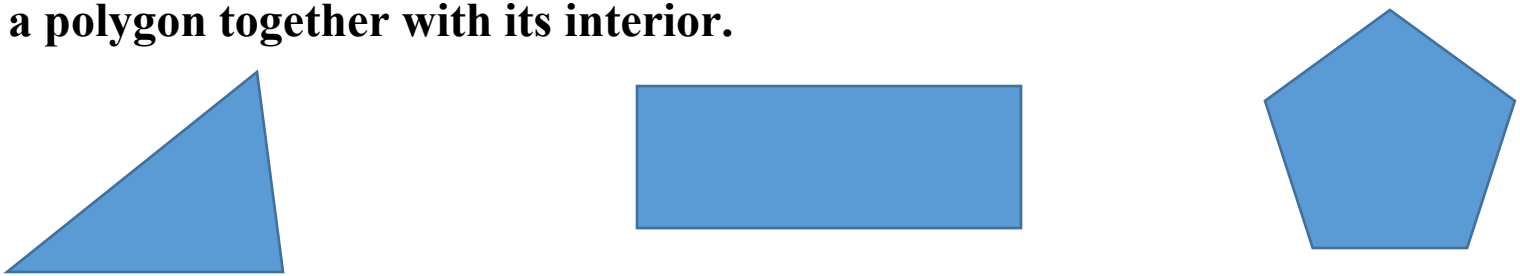
Polygonal Region:

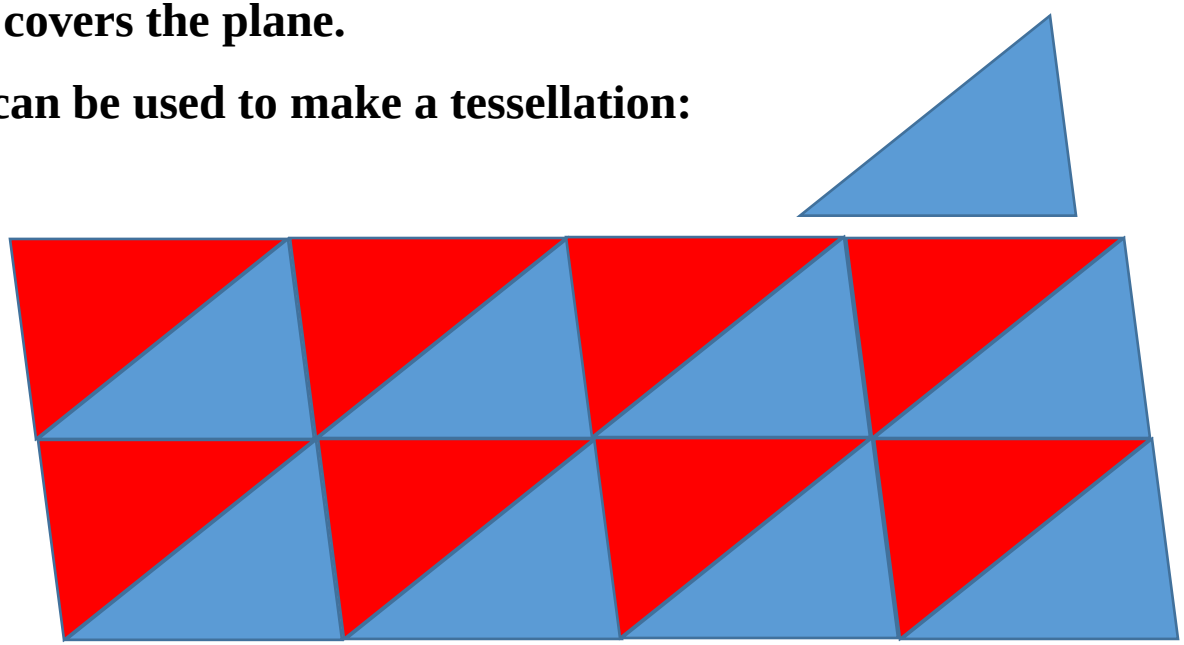
It's a polygon together with its interior.



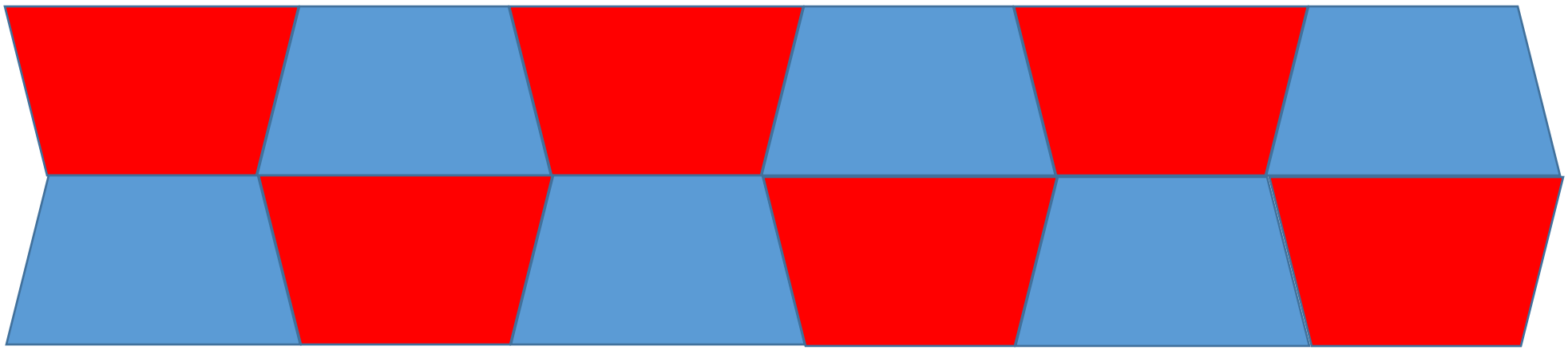
Tessellation:

It's an arrangement of polygonal regions having only sides in common that completely covers the plane.

A triangle can be used to make a tessellation:

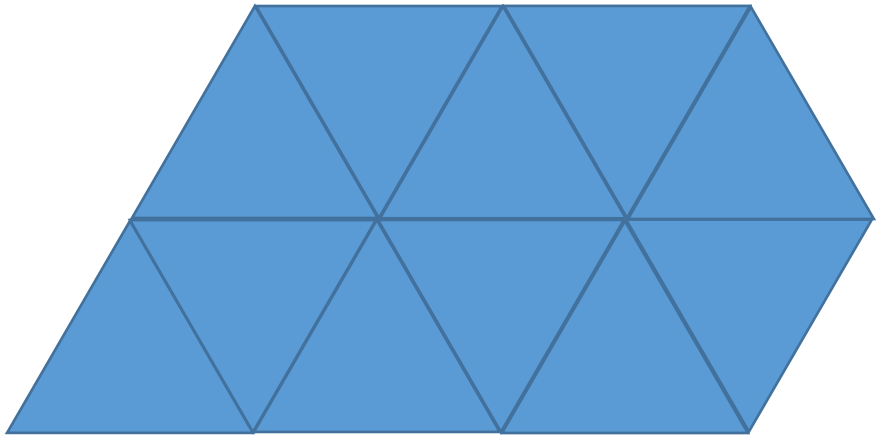


A trapezoid can be used to make a tessellation:



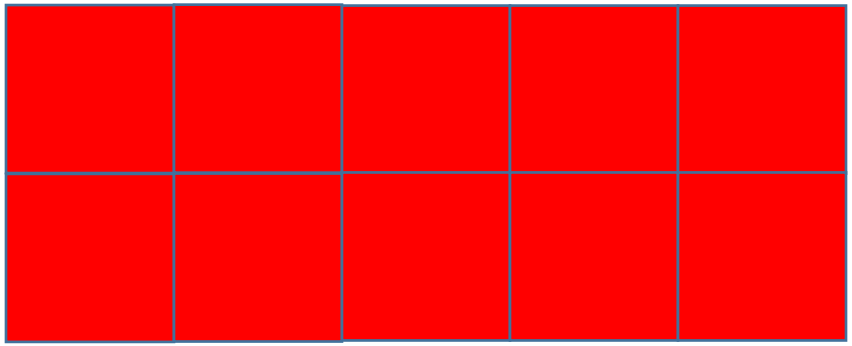
Regular Tessellation:

It's a tessellation that only uses a single type of regular polygonal region.



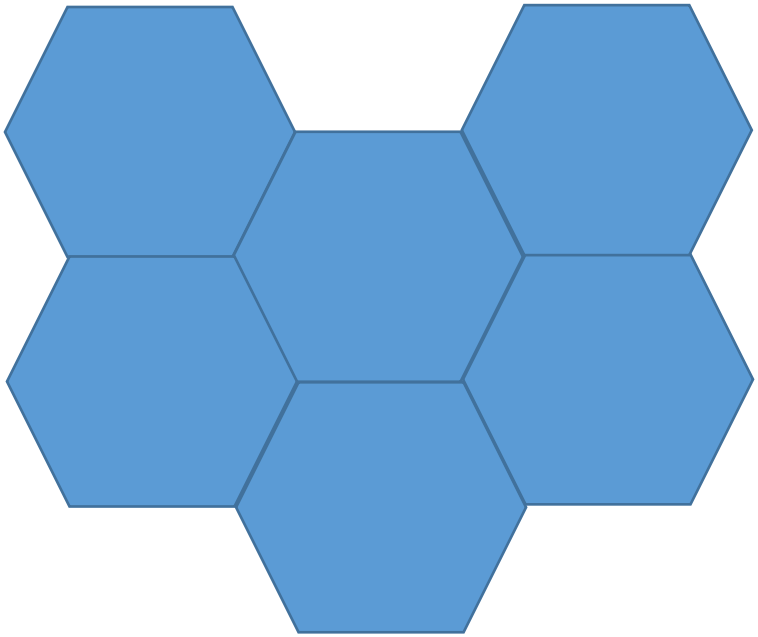
3-gon or equilateral triangle

Vertex angle is $60°$**.**



4-gon or square

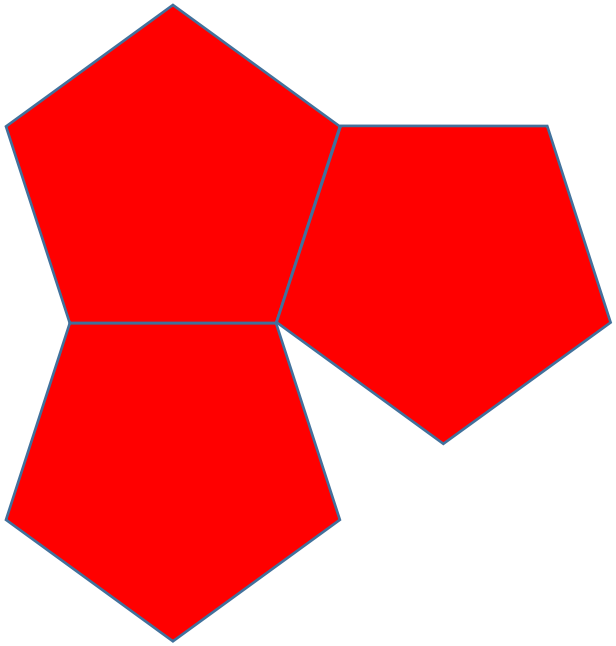
Vertex angle is $90°$**.**



6-gon or regular hexagon

Vertex angle is $120°$**.**

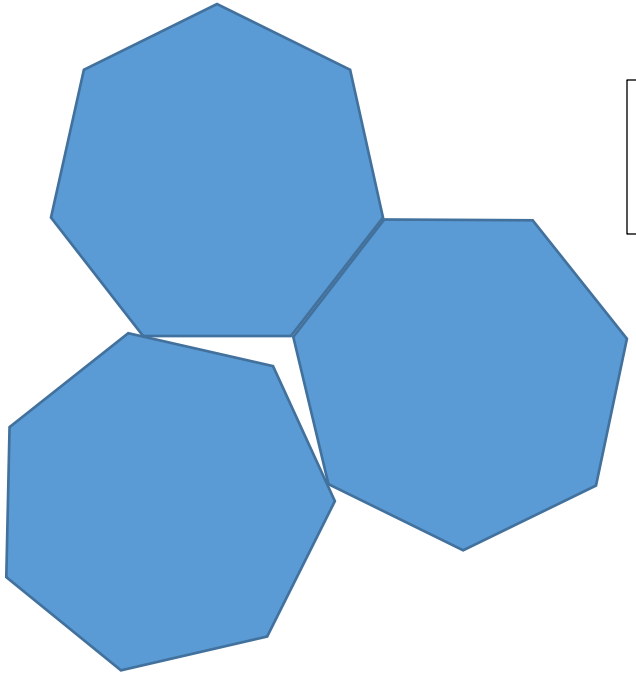
Regular tessellations can only be formed if the measure of a vertex angle of the underlying regular polygon is a factor of $360°$**.**



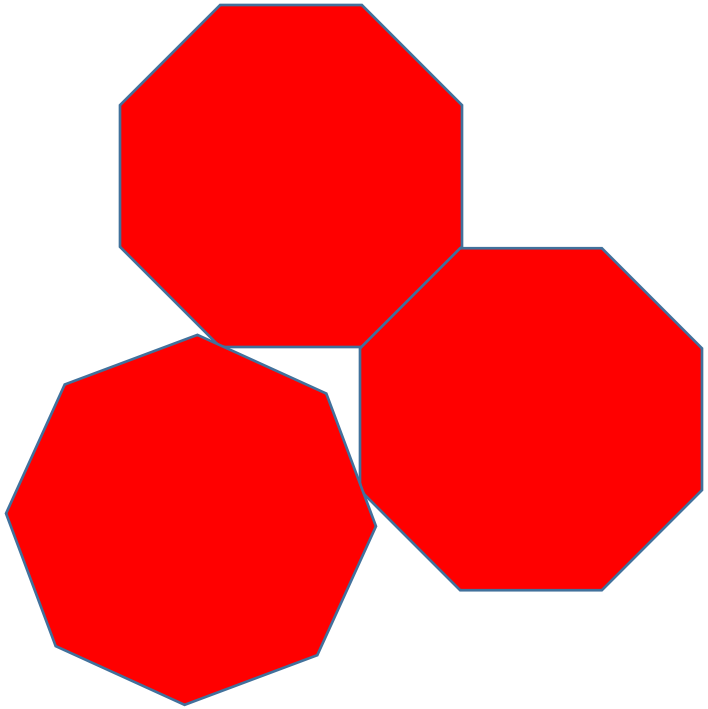
5-gon or regular pentagon

Vertex angle is $108°$**, which is not a factor of** $360°$**.**

7-gon or regular septagon



Vertex angle is $\frac{5}{7} \cdot 180° = 128.\overline{571428}°$**, which is not a factor of** $360°$**.**



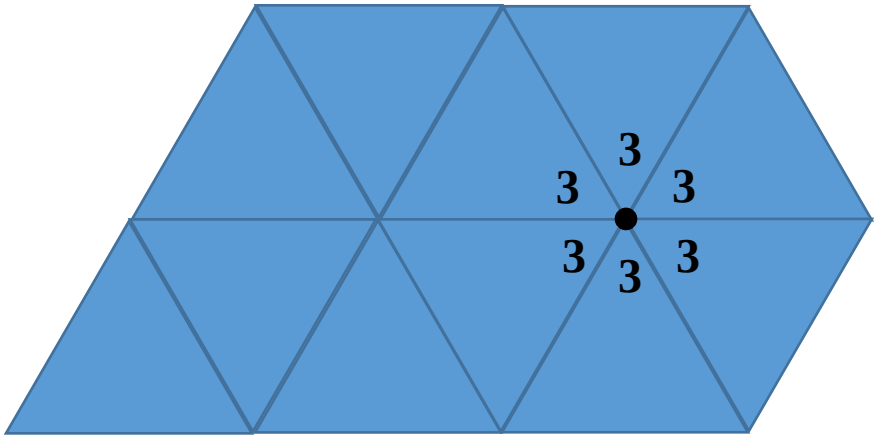
8-gon or regular octagon

Vertex angle is $135°$**, which is not a factor of** $360°$**.**

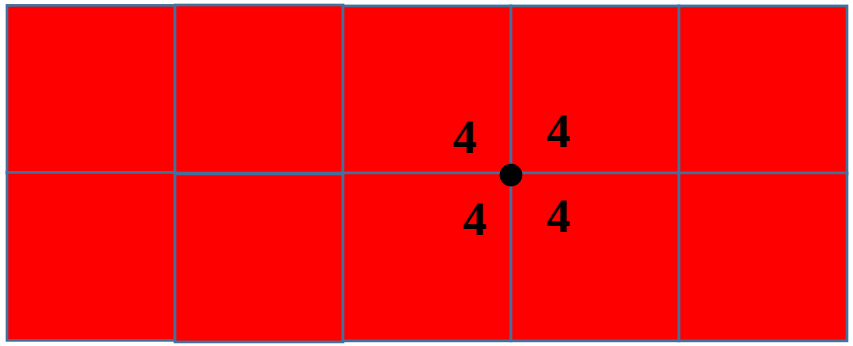
The only regular tessellations use 3-gons, 4-gons, or 6-gons.

Vertex Arrangement:

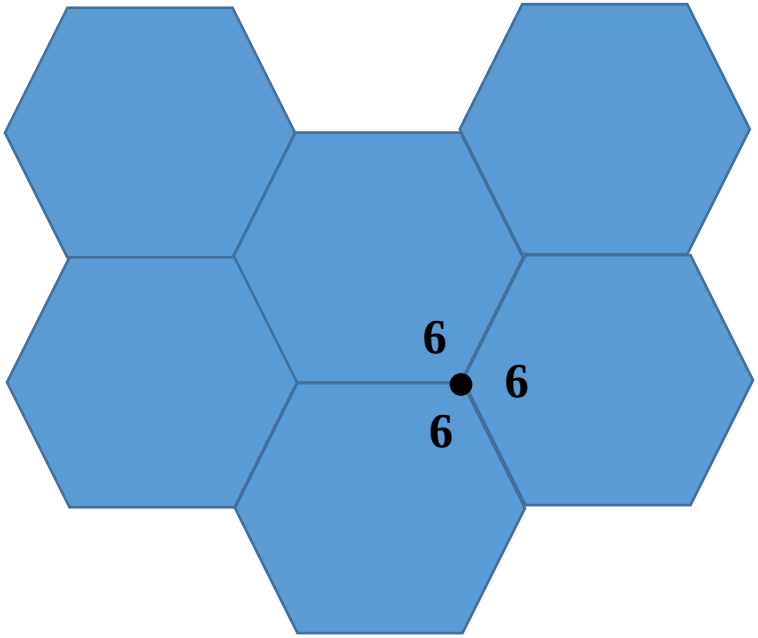
Tessellations are described by selecting a vertex and listing the number of sides of the polygonal regions that surround it in either a clockwise or a counterclockwise direction. This description is called a vertex arrangement.



The vertex arrangement is $(3,3,3,3,3,3)$**.**



The vertex arrangement is $(4,4,4,4)$.

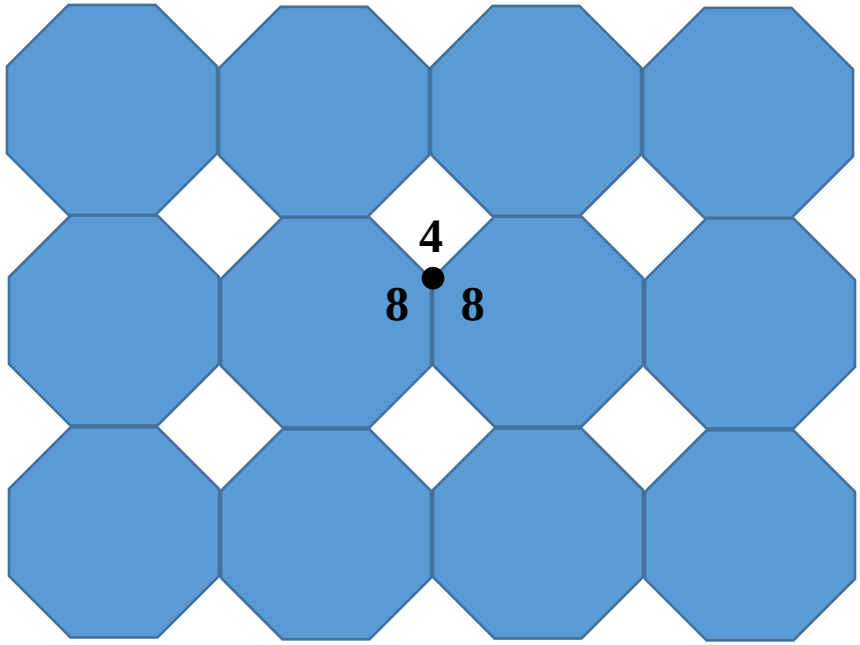


The vertex arrangement is $(6,6,6)$.

Semiregular Tessellation:

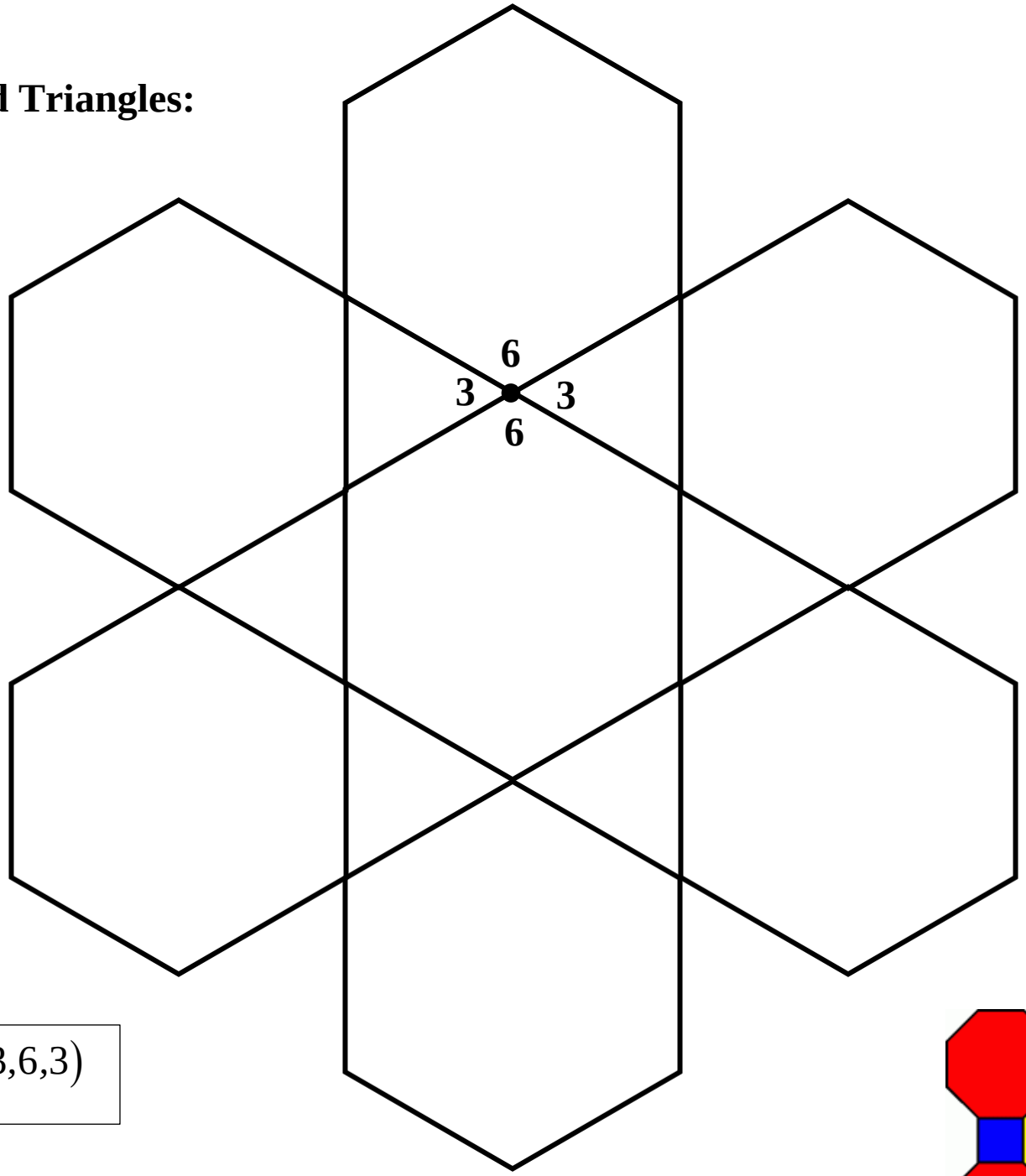
It's a tessellation that uses two or more regular polygonal regions with congruent sides with all the vertex arrangements identical.

Octagons and Squares:

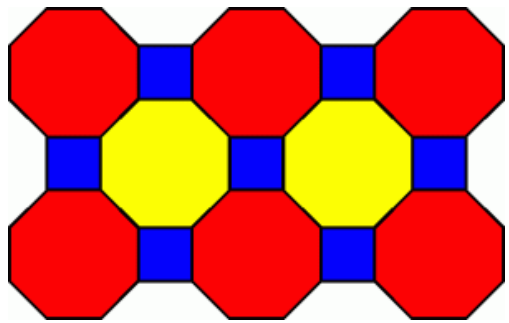


The vertex arrangement can be described by $(4,8,8)$, $(8,8,4)$ or $(8,4,8)$, but they're all the same.

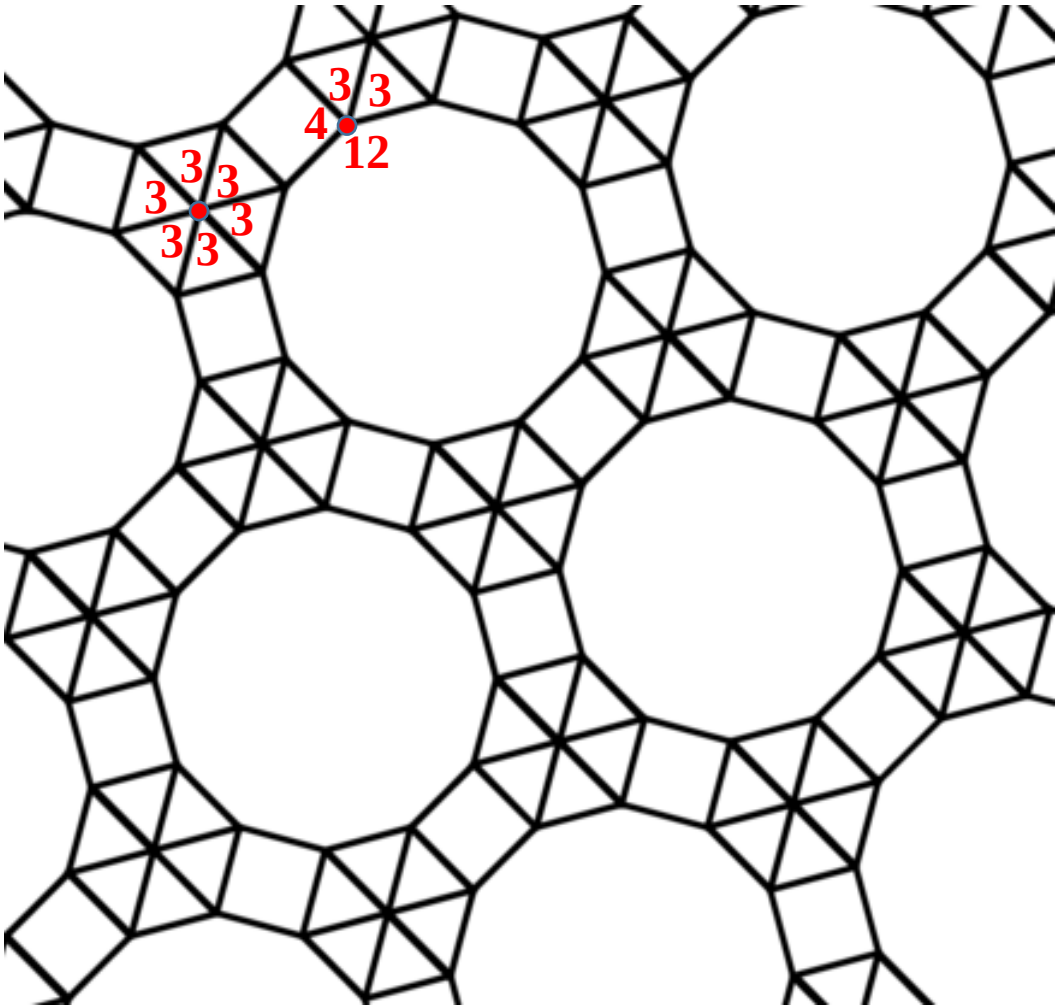
Hexagons and Triangles:

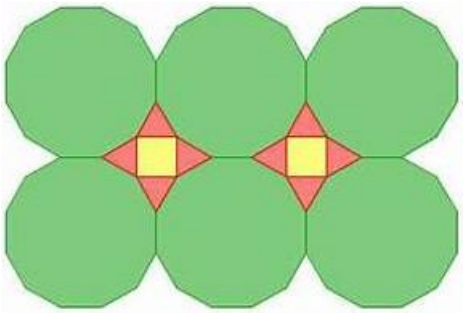


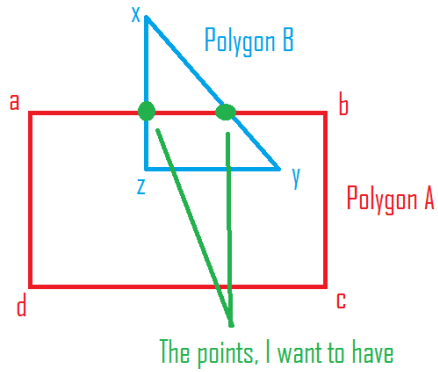
$(3,6,3,6)$ **or** $(6,3,6,3)$



There are tessellations using two or more regular polygonal regions that don't have the same vertex arrangement at each vertex.



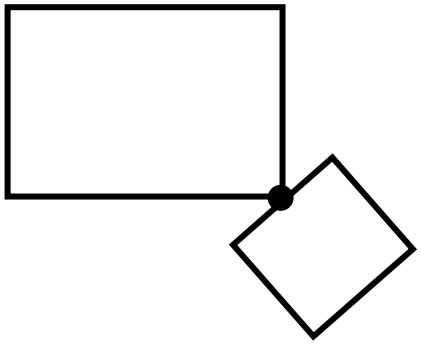




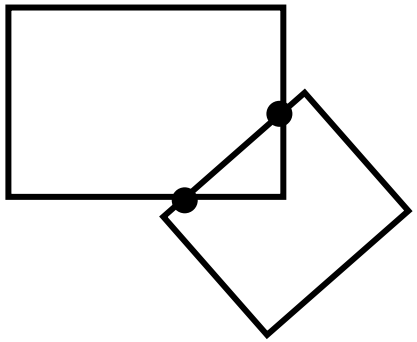
Intersections of Polygons:

Example: **Two rectangles.**

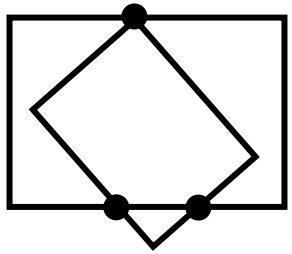
1 intersection point



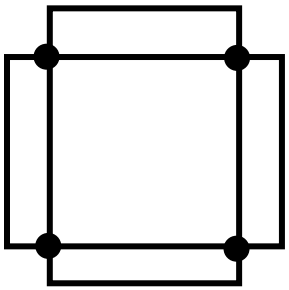
2 intersection points



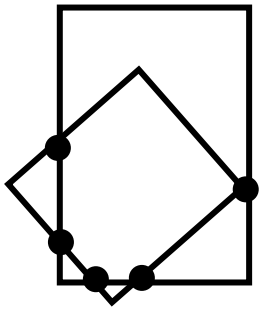
3 intersection points



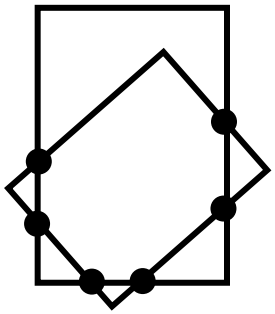
4 intersection points



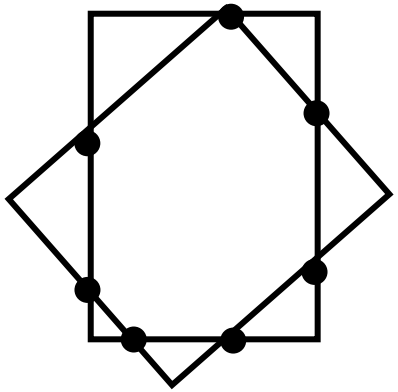
5 intersection points



6 intersection points



7 intersection points



8 intersection points