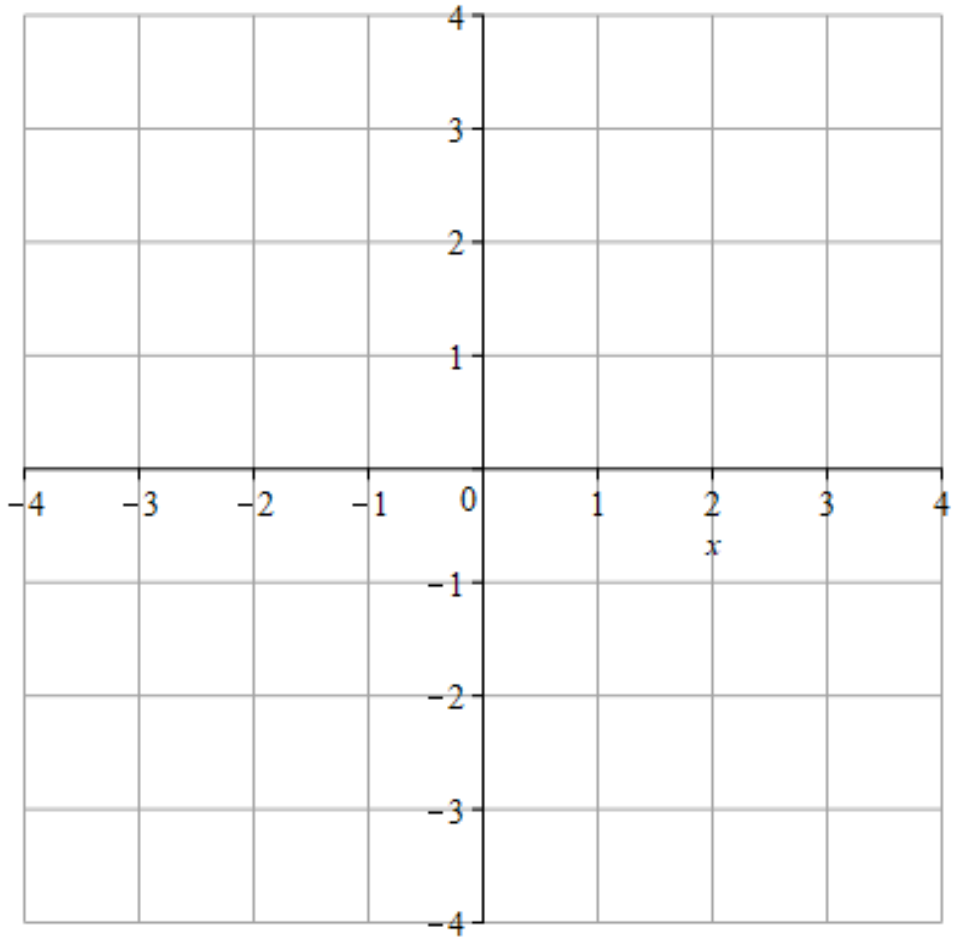


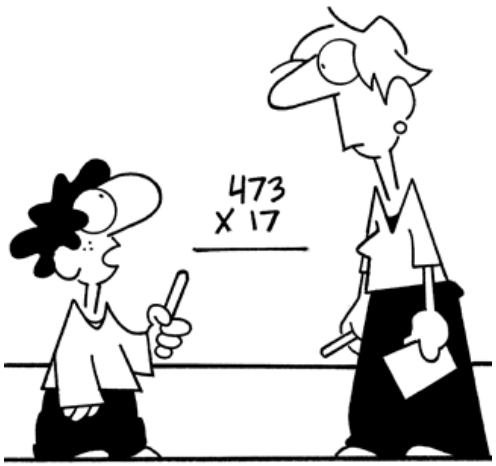


"Pretend you're starring in a reality show about a kid who can make his dreams come true if he works hard and gets good grades."

4. $f(x) = -\left(x^2 - 2\right)x^3$

{Factor first.}

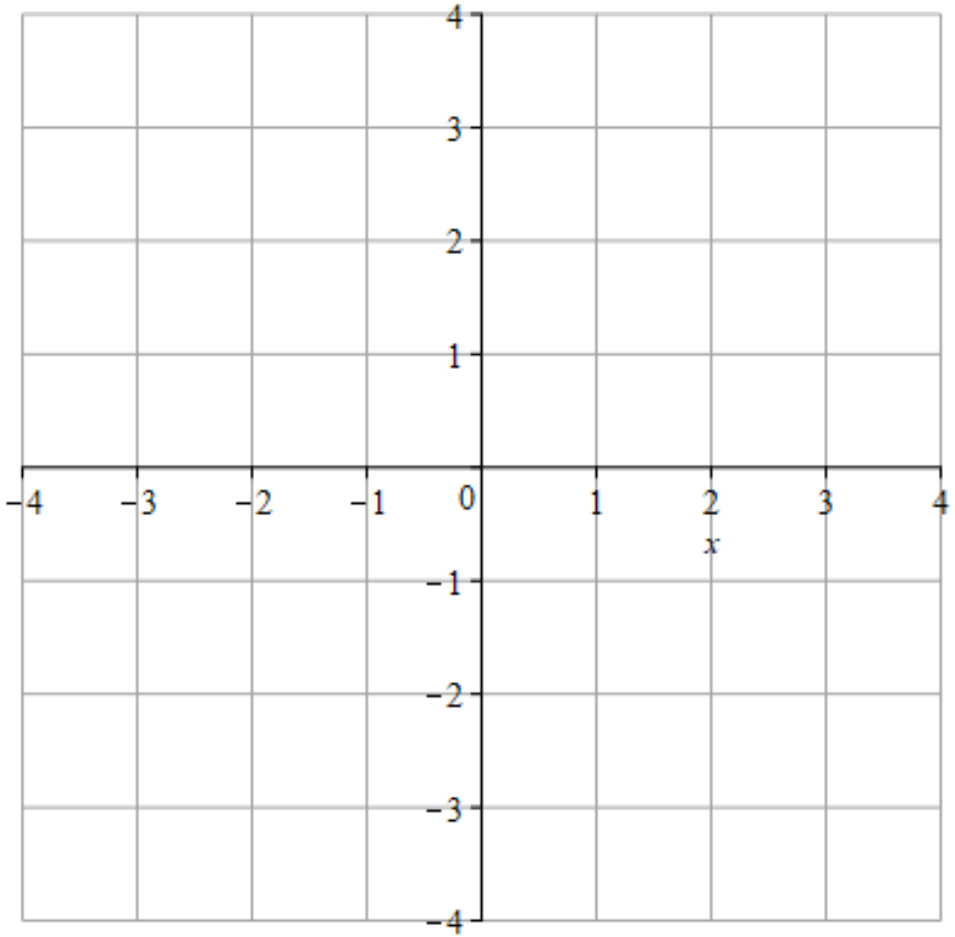


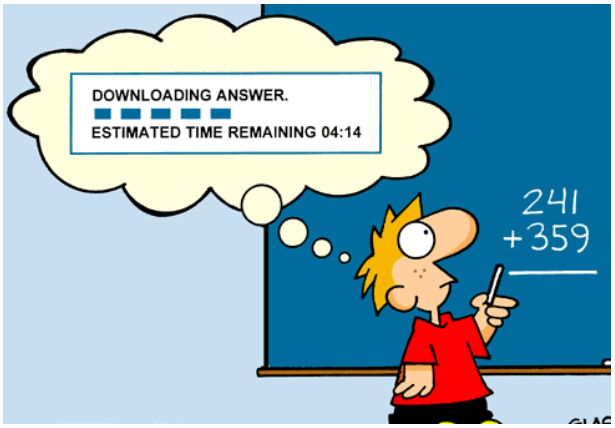


"If we learn from our mistakes, shouldn't I make as many mistakes as possible?"

5. $f(x) = x - x^3$

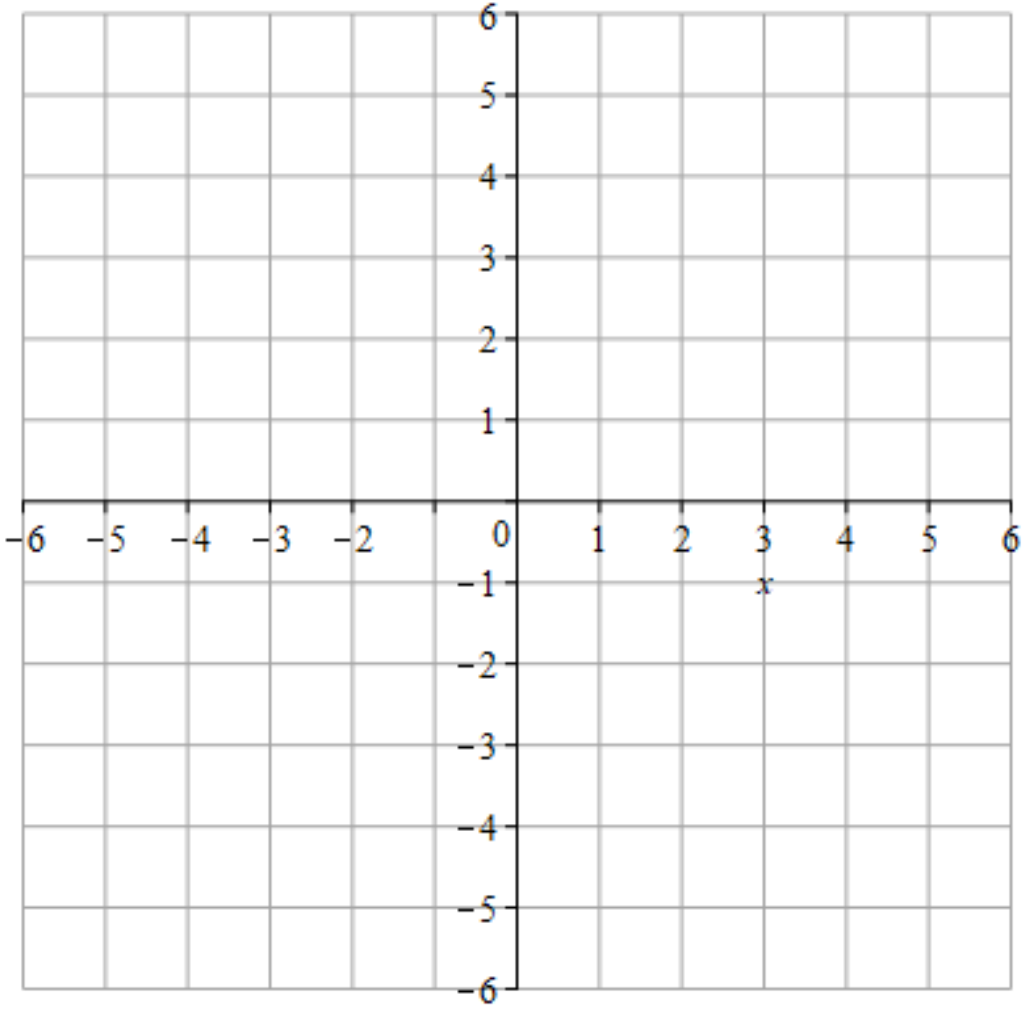
{Factor first.}

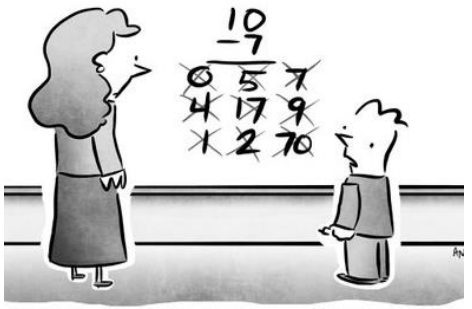




6. $f(x) = x^3 + 2x^2 - 8x$

{Factor first.}

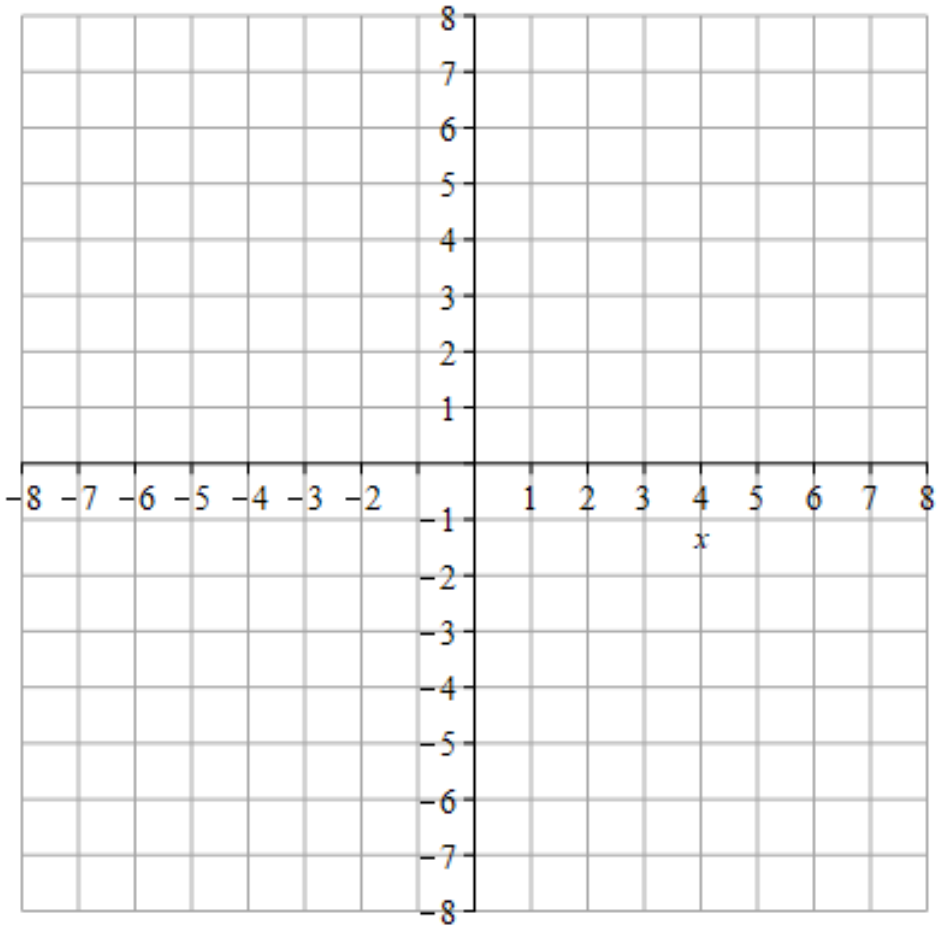


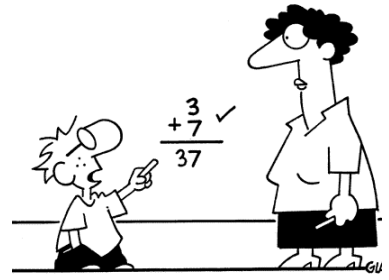


"OK, the good news is we've ruled these out."

7. $f(x) = 2x^4 + 12x^3 - 8x^2 - 48x$

{Factor first.}

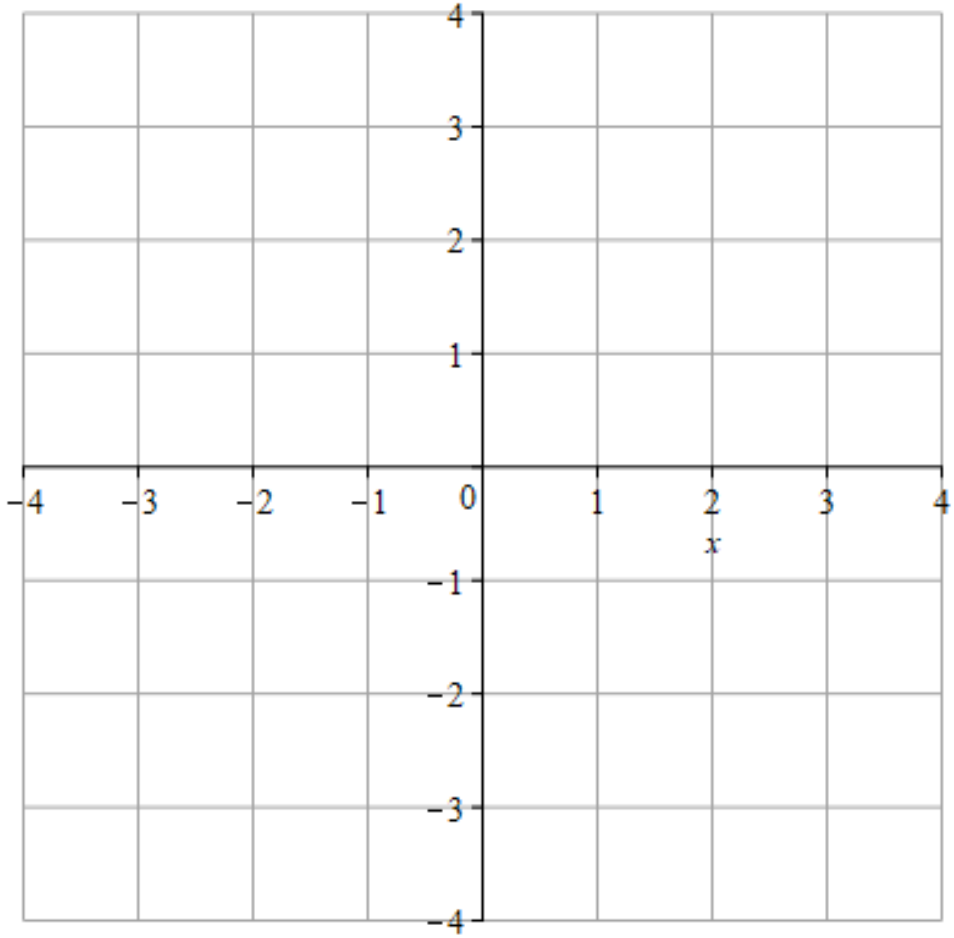




"In the corporate world they pay you big bucks for thinking outside of the box!"

8. $f(x) = x^2 - x^4$

{Factor first.}



things i haven't learned in school

how to:

pay bills
buy a house
apply for college

but thank jesus i can graph a polynomial function