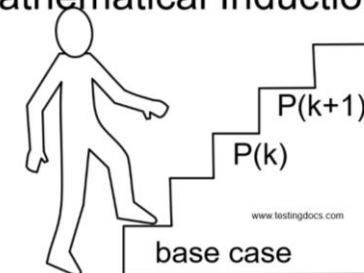


Proving Things Using the Principle of Mathematical Induction:

Mathematical Induction is used to prove statements about the natural/counting numbers.

It consists of two steps.

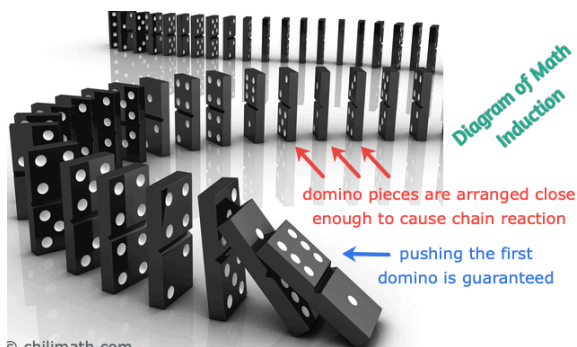
Mathematical Induction



The first step(*base step*) is to show that the statement is true for the first natural number, 1.

The second step(*induction step*) is to show that if the statement is true for a natural number greater than or equal to 1, then it must also be true for the next natural number.

The reason why this combination proves the statement for all natural numbers is the following: Choose any natural number, j . The statement must be true for j , because it's true for 1 and therefore 2, and therefore 3,....., and therefore j .



Examples:

1. Prove that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$ for all natural numbers, n .

Base Step: Show that it's true for $n = 1$.

Left side	Right side
1	$\frac{1 \cdot (1+1)}{2} =$

IF I'VE TOLD YOU N
TIMES
I'VE TOLD YOU N+1
TIMES

Induction Step: Show that if it's true for $n = k$, then it must also be true for

$$n = k + 1.$$

Assuming that it's true for $n = k$, leads to

$$1 + 2 + 3 + \cdots + k = \frac{k(k+1)}{2}.$$

Goal: $1 + 2 + 3 + \cdots + k + (k+1) = \frac{(k+1)(k+2)}{2}.$

How do you turn the current left side into the desired left side of

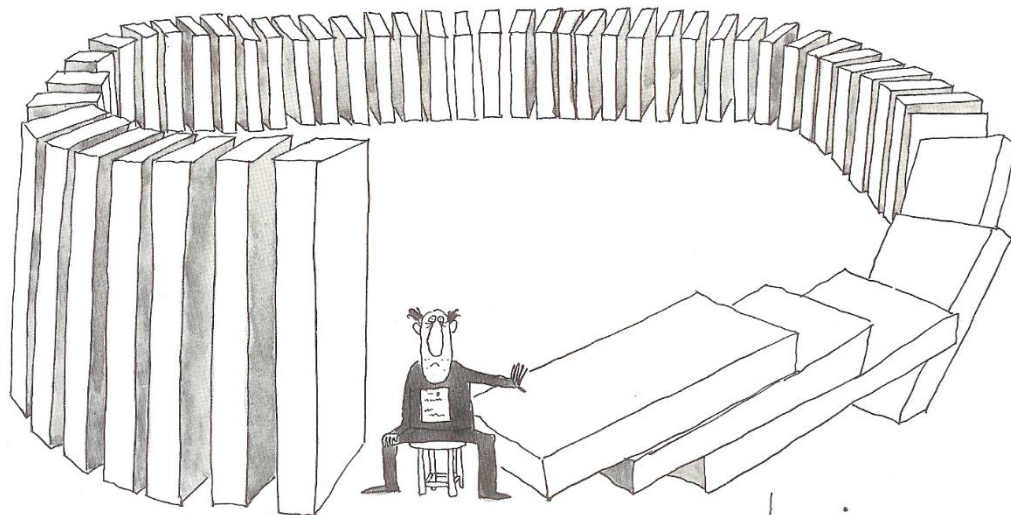
$$1 + 2 + 3 + \cdots + k + (k+1)?$$

Conclusion:

2. Prove that $1 + 2 + 2^2 + \cdots + 2^{n-1} = 2^n - 1$ for all natural numbers, n .

Base Step: Show that it's true for $n = 1$.

Left side	Right side
1	$2^1 - 1 =$



Induction Step: Show that if it's true for $n = k$, then it must also be true for

$$n = k + 1.$$

Assuming that it's true for $n = k$, leads to

$$1 + 2 + 2^2 + \cdots + 2^{k-1} = 2^k - 1.$$

Goal: $1 + 2 + 2^2 + \cdots + 2^{k-1} + 2^k = 2^{k+1} - 1.$

How do you turn the current left side into the desired left side of

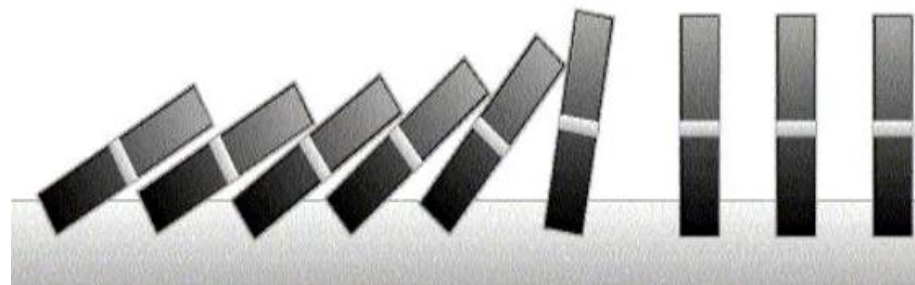
$$1 + 2 + 2^2 + \cdots + 2^{k-1} + 2^k ?$$

Conclusion:

3. Prove that $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$ **for all natural numbers, n .**

Base Step: Show that it's true for $n = 1$.

Left side	Right side
$\frac{1}{1 \cdot 2} =$	$\frac{1}{1+1} =$



Induction Step: Show that if it's true for $n = k$, then it must also be true for $n = k + 1$.

Assuming that it's true for $n = k$, leads to

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{k(k+1)} = \frac{k}{k+1}.$$

Goal: $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{k+1}{k+2}.$

How do you turn the current left side into the desired left side of

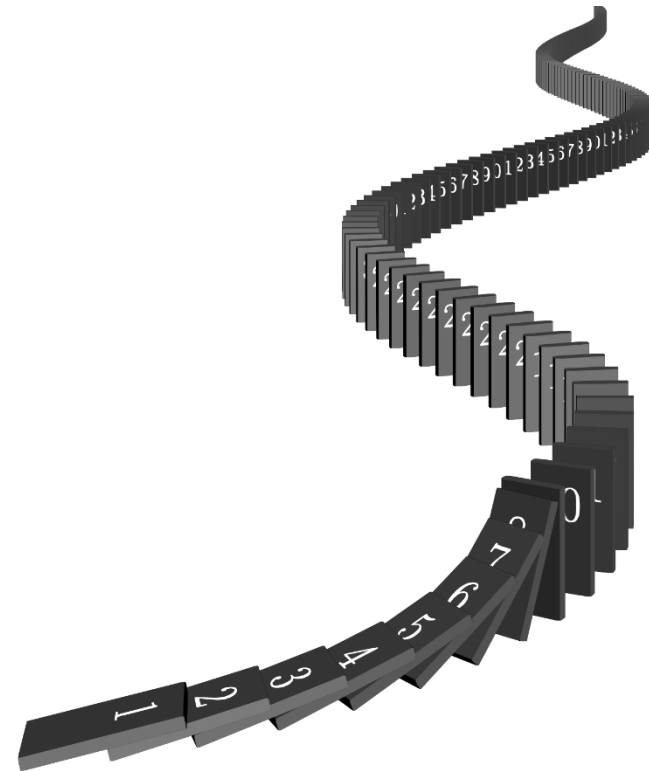
$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)}?$$

Conclusion:

4. Prove that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$ for all natural numbers, n .

Base Step: Show that it's true for $n = 1$.

Left side	Right side
$1^3 =$	$\frac{1^2(1+1)^2}{4} =$



Induction Step: Show that if it's true for $n = k$, then it must also be true for $n = k + 1$.

Assuming that it's true for $n = k$, leads to

$$1^3 + 2^3 + 3^3 + \cdots + k^3 = \frac{k^2(k+1)^2}{4}.$$

Goal: $1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3 = \frac{(k+1)^2(k+2)^2}{4}.$

How do you turn the current left side into the desired left side of

$$1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^3?$$

Conclusion:

5. Prove that $2^n > n$ for all natural numbers, n .

Base Step: Show that it's true for $n = 1$.

Left side	Right side
$2^1 =$	1

Induction Step: Show that if it's true for $n = k$, then it must also be true for

$$n = k + 1.$$

Assuming that it's true for $n = k$, leads to

$$2^k > k.$$

Goal: $2^{k+1} > k + 1.$

How do you turn the current left side into the desired left side of

$$2^{k+1} ?$$

$$\{2k = k + k \geq k + 1\}$$

Conclusion:

6. Consider the sequence $\{a_n\}$ with $a_1 = 1$ and $a_{n+1} = \sqrt{2 + a_n}$; $n \geq 1$.

a) Show that $a_n < 2$ for all natural numbers, n .

Base Step: Show that it's true for $n = 1$.

Left side	Right side
$a_1 =$	2

Induction Step: Show that if it's true for $n = k$, then it must also be true for

$$n = k + 1.$$

Assuming that it's true for $n = k$, leads to $a_k < 2$.

Goal: $a_{k+1} < 2$.

$$a_{k+1} = \sqrt{2 + a_k} < \sqrt{2 + 2} =$$

Conclusion:

b) Show that $a_{n+1} > a_n$ for all natural numbers, n .

Base Step: Show that it's true for $n = 1$.

Left side	Right side
$a_2 =$	$a_1 =$

Induction Step: Show that if it's true for $n = k$, then it must also be true for $n = k + 1$.

Assuming that it's true for $n = k$, leads to $a_{k+1} > a_k$.

Goal: $a_{k+2} > a_{k+1}$.

$$a_{k+2} = \sqrt{2 + a_{k+1}} > \sqrt{2 + a_k} =$$

Conclusion:

