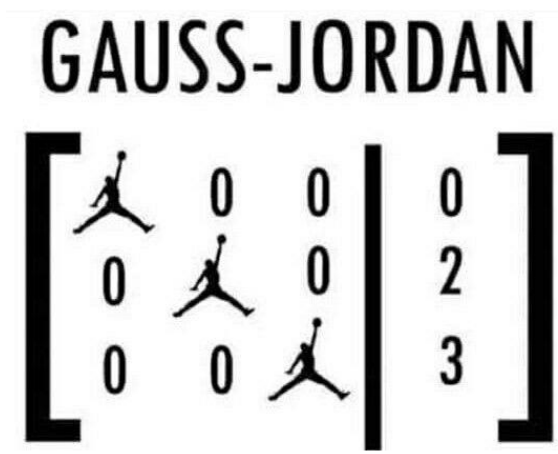
<u>Gauss-Jordan Elimination:</u>

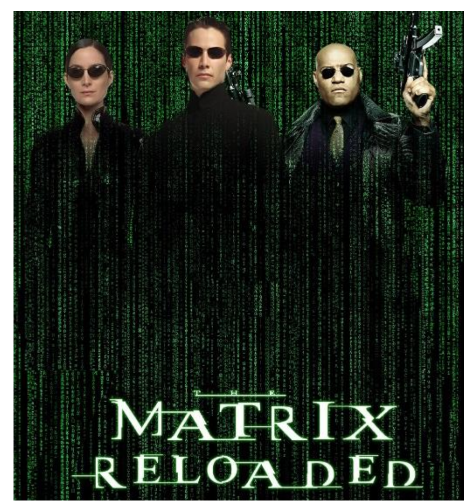
There is an extension of Gaussian Elimination called Gauss-Jordan Elimination.

In general, the goal is to use row operations to reach a matrix with the following form:

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & \cdots & 0 & \alpha \\ 0 & 1 & 0 & \cdots & 0 & \beta \\ 0 & 0 & 1 & 0 & \vdots & \gamma \\ \vdots & \vdots & 0 & 1 & 0 & \delta \\ \vdots & \vdots & \vdots & \vdots & & \varepsilon \\ 0 & 0 & 0 & 0 & & \phi \end{array}\right]$$

There are as many 1’s as possible on the diagonal with zeros both below the 1’s and above the 1’s.



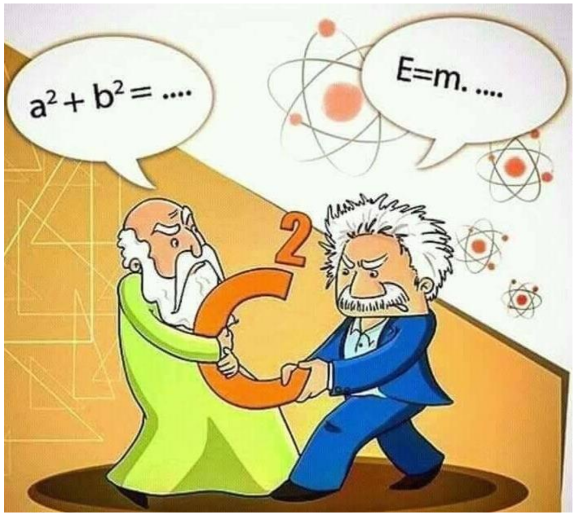


Examples:

1. $2x+3y=-1$
$3x-4y=7$

$$\left[\begin{array}{cc|c} 2 & 3 & -1 \\ 3 & -4 & 7 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cc|c} 3 & -4 & 7 \\ 2 & 3 & -1 \end{array}\right] \xrightarrow{-R_2+R_1 \to R_1} \left[\begin{array}{cc|c} 1 & -7 & 8 \\ 2 & 3 & -1 \end{array}\right] \xrightarrow{-2R_1+R_2 \to R_2} \left[\begin{array}{cc|c} 1 & -7 & 8 \\ 0 & 17 & -17 \end{array}\right] \xrightarrow{\frac{1}{17}R_2 \to R_2} \left[\begin{array}{cc|c} 1 & -7 & 8 \\ 0 & 1 & -1 \end{array}\right] \xrightarrow{7R_2+R_1 \to R_1} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right]$$

Now that the goal has been reached, you can easily see that the only solution is $x=1$ **and** $y=-1$**.**



2. $$\begin{aligned} x + y &= 1 \\ -2x - 2y &= 2 \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ -2 & -2 & 2 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 4 \end{array}\right]$$

$2R_1 + R_2 \rightarrow R_2$

Now that the goal has been reached, convert the last row back into an equation:

$$0 = 4.$$

Since this is impossible, the system has no solution. If at any time in the process of reaching the goal, you get a row with zeros to the left of the bar and a non-zero number to the right, you may stop and conclude that the system has no solution.

"There's supposed to be a fine line between a guess and an educated guess."

3. $$\begin{aligned} x+y&=1 \\ 3x+3y&=3 \end{aligned}$$

$$\left[\begin{array}{cc|c} 1 & 1 & 1 \\ 3 & 3 & 3 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$-3R_1 + R_2 \rightarrow R_2$

Now that the goal has been reached, convert the last row back into an equation:

$$0=0.$$

There is no contradiction, and we can't uniquely solve for the values of the variables from the first row(equation): $x+y=1$.

When this happens, the system has infinitely many solutions, and we represent them as follows:

Let *y* be an arbitrary real number, and solve for *x* in the first equation in terms of *y* to get $x+y=1 \Rightarrow x=1-y$.

The solutions of the system are given by $x=1-y, y=y$; where y is any real number.

4. $$\begin{aligned} x-2y+z&=2 \\ -3x+y+2z&=4 \end{aligned}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ -3 & 1 & 2 & 4 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ 0 & -5 & 5 & 10 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & -2 & 1 & 2 \\ 0 & 1 & -1 & -2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & -1 & -2 \end{array}\right]$$

$3R_1 + R_2 \rightarrow R_2$ $\qquad -\frac{1}{5}R_2 \rightarrow R_2$ $\qquad 2R_2 + R_1 \rightarrow R_1$

Now that the goal has been reached, notice that there are no contradictions, and we can't uniquely solve for the values of the variables. The system has infinitely many solutions, and we'll represent them by letting the variable furthest to the right, *z*, be an arbitrary real number.

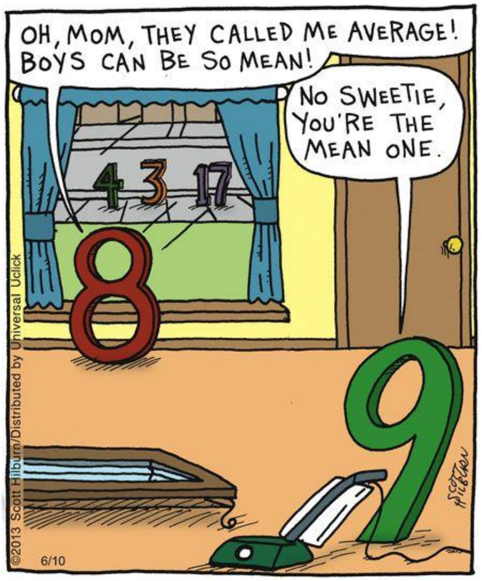
Solve for *y* in the last row(equation) in terms of *z* to get $y-z=-2 \Rightarrow y=z-2$**. Solve for *x* in the first row(equation) in terms of *z* to get** $x-z=-2 \Rightarrow x=z-2$**.**

The solutions of the system are given by

$x=z-2, y=z-2, z=z$; where z is any real number.

More Examples:

1. $$\begin{aligned} x - 2y &= 1 \\ 2x - y &= 5 \end{aligned}$$

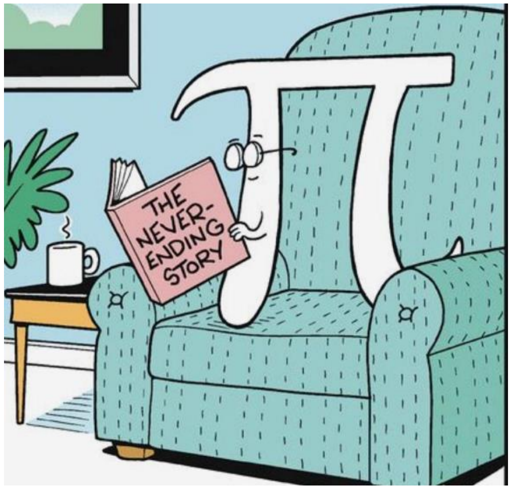


2. $$\begin{aligned} x + 2y &= 4 \\ 2x + 4y &= -8 \end{aligned}$$

"Algebra will be useful to you later in life because it teaches you to shut up and accept things that seem pointless and stupid."

3. $$\begin{aligned} 3x-6y&=-9\\ -2x+4y&=6 \end{aligned}$$

4. $$\begin{aligned} 2x + 4y - 10z &= -2 \\ 3x + 9y - 21z &= 0 \\ x + 5y - 12z &= 1 \end{aligned}$$



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5. $$\begin{aligned} 2x + 4y - 2z &= 2 \\ -3x - 6y + 3z &= -3 \end{aligned}$$