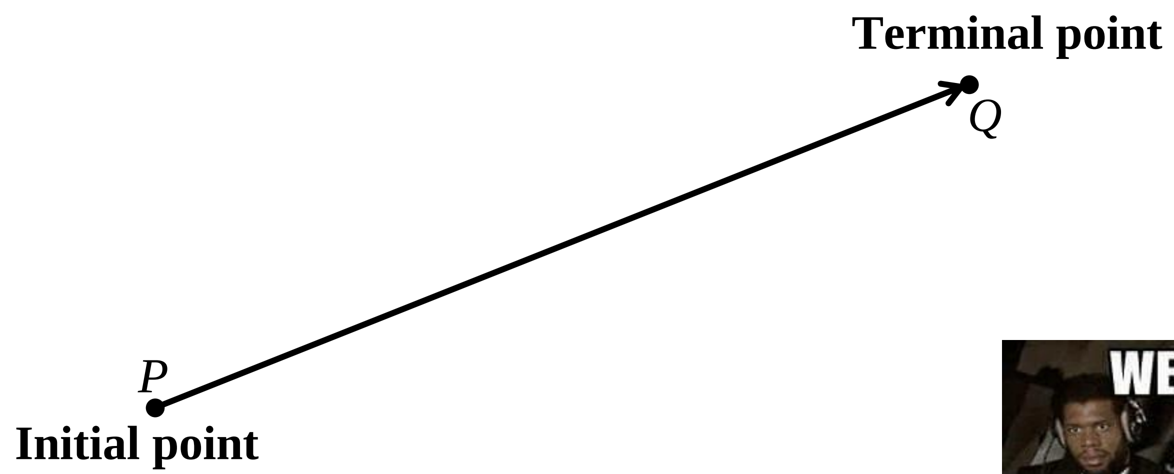
Vectors:

For us, vectors are directed line segments(arrows). They represent quantities that have both a direction and a magnitude(amount).



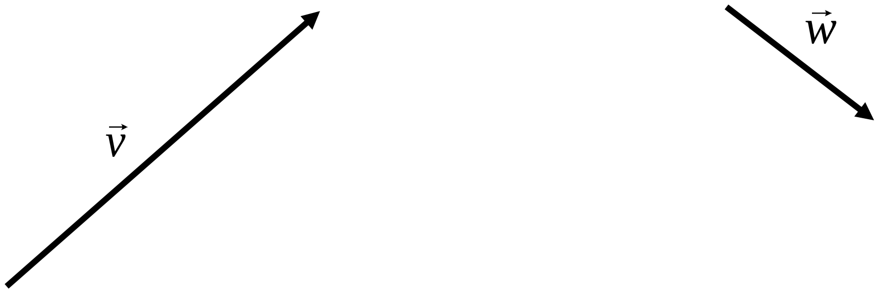
$\overrightarrow{PQ}$ or $\vec{v}$

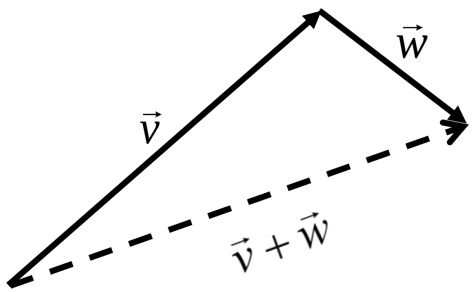
The length of a vector is its magnitude, and the direction can be interpreted as an angle or slope of the arrow. There is a special vector whose initial and terminal points are the same, i.e. its magnitude is zero. This vector is called the zero vector, $\vec{0}$.

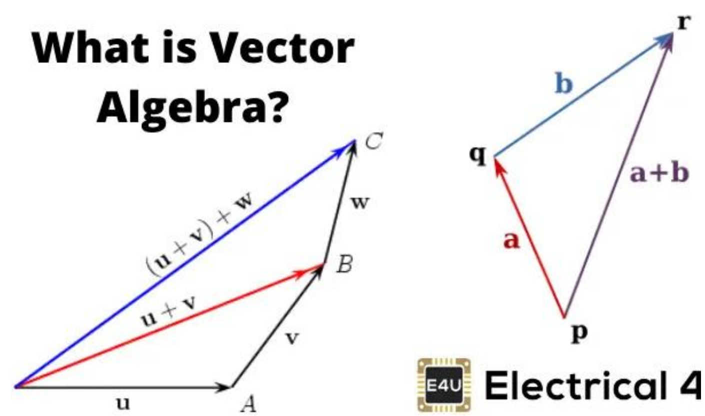
VECTOR: I'm applying for a villain loan. I go by Vector. It's a mathematical term, represented by an arrow with both direction and magnitude. Vector! That's me, because I commit crimes with both direction and magnitude. Oh yeah!

<u>The Sum of Two Vectors:</u>

$\vec{v} + \vec{w}$

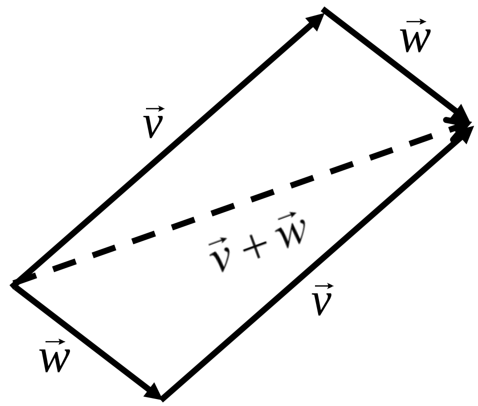






Properties of Vector Addition:

1. $\vec{v}+\vec{w}=\vec{w}+\vec{v}$**, Commutative Property.**

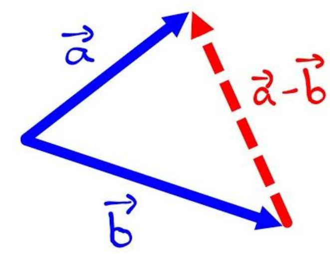


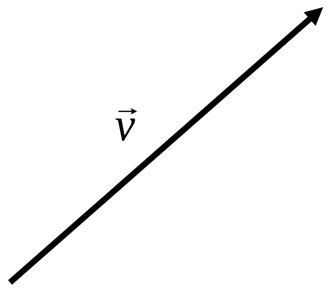
"It's a discipline note from my teacher. I solved the math problems in my head and I was supposed to use a calculator."

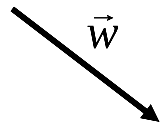
2. $\vec{u}+\left(\vec{v}+\vec{w}\right)=\left(\vec{u}+\vec{v}\right)+\vec{w}$**, Associative Property**

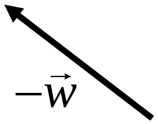
3. $\vec{v}+\vec{0}=\vec{0}+\vec{v}=\vec{v}$**, Zero Vector Property**

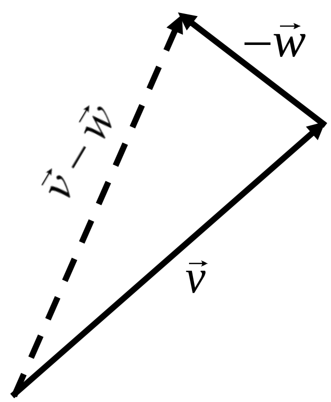
4. $\vec{v}-\vec{w}=\vec{v}+\left(-\vec{w}\right)$**, where** $-\vec{w}$ **is a vector with same magnitude as** $\vec{w}$ **but the opposite direction. Vector Subtraction.**

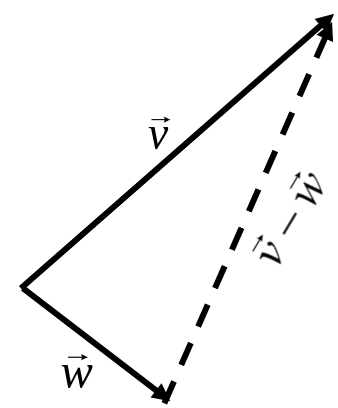


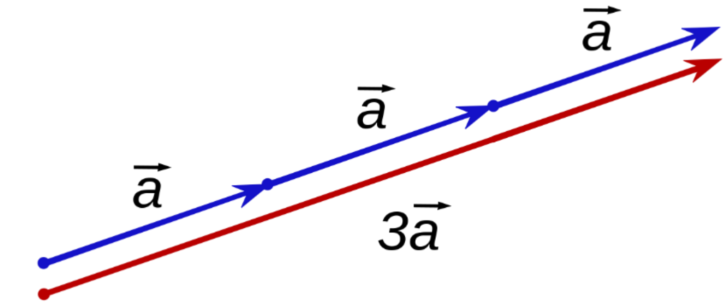








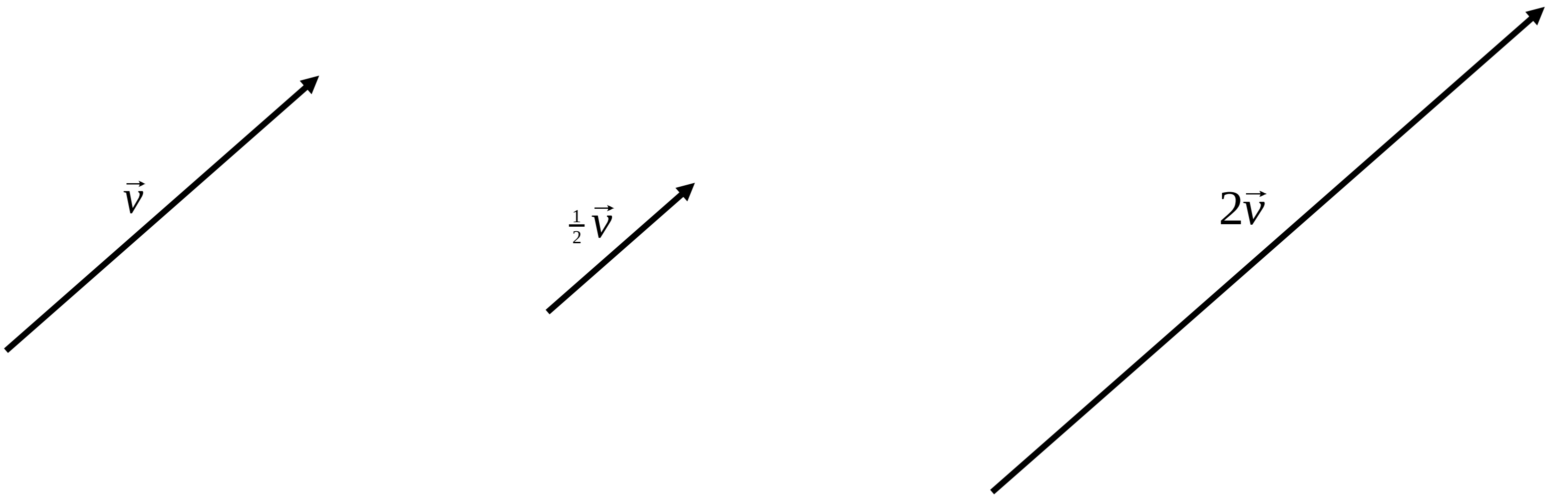


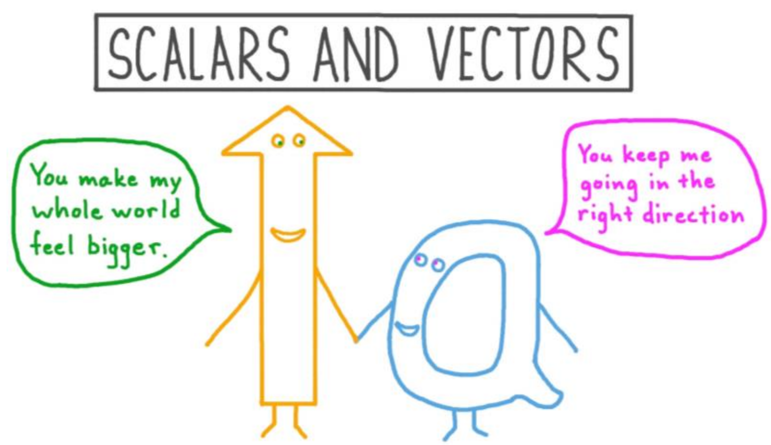


Multiplying Vectors by Numbers:

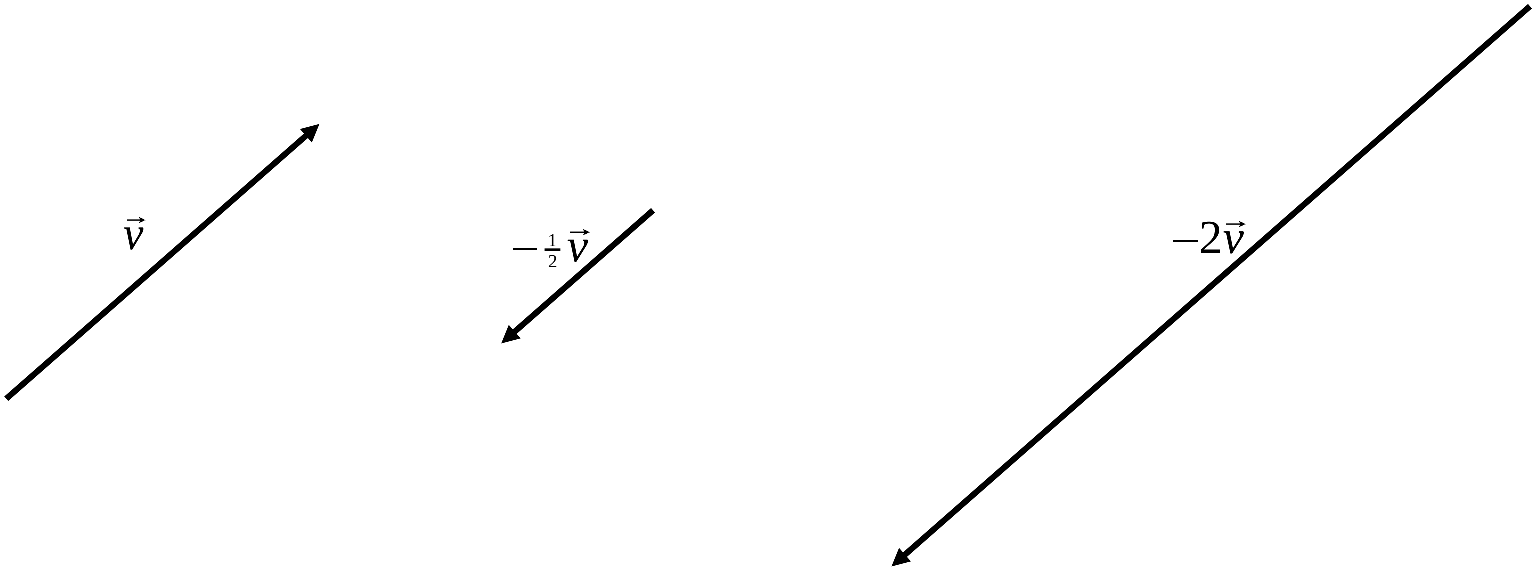
The process is called scalar multiplication.

If $\alpha > 0$, then the vector $\alpha\vec{v}$ is the vector with the same direction as $\vec{v}$, but whose magnitude is α times the magnitude of $\vec{v}$.





If $\alpha < 0$, then the vector $\alpha\vec{v}$ is the vector with the opposite direction as $\vec{v}$, but whose magnitude is $|\alpha|$ times the magnitude of $\vec{v}$.



If $\alpha = 0$ or $\vec{v} = \vec{0}$ then $\alpha\vec{v} = \vec{0}$.

"You can feel good knowing your lesson plans put up a good fight."

Properties of Scalar Multiplication:

1. $0\vec{v} = \vec{0}$

2. $1\vec{v} = \vec{v}$

3. $-1\vec{v} = -\vec{v}$

4. $(\alpha + \beta)\vec{v} = \alpha\vec{v} + \beta\vec{v}$

5. $\alpha(\vec{v} + \vec{w}) = \alpha\vec{v} + \alpha\vec{w}$

6. $\alpha(\beta\vec{v}) = (\alpha\beta)\vec{v}$

The games get pretty crazy
at English teachers' parties.

<u>Magnitude of a Vector:</u>

The abbreviation for the magnitude of the vector $\vec{v}$, is $\|\vec{v}\|$.

<u>Properties of Magnitude:</u>

1. $\|\vec{v}\| \geq 0$

2. $\|\vec{v}\| = 0$ **if and only if** $\vec{v} = \vec{0}$

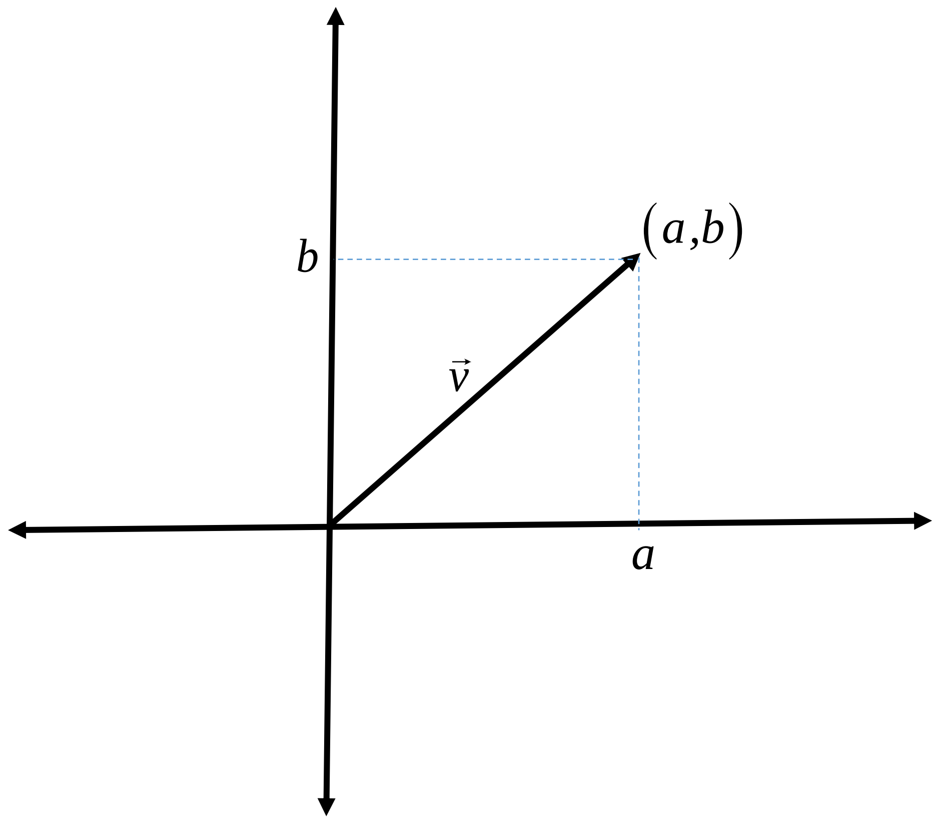
3. $\|-\vec{v}\| = \|\vec{v}\|$

4. $\|\alpha\vec{v}\| = |\alpha|\|\vec{v}\|$

If $\|\vec{v}\| = 1$, then $\vec{v}$ is called a <u>unit vector</u>.

<u>Algebraic or Component Form of Vectors:</u>

Place the initial point of the vector $\vec{v}$ at the origin.



The component form of the vector $\vec{v}$ is $\langle a,b \rangle$, i.e. $\vec{v} = \langle a,b \rangle$. In general, if you know the coordinates of the initial point, (x_1, y_1), and terminal point, (x_2, y_2), of $\vec{v}$, then $\vec{v} = \langle x_2 - x_1, y_2 - y_1 \rangle$.

Example:

Find the component form of the vector that connects the point $(1,-2)$ to the point $(-3,4)$.

There is an alternative method of representing the component form of a vector using the special unit vectors $\vec{i}$ and $\vec{j}$.

$\vec{i}$ **is a unit vector pointing in the same direction as the positive *x*-axis, and** $\vec{j}$ **is a unit vector pointing in the same direction as the positive *y*-axis.**

So if $\vec{v} = \langle a, b \rangle$**, then** $\vec{v} = a\vec{i} + b\vec{j}$ **.**

<u>Vector Operations Using Algebraic Vectors:</u>

If $\vec{v} = \langle a_1, b_1 \rangle$ **and** $\vec{w} = \langle a_2, b_2 \rangle$ **then**

1. $\vec{v} + \vec{w} = \langle a_1 + a_2, b_1 + b_2 \rangle$

2. $\vec{v} - \vec{w} = \langle a_1 - a_2, b_1 - b_2 \rangle$

3. $\alpha\vec{v} = \langle \alpha a_1, \alpha b_1 \rangle$

4. $\|\vec{v}\| = \sqrt{a_1^2 + b_1^2}$

Examples:

If $\vec{v} = \langle 2, -3 \rangle$ **and** $\vec{w} = \langle 4, 5 \rangle$ **then find**

$$\vec{v} + \vec{w}$$

$$\vec{v} - \vec{w}$$

$$5\vec{v}$$

$$3\vec{v} - 2\vec{w}$$

$$\|\vec{v}\|$$

Finding Unit Vectors with the Same Direction:

If $\vec{v}$ **is a non-zero vector, then the vector** $\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}$ **is a unit vector with same direction as** $\vec{v}$**.**

Show why.

Example:

Find a unit vector with the same direction as the vector $\vec{v} = \langle 2, -3 \rangle$**.**

If $\vec{u}$ is a unit vector with the same direction as $\vec{v}$, then $\vec{v} = \|\vec{v}\|\vec{u}$.

If you place the initial point of a unit vector, $\vec{u}$, at the origin, its terminal point will lie on the unit circle. It will determine an angle α with the positive *x*-axis with $0 \le \alpha < 2\pi$.

α is called the direction angle of the unit vector $\vec{u}$, and
$\vec{u} = \langle \cos\alpha, \sin\alpha \rangle = \cos\alpha\vec{i} + \sin\alpha\vec{j}$.

So if you are given the magnitude $\|\vec{v}\|$ of a non-zero vector $\vec{v}$ and a direction angle α, then $\vec{v} = \|\vec{v}\|\langle \cos\alpha, \sin\alpha \rangle = \|\vec{v}\|\left(\cos\alpha\vec{i} + \sin\alpha\vec{j}\right)$.

Examples:

1. Find the direction angle of the vector $\vec{v} = \left\langle 1, -\sqrt{3} \right\rangle$**.**

2. Find the vector of magnitude 6 and direction angle of $\frac{5\pi}{6}$**.**