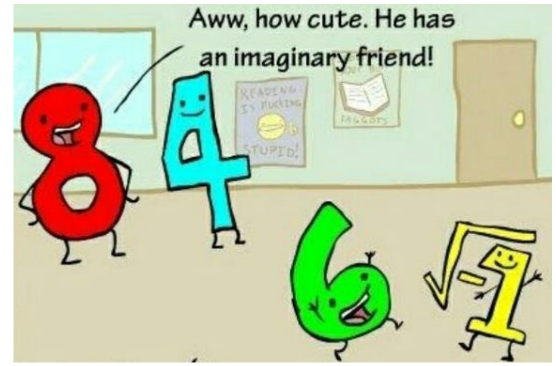
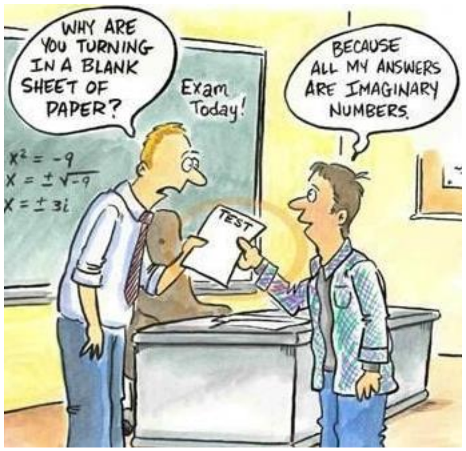
__Review of Complex Numbers:__

The standard form of a complex number is $a+bi$, where *a* and *b* are real numbers and $i^2=-1$.



<u>Basic Operations:</u>



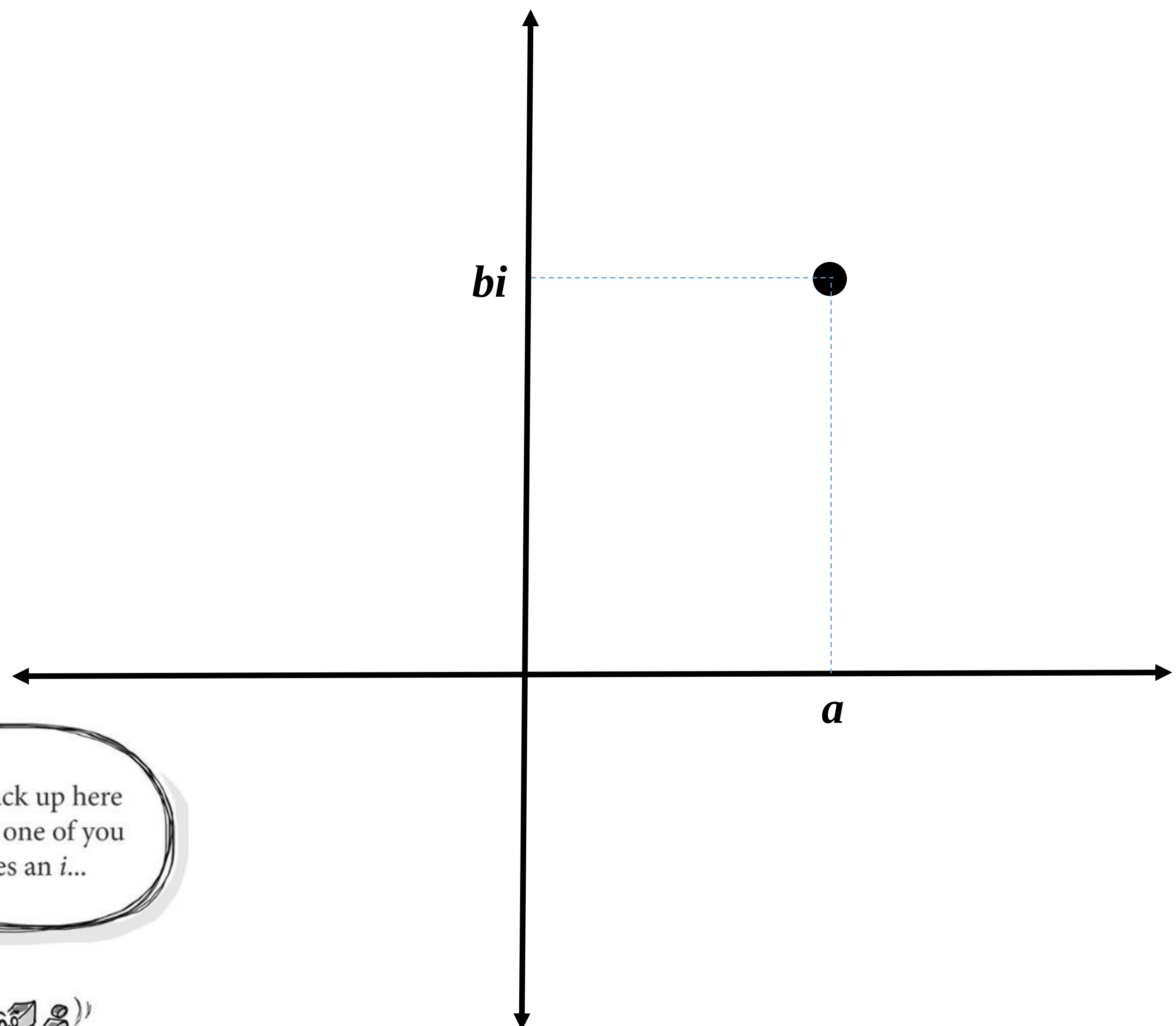
$$(2+3i)+(5-7i)$$

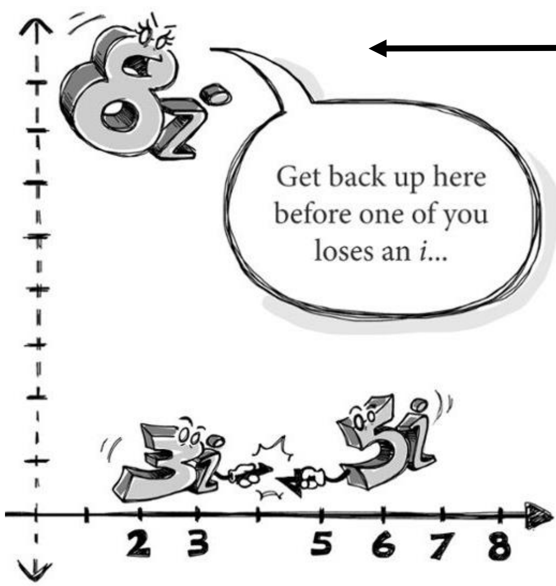
$$(-4+3i)-(6-7i)$$

$$(2+3i)(5-7i)$$

$$(2+3i)\div(1+2i)$$

The standard form is also known as the rectangular form, since the complex number $a + bi$ can be thought of as a point in the complex plane.





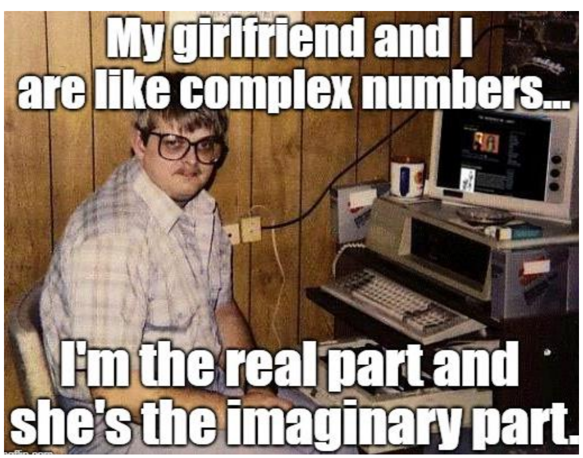
The distance that a complex number, $z = a + bi$, is in the complex plane from the origin is called its magnitude or modulus, $|z| = |a + bi| = \sqrt{a^2 + b^2}$.

Example:

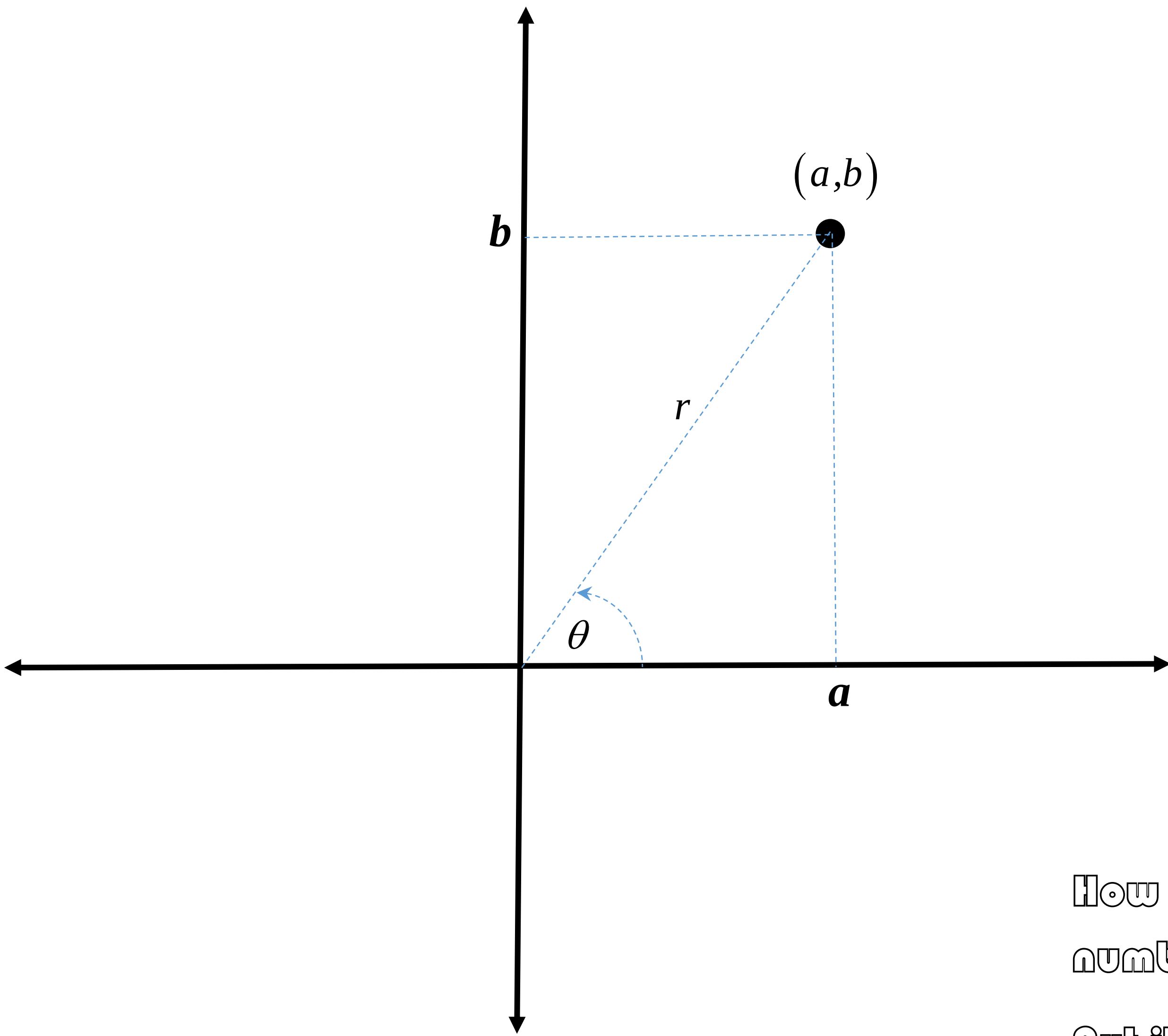
$$|3 - 4i| =$$

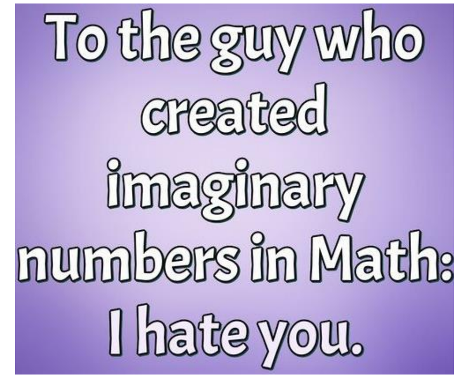
The conjugate or complex conjugate of $a + bi$ is $a - bi$, and the notation is $\overline{a + bi} = a - bi$. For any complex number, *z*, $z\overline{z} = |z|^2$.

Show why.



There is an alternative method for locating and describing complex numbers in the complex plane called polar form-just like polar coordinates.





How do you convert a complex number into polar form?

Put it in the freezer.

Standard polar form has $r \geq 0$ **,** $0 \leq \theta < 2\pi$**, and is written as** $a + bi = r(cos\,\theta + i\,sin\,\theta)$**, where** $r = \sqrt{a^2 + b^2}$**.**

Examples:

Find the standard polar form of $1+i$**.**

Find the standard polar form of $1-\sqrt{3}i$**.**

"EINSTEIN, THE CHILDREN ARE GETTING TOO COMPLEX FOR ME."

Products and Quotients of Complex Numbers in Polar Form:

We'll need some trig. identities: $cos(\alpha+\beta)=cos\alpha\, cos\beta - sin\alpha\, sin\beta$ **and** $sin(\alpha+\beta)=cos\alpha\, sin\beta + cos\beta\, sin\alpha$

$z_1 = r_1(cos\theta_1 + i\,sin\theta_1)$ **and** $z_2 = r_2(cos\theta_2 + i\,sin\theta_2)$

Product:

$$\begin{aligned} z_1 z_2 &= r_1(cos\theta_1 + i\,sin\theta_1)\cdot r_2(cos\theta_2 + i\,sin\theta_2) \\ &= r_1 r_2\left[(cos\theta_1\, cos\theta_2 - sin\theta_1\, sin\theta_2) + i(cos\theta_1\, sin\theta_2 + cos\theta_2\, sin\theta_1)\right] \\ &= r_1 r_2\left[cos(\theta_1+\theta_2) + i\,sin(\theta_1+\theta_2)\right] \end{aligned}$$

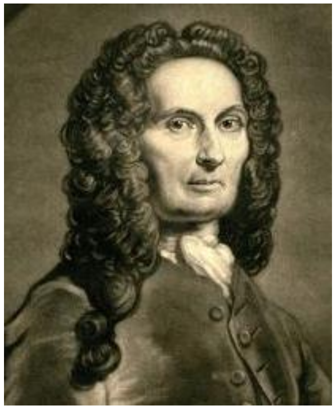
Similarly,

Quotient: $\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2}\left[cos(\theta_1-\theta_2) + i\,sin(\theta_1-\theta_2)\right]$

Examples:

$$\left[2\left(cos\frac{5\pi}{12}+i\,sin\frac{5\pi}{12}\right)\right]\cdot\left[3\left(cos\frac{\pi}{12}+i\,sin\frac{\pi}{12}\right)\right]=$$

$$\left[2\left(cos\frac{5\pi}{12}+i\,sin\frac{5\pi}{12}\right)\right]\div\left[3\left(cos\frac{\pi}{12}+i\,sin\frac{\pi}{12}\right)\right]=$$



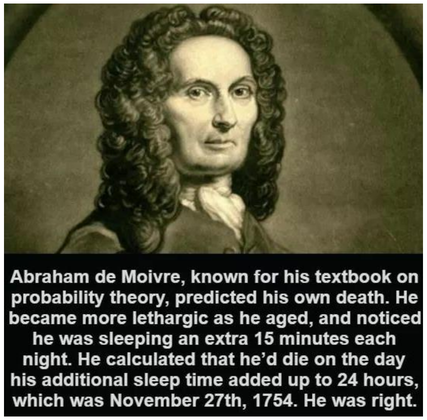
De Moivre's Theorem:

For $z = r(\cos\theta + i\sin\theta)$ **and** ***n*****, an integer,** $z^n = r^n\left[\cos(n\theta) + i\sin(n\theta)\right]$**.**

Examples:

Find $(1+i)^8$**.**

Find $\left(\sqrt{3} - i\right)^6$**.**



Let's use De Moivre's Theorem to find some roots of imaginary numbers.

Find all the square-roots of *i*.

We want to find a complex number, $z=|z|(\cos\theta+i\sin\theta);0\le\theta<2\pi$**, so that** $z^2=i$**.**

This means that

$$|z|^2(\cos\theta+i\sin\theta)^2=0+i\Rightarrow|z|^2(\cos2\theta+i\sin2\theta)=0+i$$

$$\Rightarrow|z|^2(\cos2\theta+i\sin2\theta)=\cos\left(2n\pi+\frac{\pi}{2}\right)+i\sin\left(2n\pi+\frac{\pi}{2}\right)$$

So $2\theta=2n\pi+\frac{\pi}{2}=\frac{\pi}{2},2\pi+\frac{\pi}{2},4\pi+\frac{\pi}{2},\ldots\Rightarrow\theta=n\pi+\frac{\pi}{4}=\frac{\pi}{4},\pi+\frac{\pi}{4},2\pi+\frac{\pi}{4},\ldots$**, and** $|z|=1$**. The only values of** θ **with** $0\le\theta<2\pi$ **are** $\frac{\pi}{4}$ **and** $\frac{5\pi}{4}$**.**

So the two square-roots of *i* are $\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}$ **and** $\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}=\frac{-\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}$**.**

Find all the cube-roots of $4\sqrt{3}+4i$**.**

We want to find a complex number, $z=|z|(\cos\theta+i\sin\theta);0\le\theta<2\pi$**, so that** $z^3=4\sqrt{3}+4i$**.**

This means that

$$\left[|z|(\cos\theta+i\sin\theta)\right]^3=4\sqrt{3}+4i\Rightarrow|z|^3(\cos3\theta+i\sin3\theta)=4\sqrt{3}+4i$$

$$\Rightarrow|z|^3(\cos3\theta+i\sin3\theta)=8\left[\cos\left(2n\pi+\frac{\pi}{6}\right)+i\sin\left(2n\pi+\frac{\pi}{6}\right)\right]$$

So

$$3\theta=2n\pi+\frac{\pi}{6}=\frac{\pi}{6},2\pi+\frac{\pi}{6},4\pi+\frac{\pi}{6},\ldots\Rightarrow\theta=\frac{2n\pi}{3}+\frac{\pi}{18}=\frac{\pi}{18},\frac{2\pi}{3}+\frac{\pi}{18},\frac{4\pi}{3}+\frac{\pi}{18},2\pi+\frac{\pi}{18},\ldots$$

, and the only values of θ **with** $0\le\theta<2\pi$ **are** $\frac{\pi}{18}$**,** $\frac{2\pi}{3}+\frac{\pi}{18}=\frac{13\pi}{18}$**, and** $\frac{4\pi}{3}+\frac{\pi}{18}=\frac{25\pi}{18}$**.**

We also need to have that $|z|=2$**. So the three cube-roots of** $4\sqrt{3}+4i$ **are** $2\left(\cos\frac{\pi}{18}+i\sin\frac{\pi}{18}\right)$**,** $2\left(\cos\frac{13\pi}{18}+i\sin\frac{13\pi}{18}\right)$**, and** $2\left(\cos\frac{25\pi}{18}+i\sin\frac{25\pi}{18}\right)$**.**