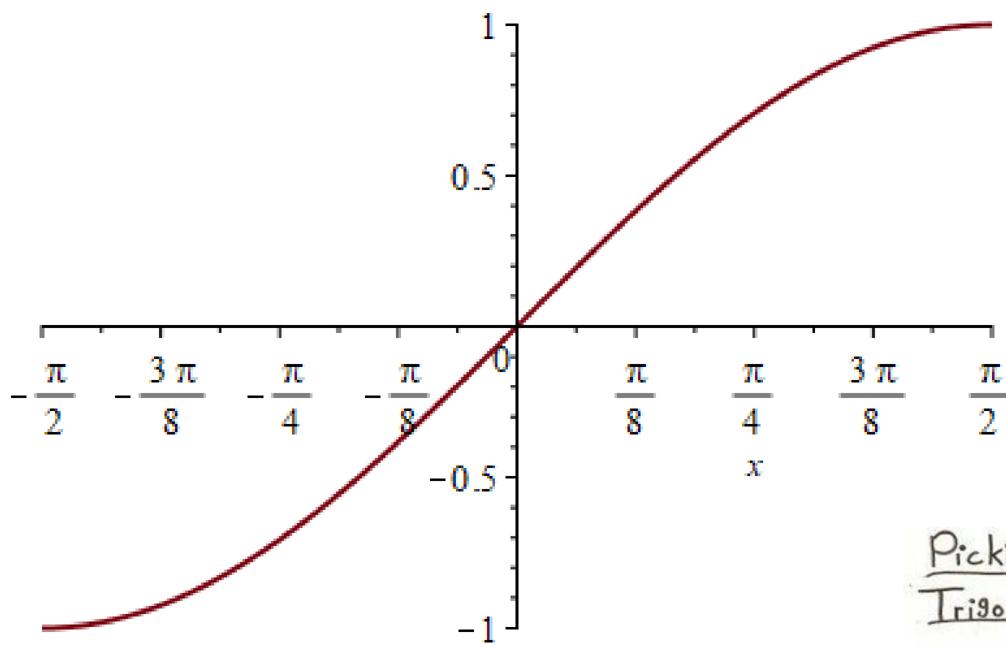
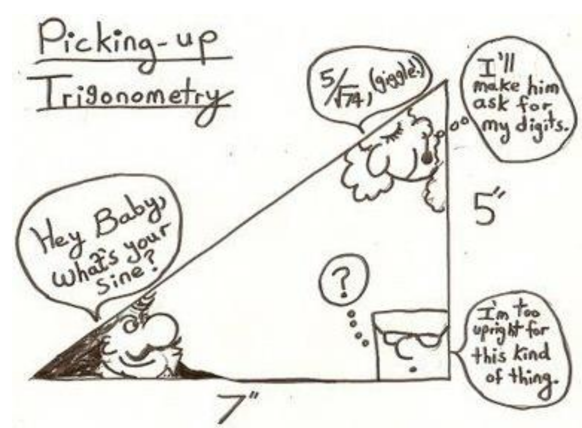
Review of Inverse Trigonometric Functions:

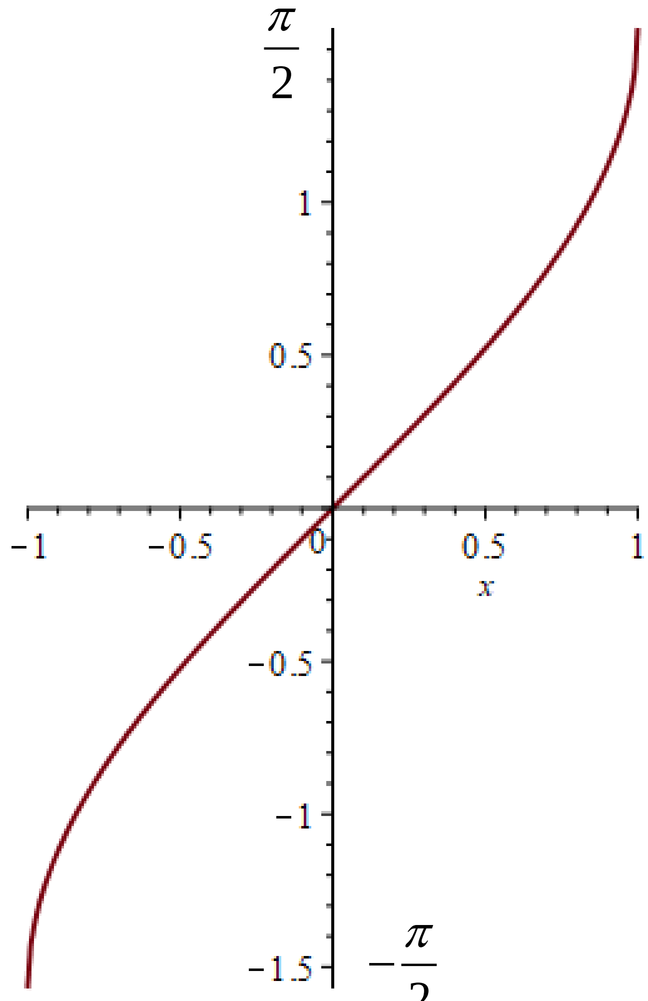
None of the Trigonometric Functions is 1-1, so none of them have inverses without restricting their domains.

Inverse Sine: **The restriction on the domain of the sine function is** $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$**.**

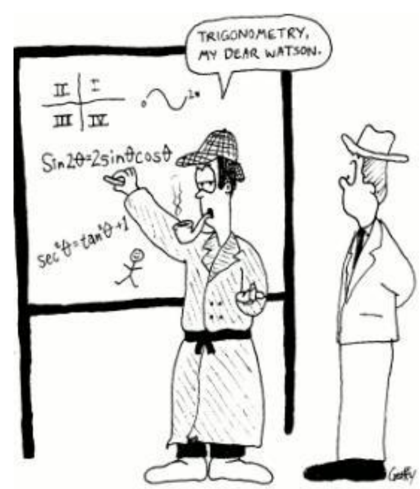




The graph of the inverse sine function is found by reflection about the line $y = x$.



$sin^{-1}\, x$ **is the angle in** $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ **whose sine is** ***x*****.**



Examples:

1. $sin^{-1}(-1)$

2. $sin^{-1}\left(-\frac{1}{2}\right)$

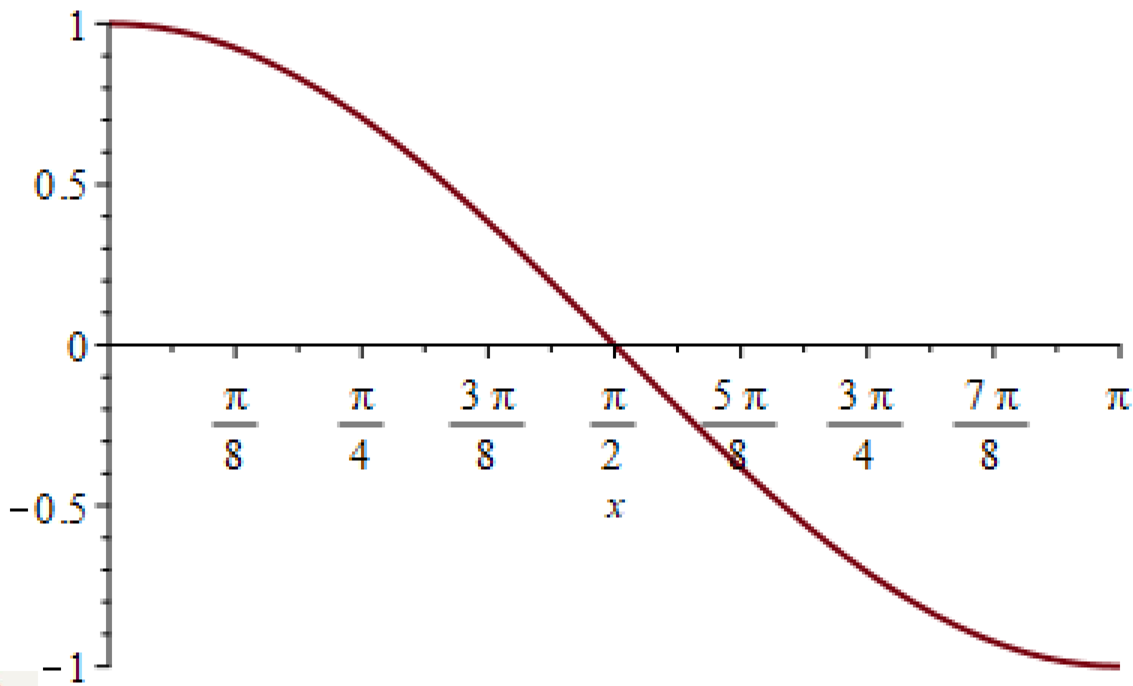
3. $sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

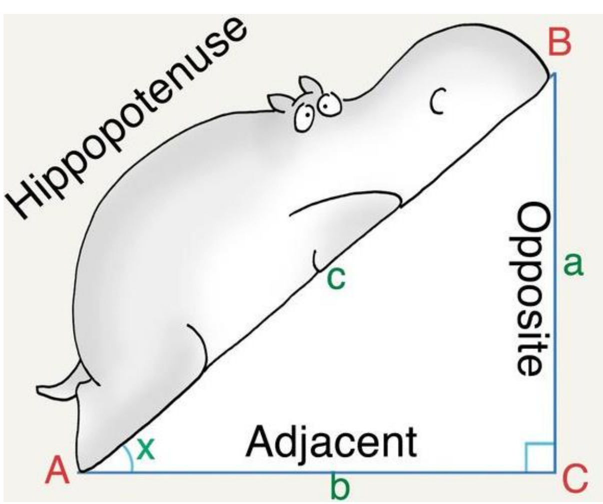
4. $sin\left(sin^{-1}\left(\frac{1}{3}\right)\right)$

5. $sin^{-1}\left(sin\left(\frac{6\pi}{7}\right)\right)$

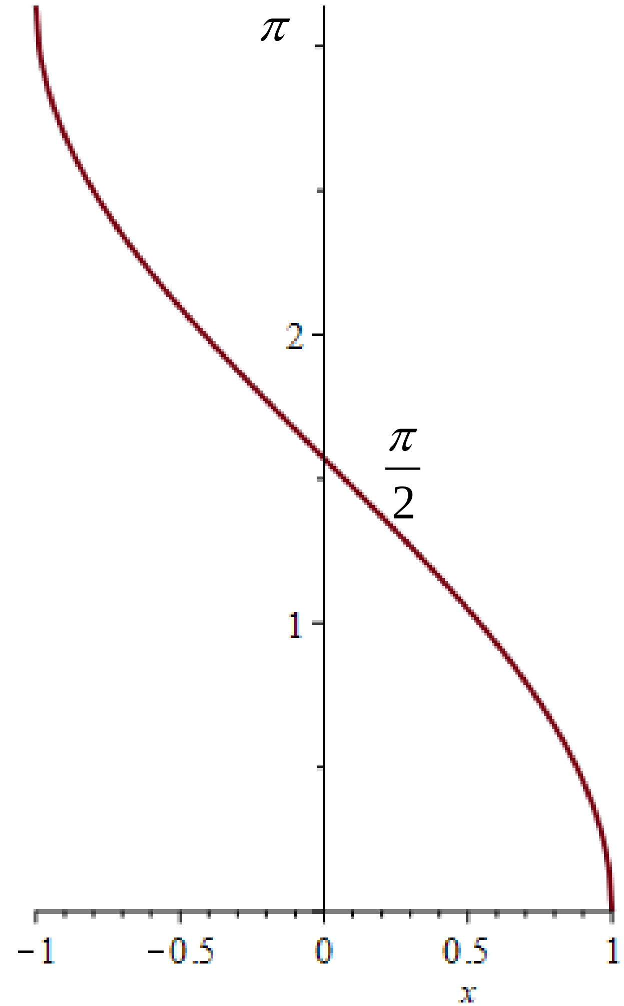
6. $cos\left(sin^{-1}\left(-\frac{3}{5}\right)\right)$

<u>Inverse Cosine:</u> **The restriction on the domain of the cosine function is** $[0,\pi]$**.**

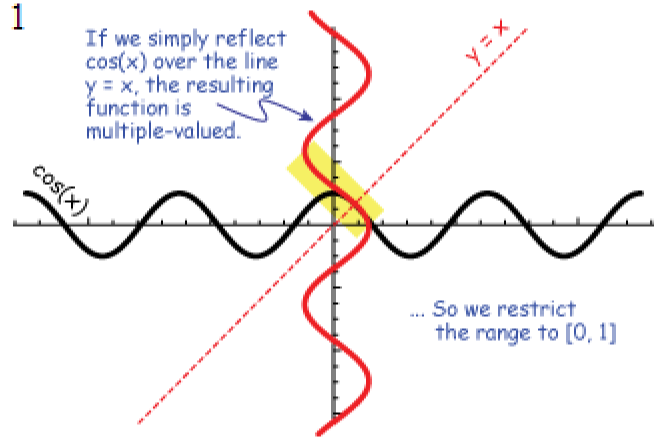




The graph of the inverse cosine function is found by reflection about the line $y = x$.



$\cos^{-1} x$ **is the angle in $[0, \pi]$ whose cosine is x.**



Examples:

1. $cos^{-1}(1)$

2. $cos^{-1}\left(-\frac{1}{2}\right)$

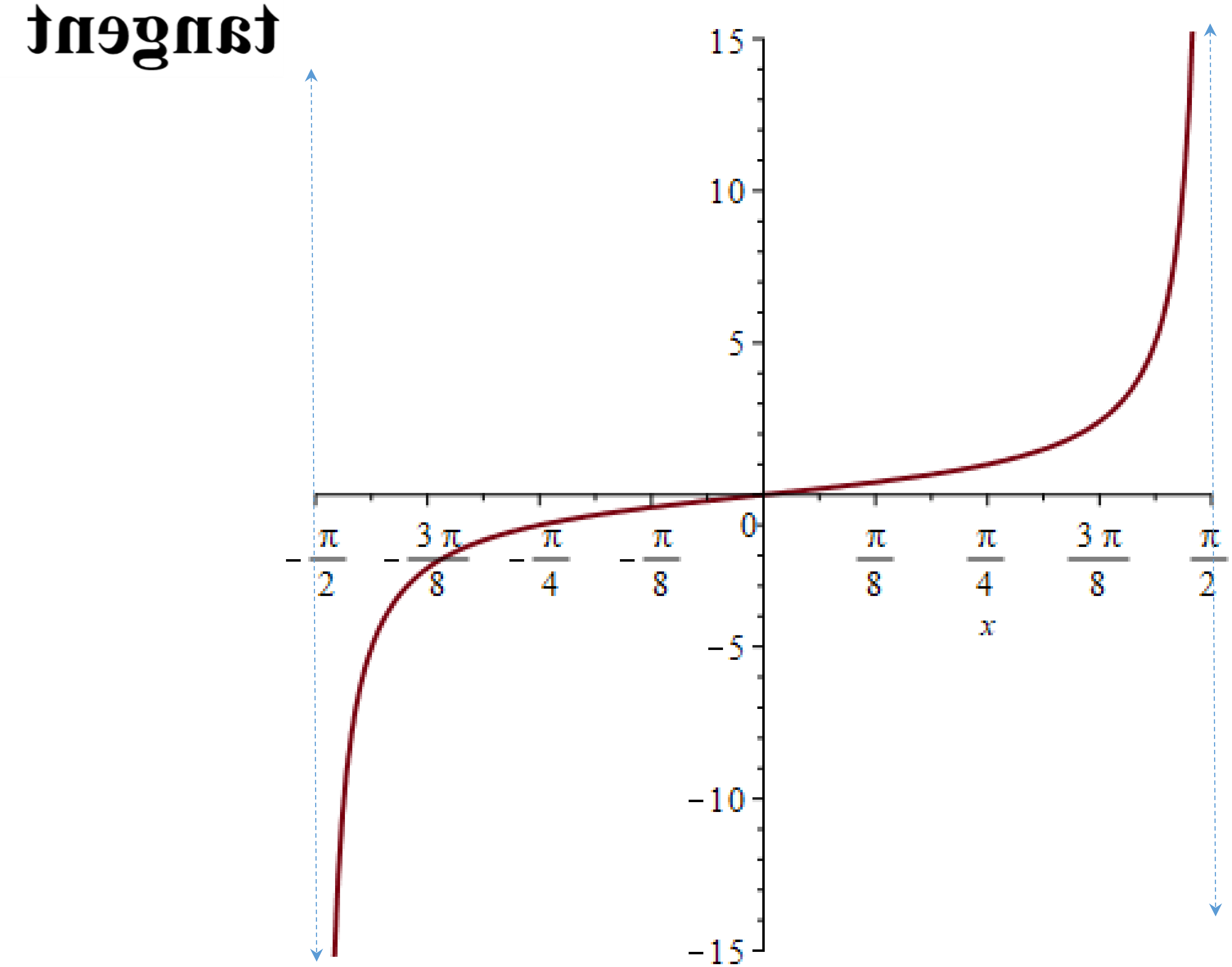
3. $cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

4. $cos\left(cos^{-1}\left(-\frac{1}{3}\right)\right)$

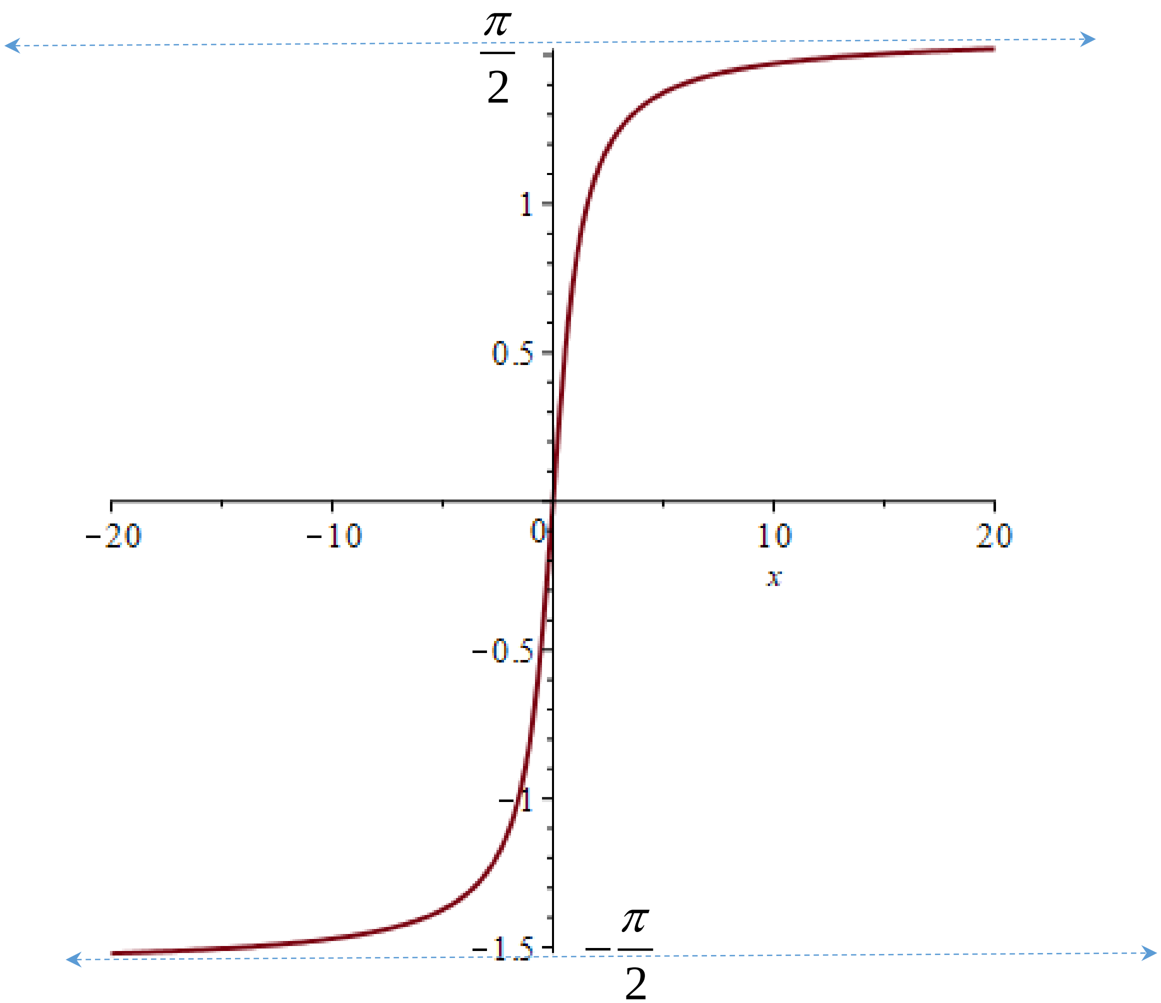
5. $cos^{-1}\left(cos\left(-\frac{\pi}{4}\right)\right)$

6. $sin\left(cos^{-1}\left(\frac{4}{5}\right)\right)$

<u>Inverse Tangent:</u> The restriction on the domain of the tangent function is $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$**.**



The graph of the inverse tangent function is found by reflection about the line $y = x$**.**



$tan^{-1}\, x$ **is the angle in** $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ **whose tangent is** x**.**

Examples:

1. $tan^{-1}\left(-1\right)$

2. $tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$

3. $tan^{-1}\left(\sqrt{3}\right)$

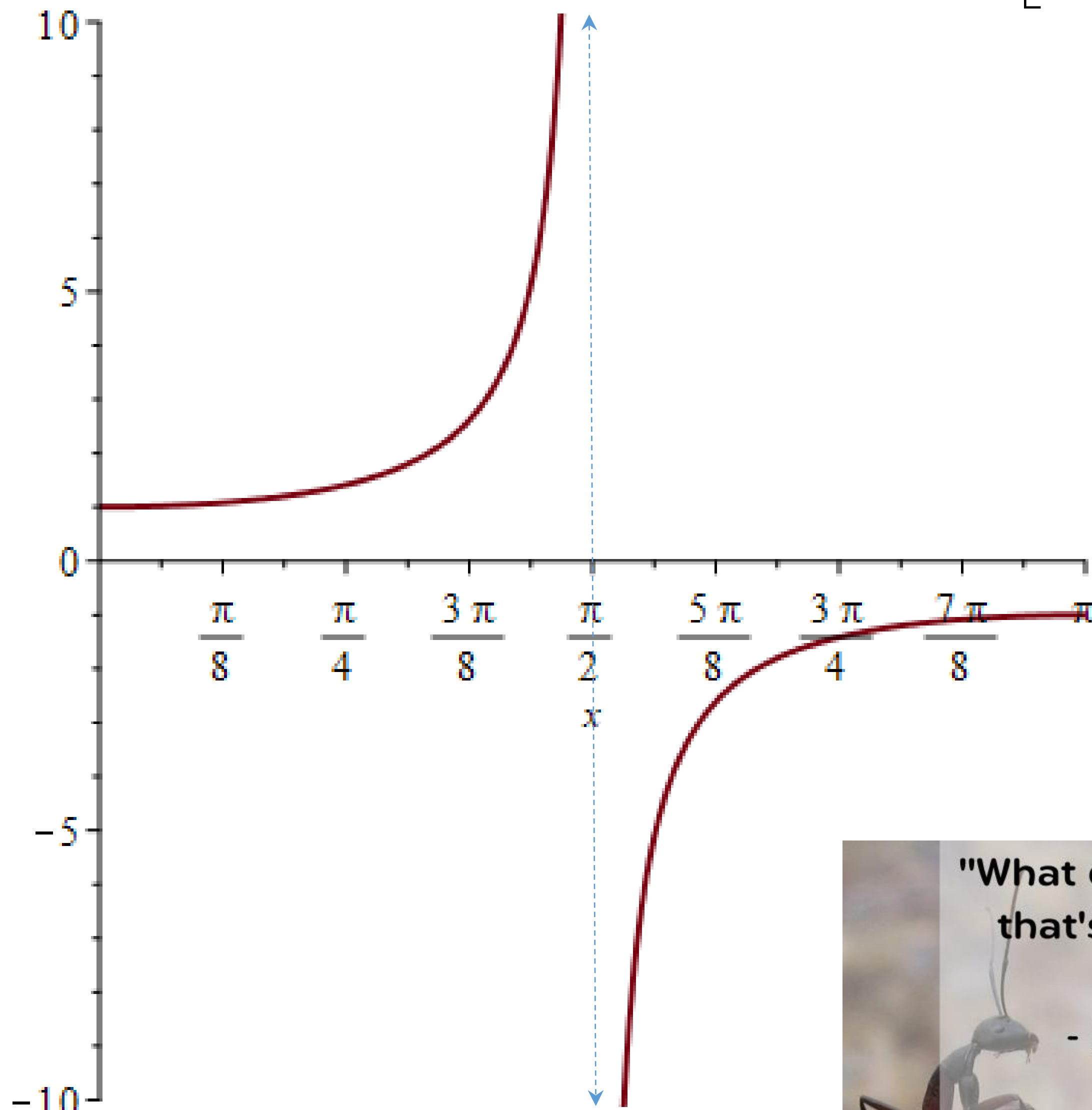
4. $tan\left(tan^{-1}\left(-\frac{1}{3}\right)\right)$

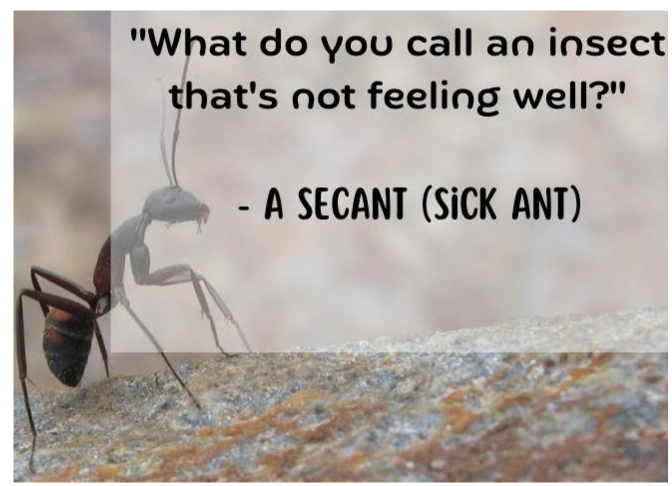
5. $tan^{-1}\left(tan\left(\frac{4\pi}{5}\right)\right)$

6. $sin\left(tan^{-1}\left(\frac{3}{4}\right)\right)$

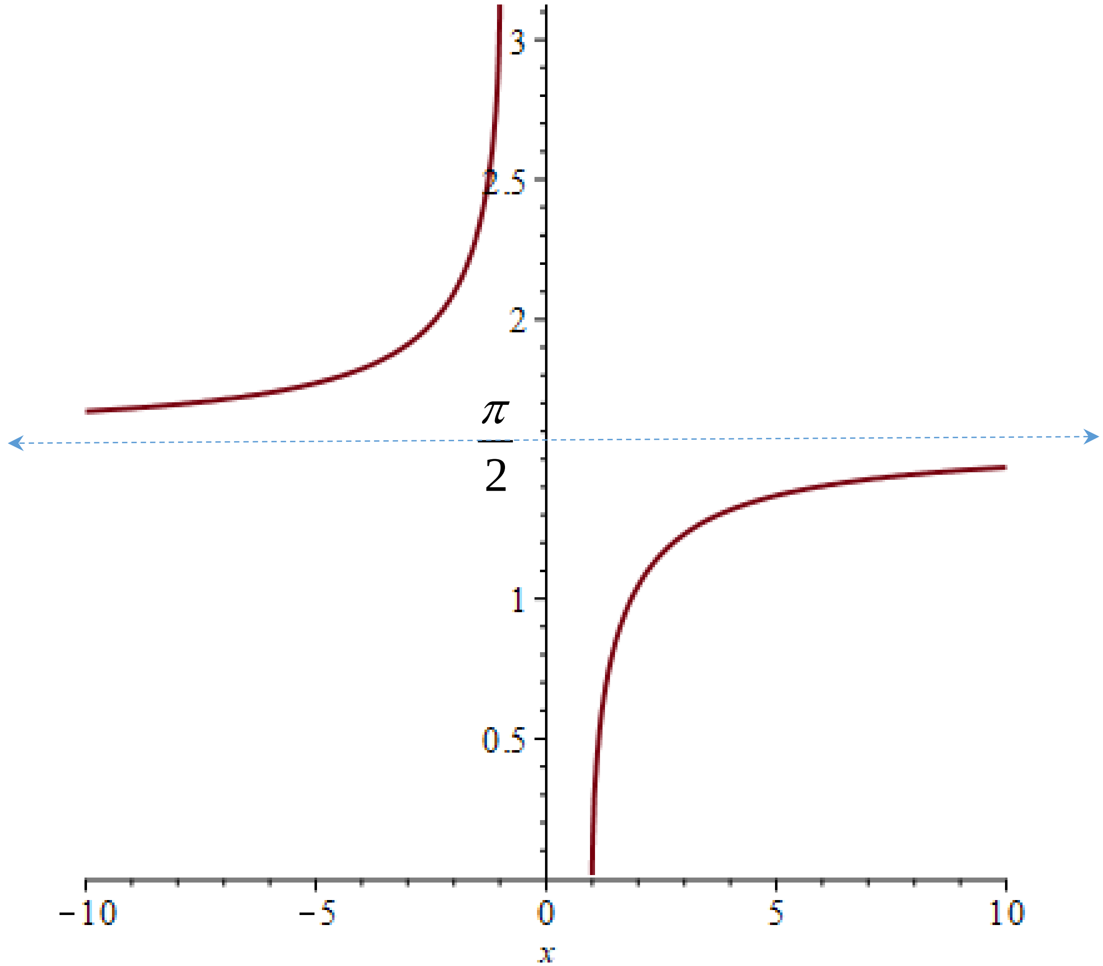
Inverse Secant: **The restriction on the domain of the secant function is** $\left[0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\pi\right]$

secant.





The graph of the inverse secant function is found by reflection about the line $y = x$**.**



$sec^{-1}\,x$ **is the angle in** $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$ **whose secant is *x*.**

Examples:

1. $sec^{-1}(-1)$

2. $sec^{-1}\left(\sqrt{2}\right)$

3. $sec^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$

4. $sec\left(sec^{-1}(-3)\right)$

5. $sec^{-1}\left(sec\left(-\tfrac{2\pi}{9}\right)\right)$

6. $sin\left(sec^{-1}\left(-\tfrac{5}{3}\right)\right)$

<u>Inverse Cotangent:</u> The restriction on the domain of the cotangent function is $(0,\pi)$.

cotangent

$$cot^{-1}(1)$$

$$cot^{-1}(-1)$$

<u>Inverse Cosecant:</u> The restriction on the domain of the cosecant function is $\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$.

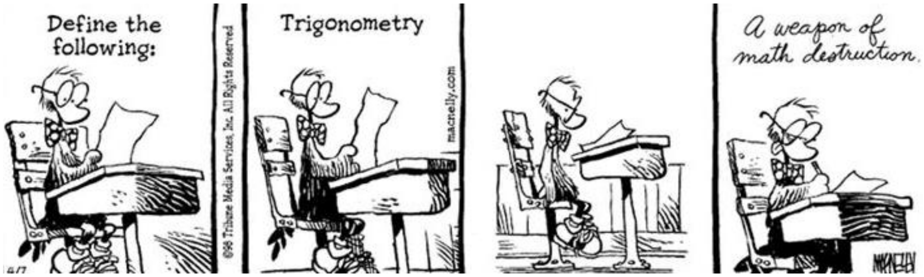
cosecant

$$csc^{-1}(2)$$

$$csc^{-1}\left(-\frac{2}{\sqrt{3}}\right)$$

Trigonometric Equations:

Because of the periodicity of the trigonometric functions, trig. equations will generally have infinitely many solutions. Unless it's stated otherwise, we'll just find the solutions in the interval $[0,2\pi)$.

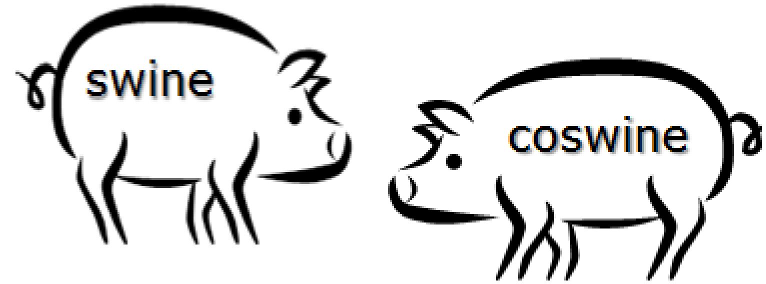


Examples:

1. $2\sin\theta+3=2$

2. $\tan^2\theta=1$

Pigonometry



3. $\cos 2\theta = -1$

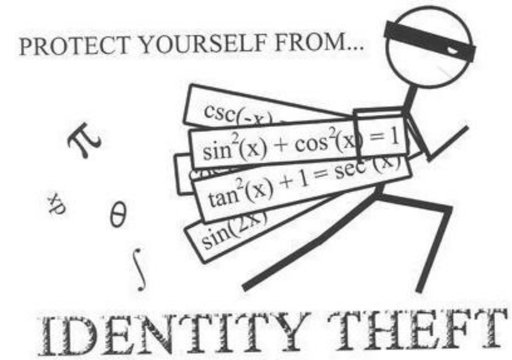
4. $\csc \frac{\theta}{3} = \frac{2}{\sqrt{3}}$

5. $sin\left(2\theta-\frac{\pi}{2}\right)=-1$

$$2\theta-\frac{\pi}{2}=\frac{3\pi}{2},\frac{3\pi}{2}+2\pi,\frac{3\pi}{2}+4\pi,\ldots$$

$$\frac{3\pi}{2}-2\pi,\frac{3\pi}{2}-4\pi,\ldots$$

6. $2\,sin^2\theta-sin\,\theta=0$



7. $2\cos^2\theta + \cos\theta - 1 = 0$

8. $\sin^2\theta - \cos^2\theta = 1 + \cos\theta$

{Trig. identity: $\sin^2\theta = 1 - \cos^2\theta$ **.}**

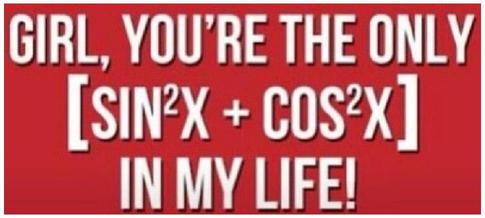
9. $\cos\theta - \sin(-\theta) = 0$

{Trig. identity: $\sin(-\theta) = -\sin\theta$**.}**

10. $2\sin^2\theta - 5\sin\theta + 2 = 0$

11. $3\sin^2\theta + 2\sin\theta - 1 = 0$

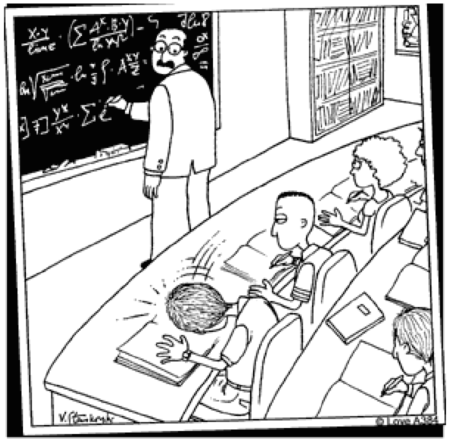
12. $\sin\theta - \sqrt{3}\cos\theta = 1$ **{Square both sides, and use** $\sin^2\theta + \cos^2\theta = 1$**.}**



13. $sin 2\theta = cos\theta$ ***{Trig. identity:*** $sin 2\theta = 2 sin\theta cos\theta$***.}***

14. $cos 2\theta + 6 sin^2\theta = 4$

{Trig. identity: $cos 2\theta = cos^2\theta - sin^2\theta$ or $2cos^2\theta - 1$ or $1 - 2sin^2\theta$***.}***



Professor Herman stopped when he heard that unmistakable thud -- another brain had imploded.

15. $sin 2\theta - sin 4\theta = 0$

{Trig. identity: $sin 4\theta = 2\, sin 2\theta \cos 2\theta$***.}***

16. $cos^2 \theta + 2 \cos\theta - 4 = 0$

17. $sin^{14}\theta + cos^{8}\theta + 1 = 4$

$$0 \le sin^{14}\theta \le 1,\ 0 \le cos^{8}\theta \le 1$$

So $\square \le sin^{14}\theta + cos^{8}\theta \le \square$

So $\square \le sin^{14}\theta + cos^{8}\theta + 1 \le \square$

So?

Find the Values of x Satisfying

$$\sin x > \frac{1}{2} \quad \cos x \ge -\frac{1}{2}$$

18. $\left(1+sin^{12}\theta\right)\left(2-cos^{3}\theta\right)=7$

$$1\leq 1+sin^{12}\theta\leq 2,\ 1\leq 2-cos^{3}\theta\leq 3$$

So $\square \leq \left(1+sin^{12}\theta\right)\left(2-cos^{3}\theta\right)\leq \square$

So?