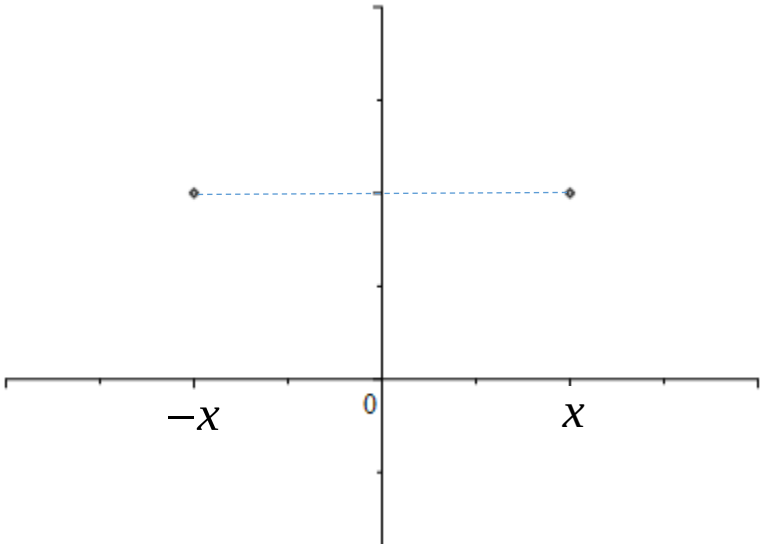
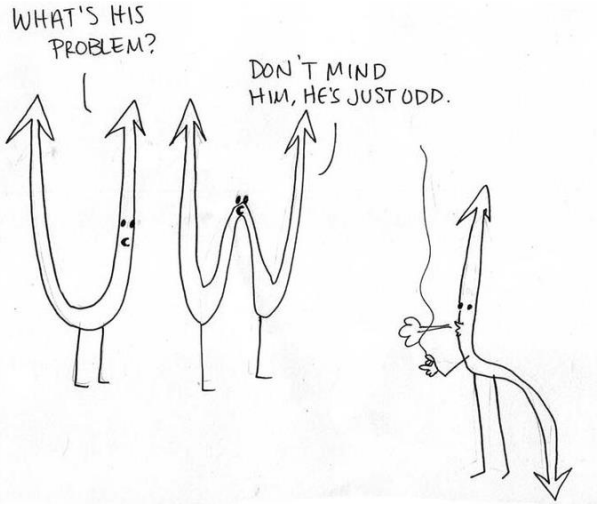
Review of Even and Odd Functions:

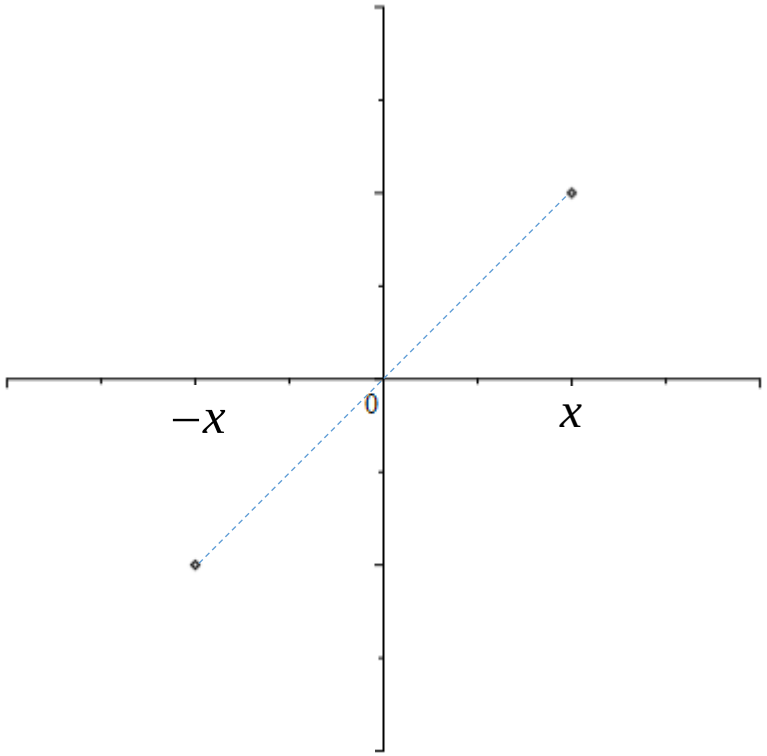
A function *f* is even if $f(-x) = f(x)$ for all *x* in the domain of *f*.

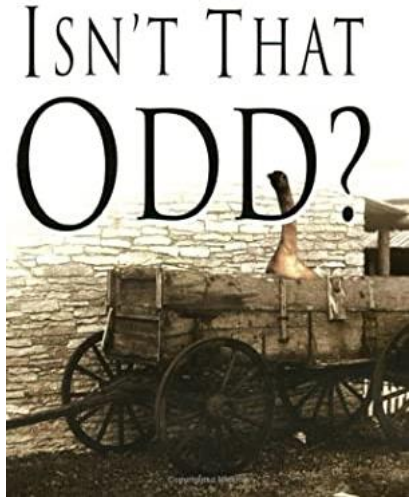


The graph has *y*-axis symmetry.

A function *f* is odd if $f(-x) = -f(x)$ for all *x* in the domain of *f*.





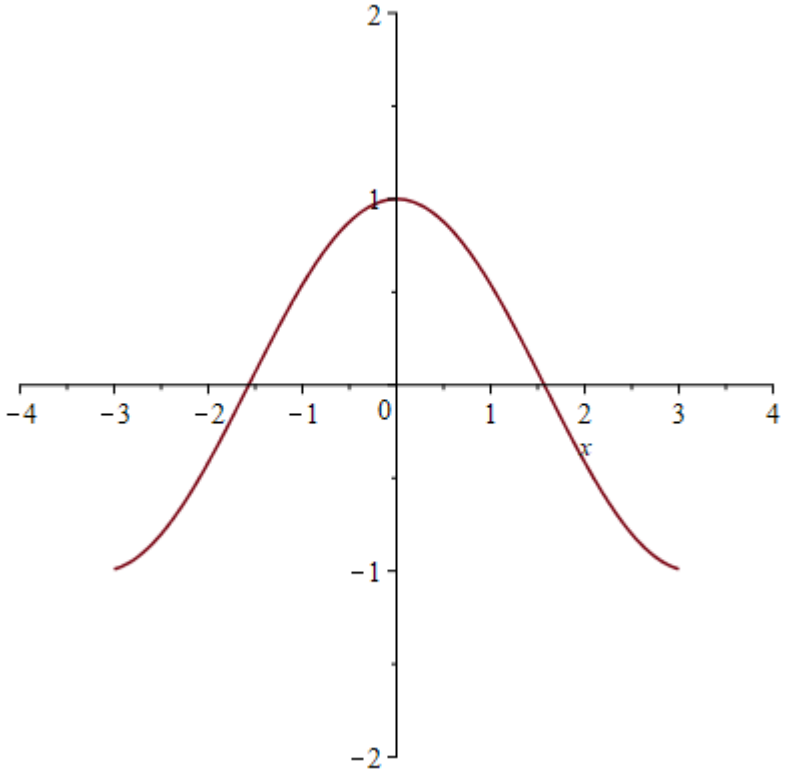


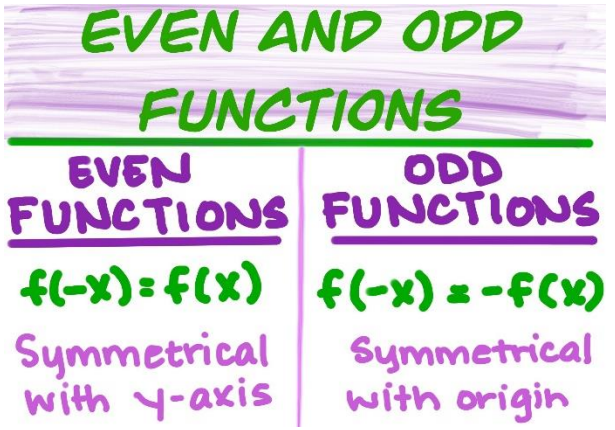
The graph has origin symmetry.

If zero is in the domain of an odd function, *f*, what must be the value of $f(0)$?

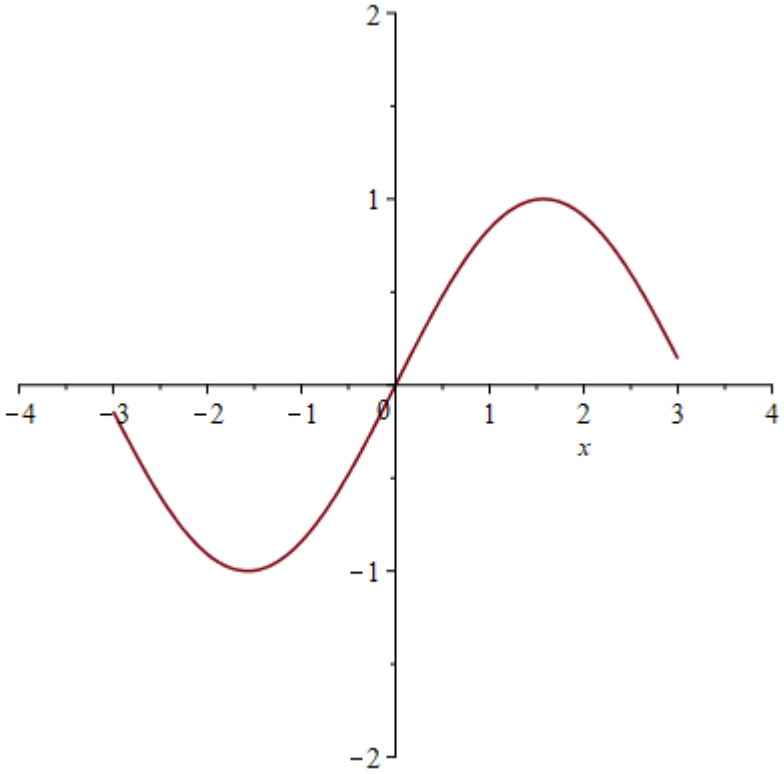
Determine if the following functions are odd, even, neither, or both.

1.

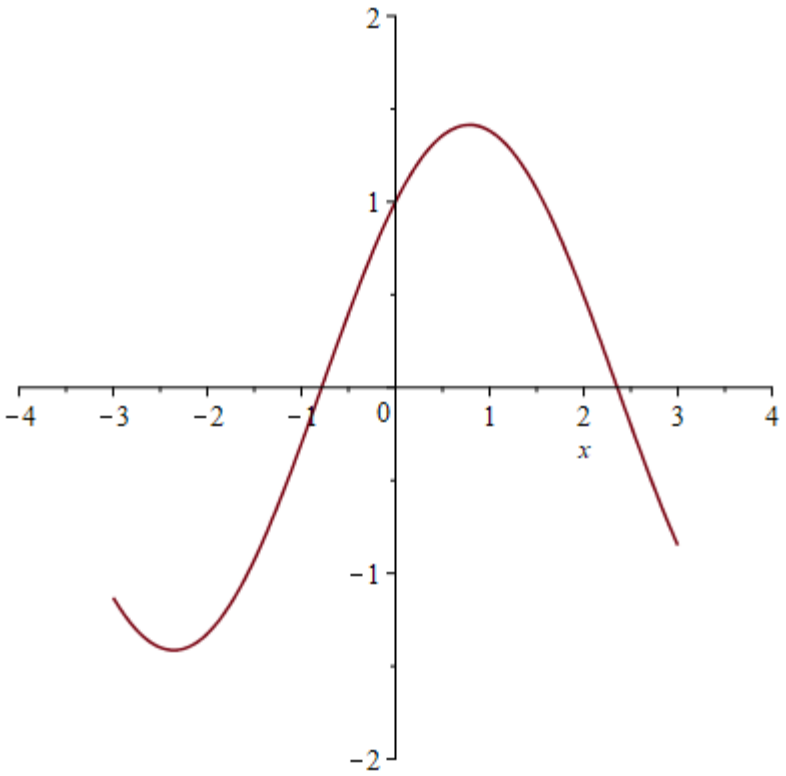




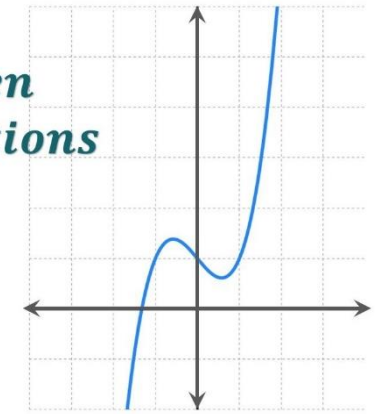
2.



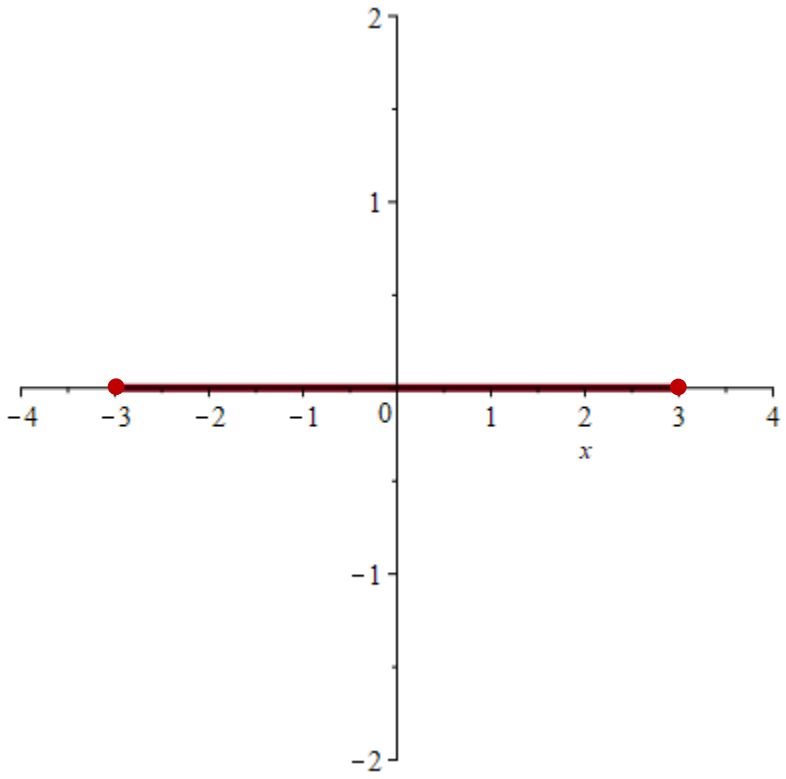
3.



Neither Even nor Odd Functions



4.



5. $f(x) = 2x^4 - x^2$

$f(-x) = 2(-x)^4 - (-x)^2$

$=$

6. $f(x) = x^3 + x$

$f(-x) = (-x)^3 + (-x)$

$=$

7. $f\left(x\right)=x+x^{2}$

8. $f\left(x\right)=\left(x+1\right)^{2}-\left(x-1\right)^{2}-4x$ ***{Simplify the formula, first!}***

9. $f(x)=\begin{cases} x^3 \ ; x\geq 0 \\ -x^3 ; x\leq 0 \end{cases} \Rightarrow f(-x)=\begin{cases} (-x)^3 \ ; -x\geq 0 \\ -(-x)^3 ; -x\leq 0 \end{cases}=\begin{cases} -x^3 \ ; x\leq 0 \\ x^3 \ ; x\geq 0 \end{cases}$

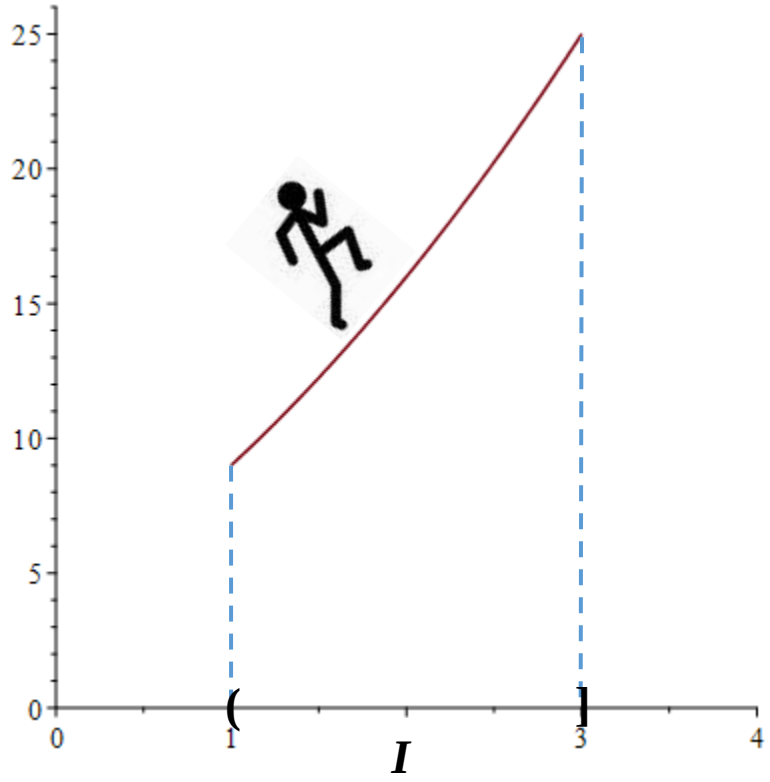
$=\begin{cases} x^3 \ ; x\geq 0 \\ -x^3 \ ; x\leq 0 \end{cases}$

$=$

10. $f(x)=\begin{cases} x-2; 1\leq x\leq 3 \\ x+2; -3\leq x\leq -1 \end{cases} \Rightarrow f(-x)=\begin{cases} (-x)-2; 1\leq -x\leq 3 \\ (-x)+2; -3\leq -x\leq -1 \end{cases}=$

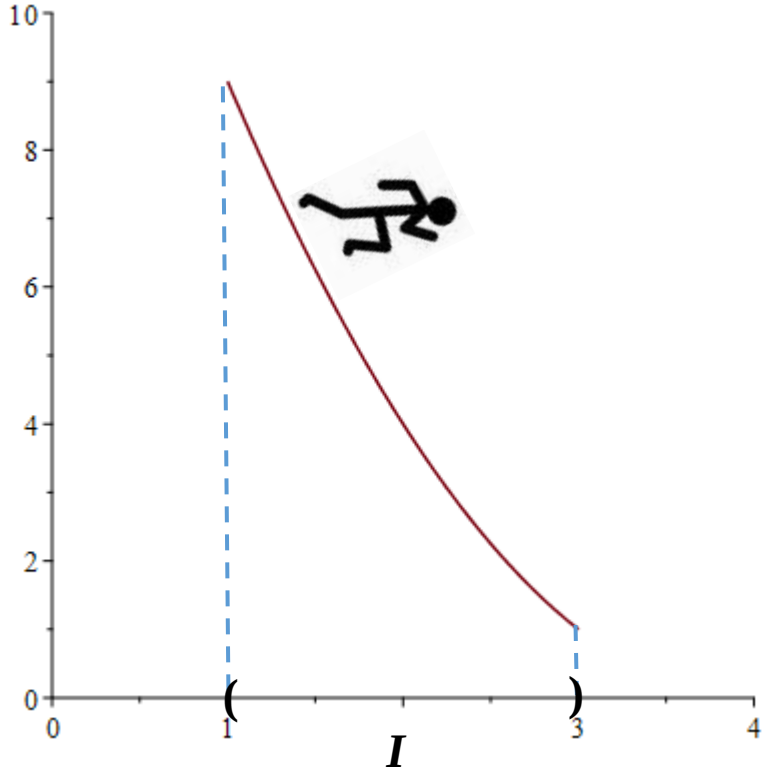
Increasing, Decreasing, Constant:

A function *f* is increasing on an interval *I*, if for x, y in *I* with $x < y$, then $f(x) < f(y)$.



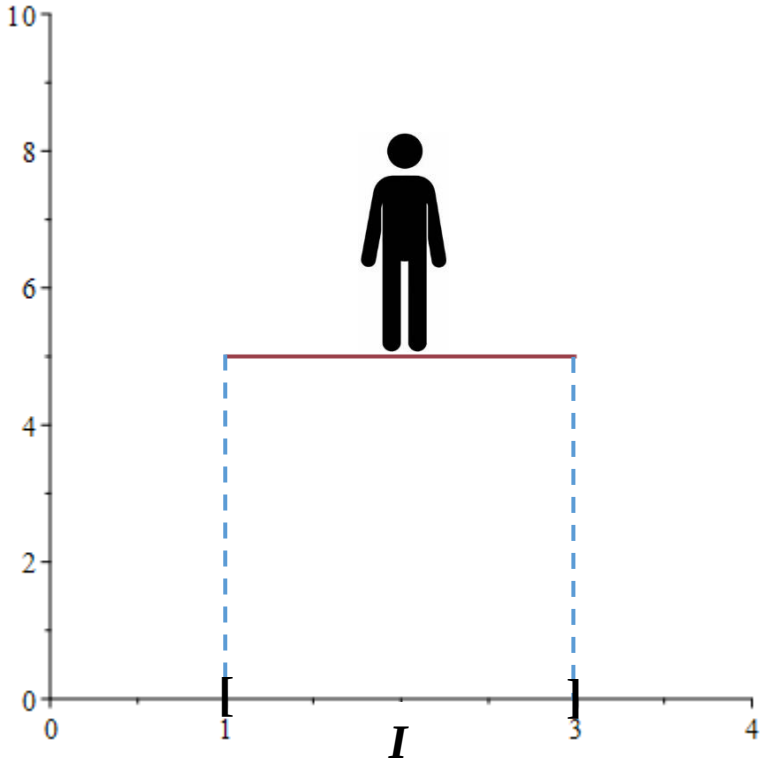
Moving up from left to right!

A function *f* is decreasing on an interval *I*, if for x, y in *I* with $x < y$, then $f(x) > f(y)$.

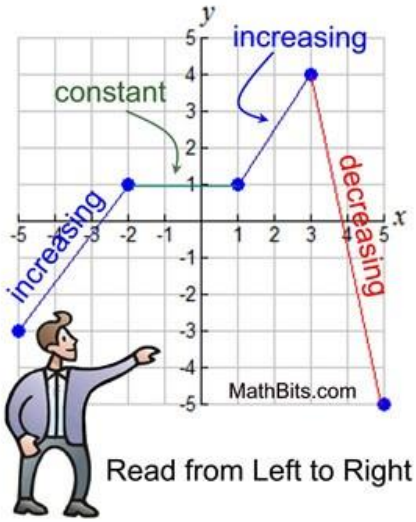


Moving down from left to right!

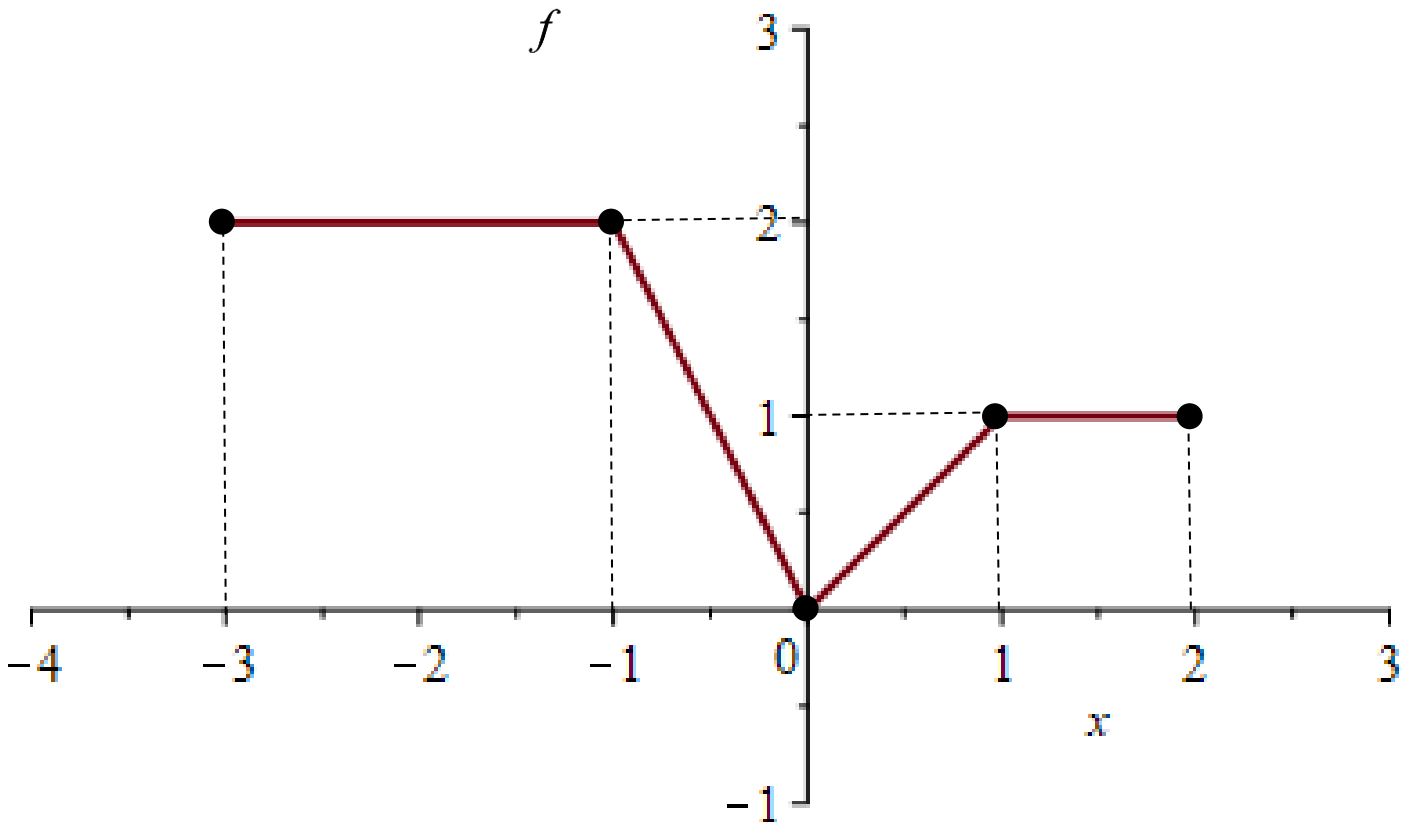
A function *f* is constant on an interval *I*, if for x, y in *I*, then $f(x) = f(y)$.



Level ground!



Determine the intervals where *f* is increasing, decreasing, and constant.

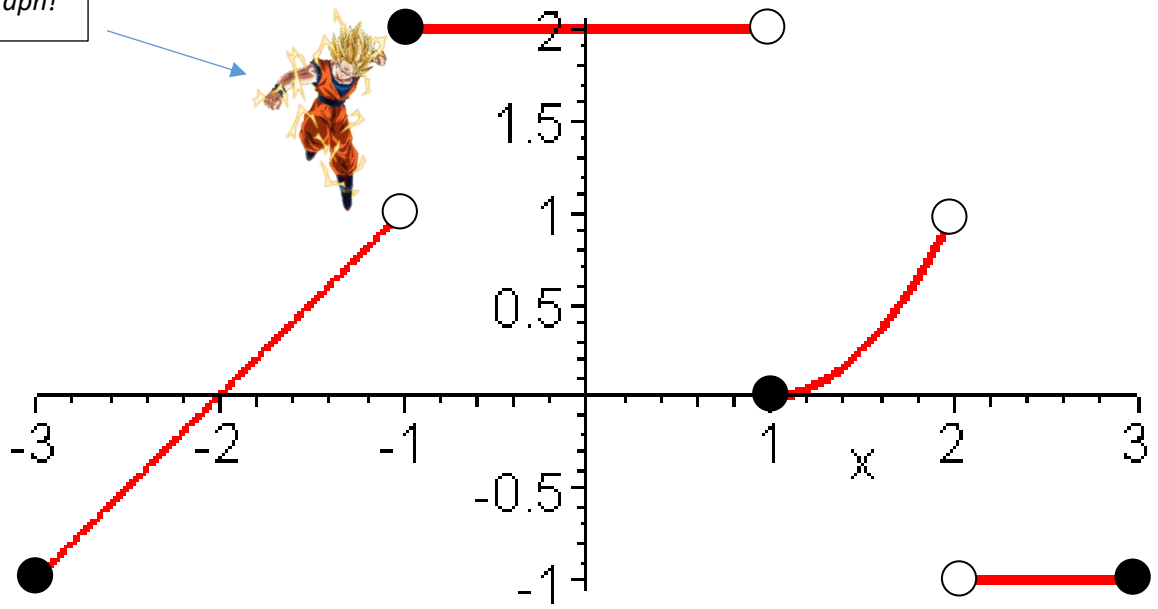


Increasing **Decreasing** **Constant**

Determine the intervals where *g* is increasing, decreasing, and constant.

g

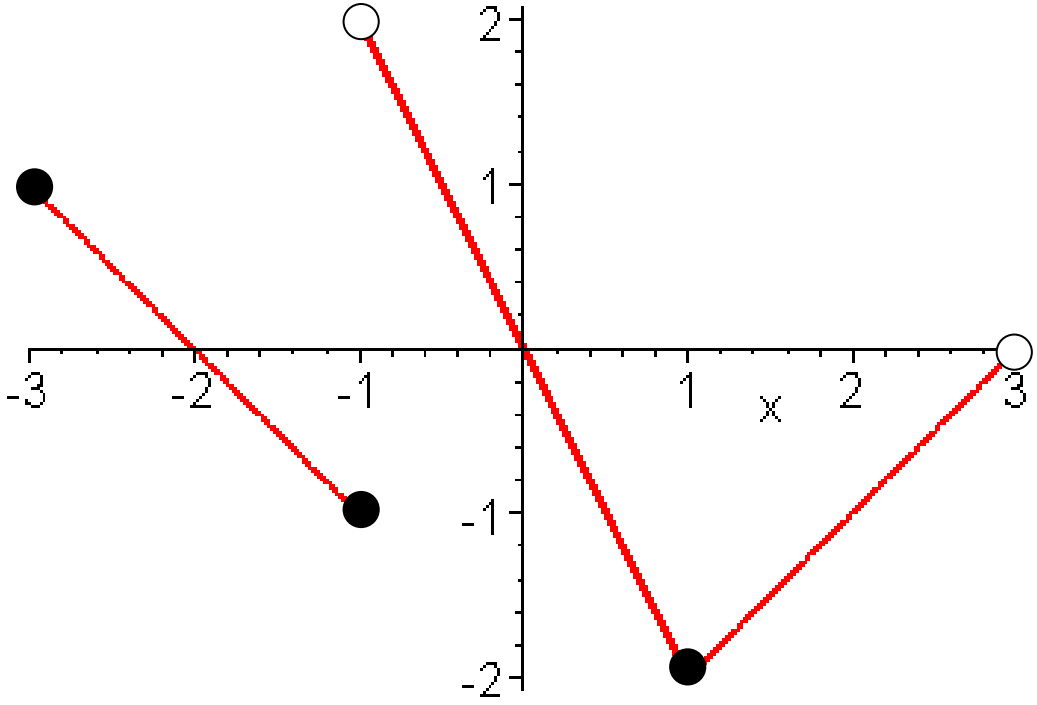
Note: Goku is not part of the graph!



<u>Increasing</u> **<u>Decreasing</u>** **<u>Constant</u>**

Determine the intervals where *h* is increasing, decreasing, and constant.

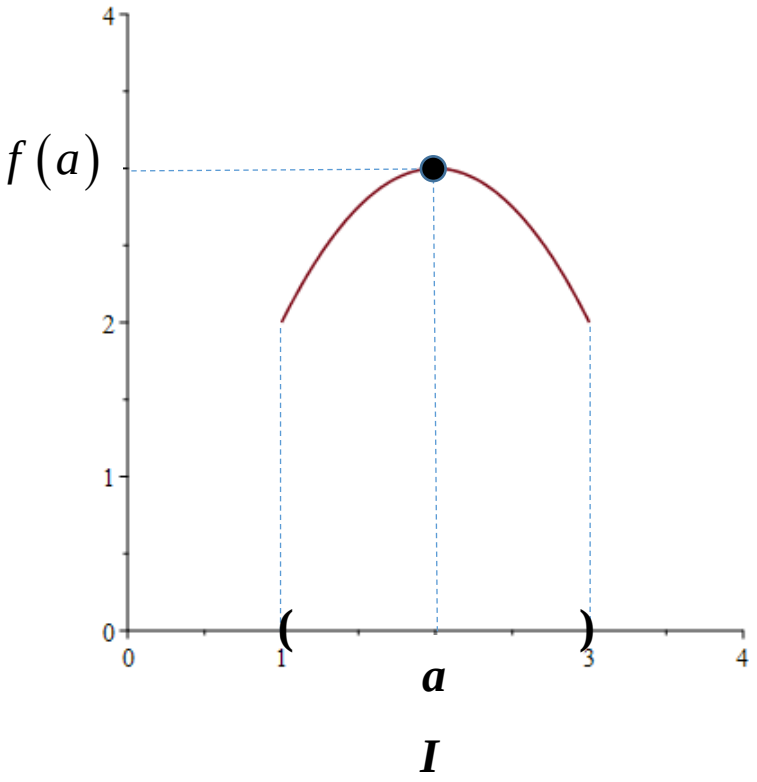
h

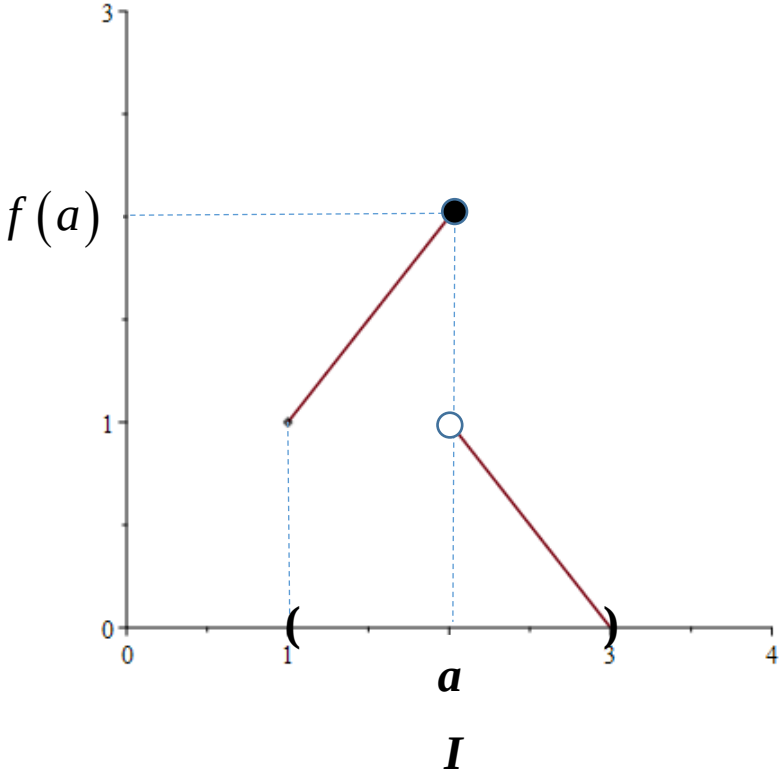


<u>Increasing</u> **<u>Decreasing</u>** **<u>Constant</u>**

Local Extrema:

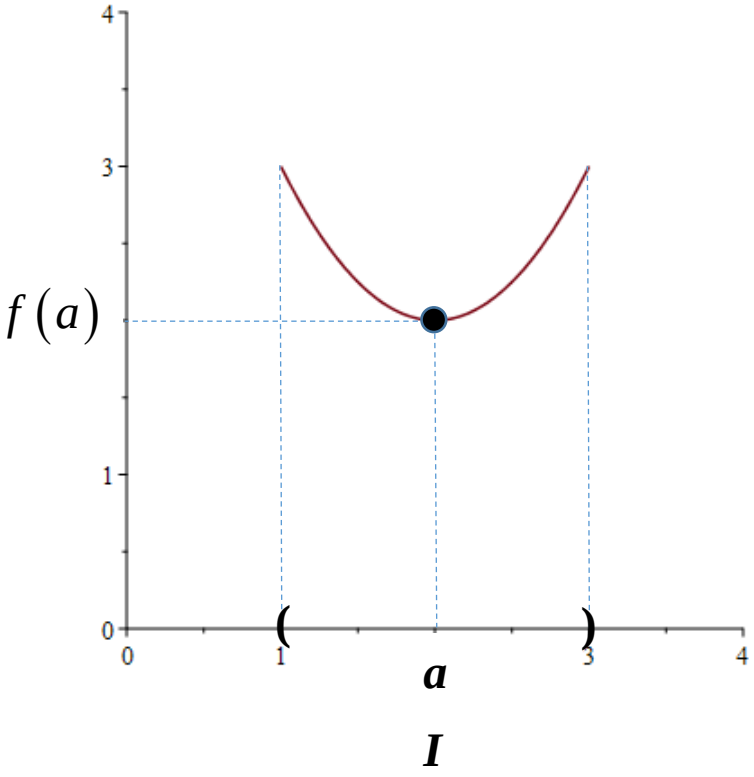
A function *f* has a local maximum at *a*, if there is an open interval *I* containing *a* with $f(x) < f(a)$ for all *x* in *I* with $x \neq a$.

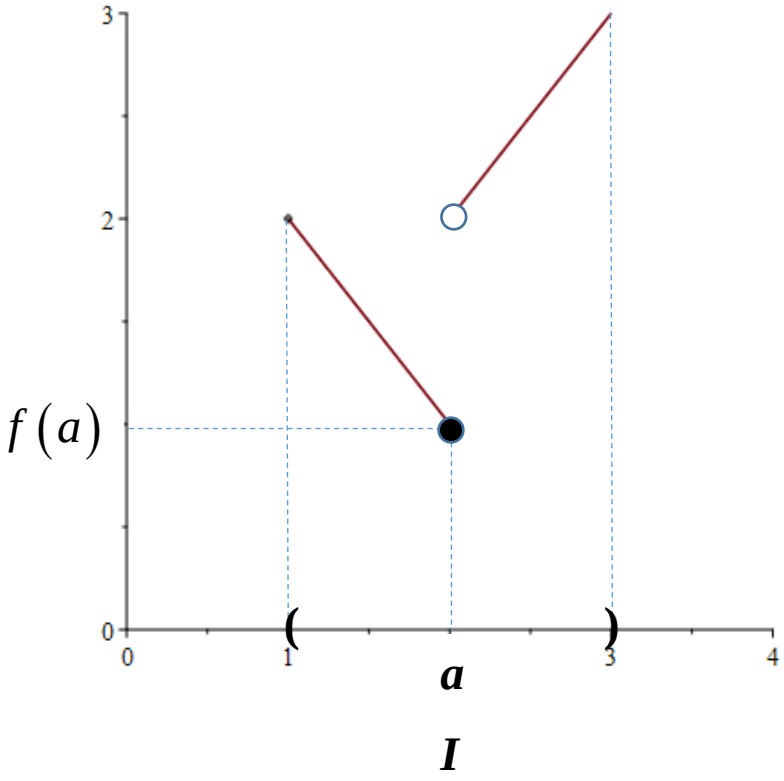




A local maximum corresponds to a high spot in the graph of the function!

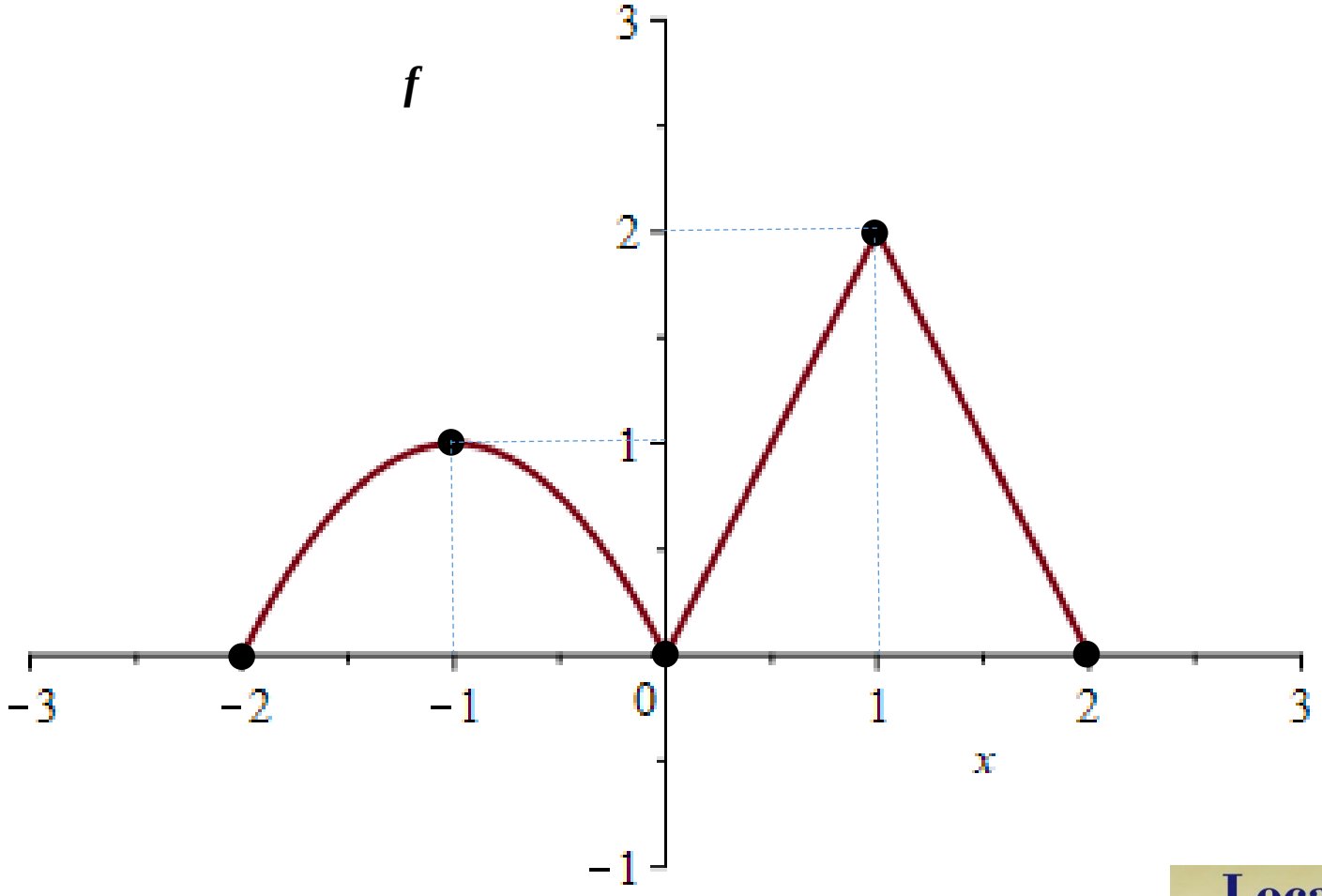
A function *f* has a local minimum at *a*, if there is an open interval *I* containing *a* with $f(x) > f(a)$ for all *x* in *I* with $x \neq a$.





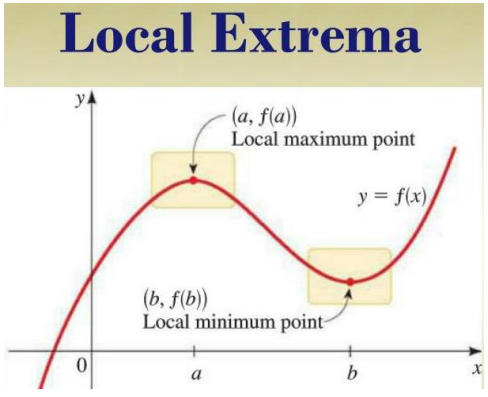
A local minimum corresponds to a low spot in the graph of the function!

Find all the local extrema of the function *f*.



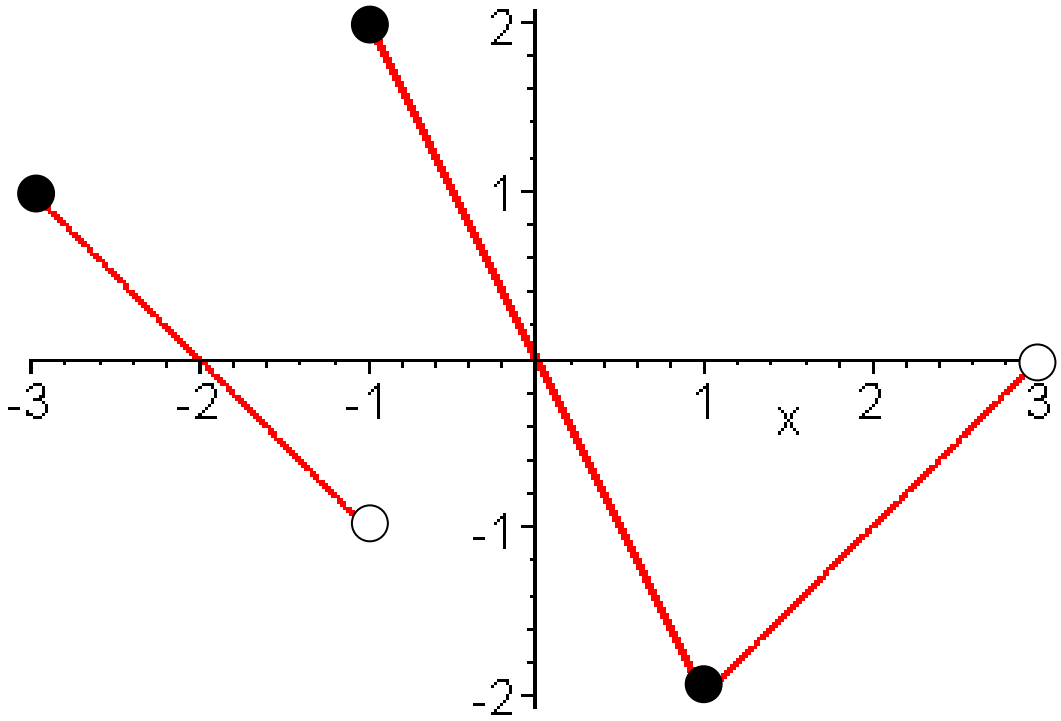
Local Maxima

Local Minima



Find all the local extrema of the function *g*.

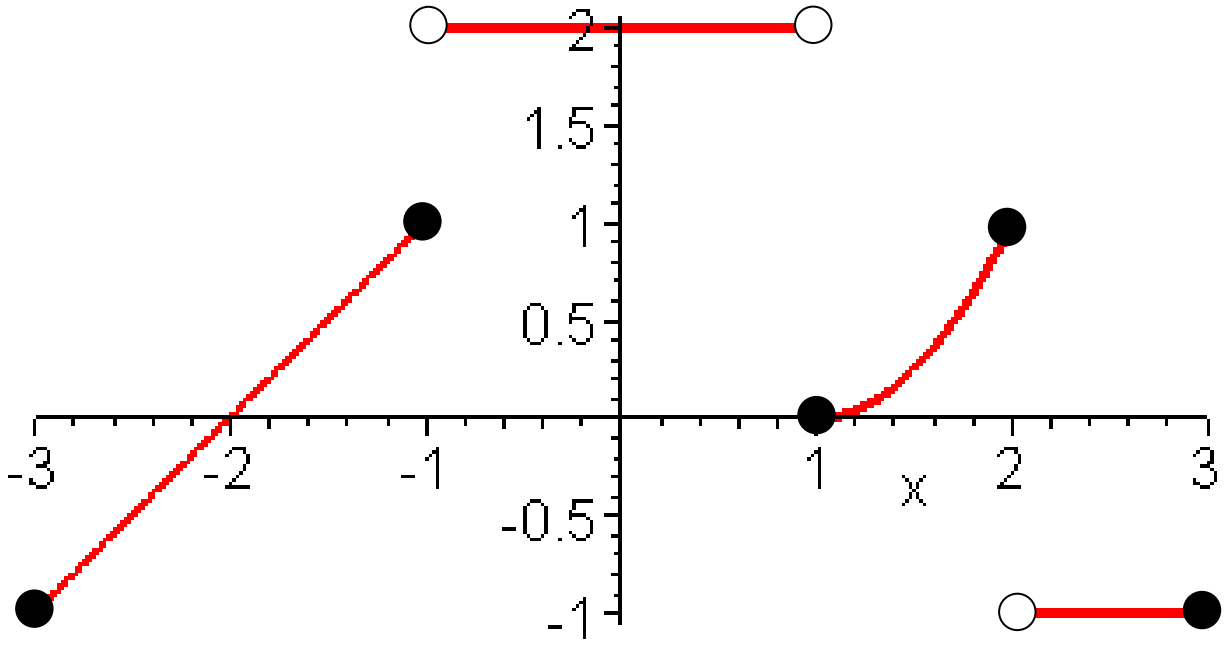
g



<u>Local Maxima</u> **<u>Local Minima</u>**

Find all the local extrema of the function *h*.

h



Local Maxima

Local Minima

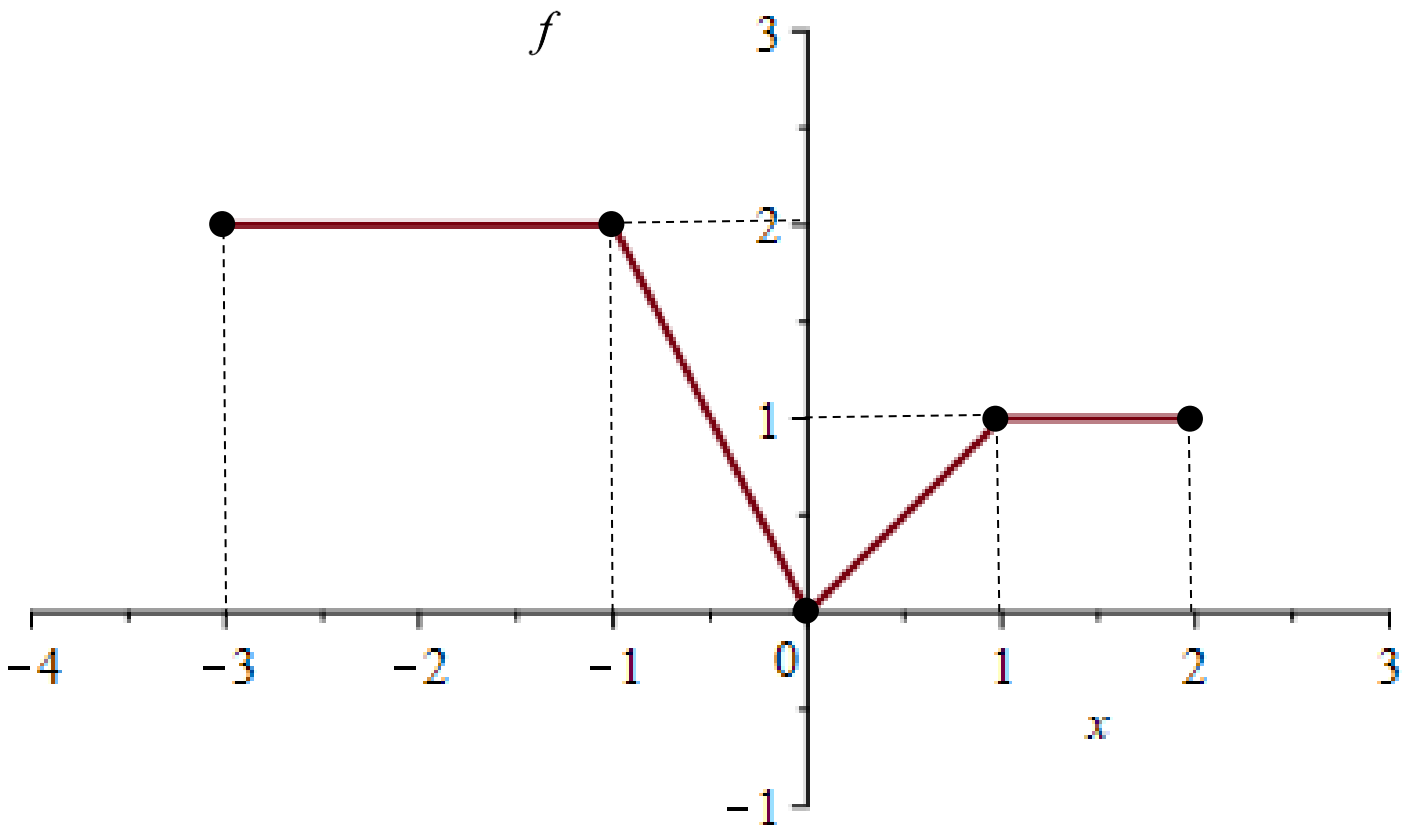
Absolute Extrema:

A function *f* has an absolute maximum at *a*, if $f(x) \leq f(a)$ for all *x* in the domain of *f*. In this case, $f(a)$ is called the absolute maximum value of the function.

If the graph of a function has a highest point, then it corresponds to an absolute maximum. The absolute maximum can occur at more than one point, but its value is unique.

A function *f* has an absolute minimum at *a*, if $f(x) \geq f(a)$ for all *x* in the domain of *f*. In this case, $f(a)$ is called the absolute minimum value of the function.

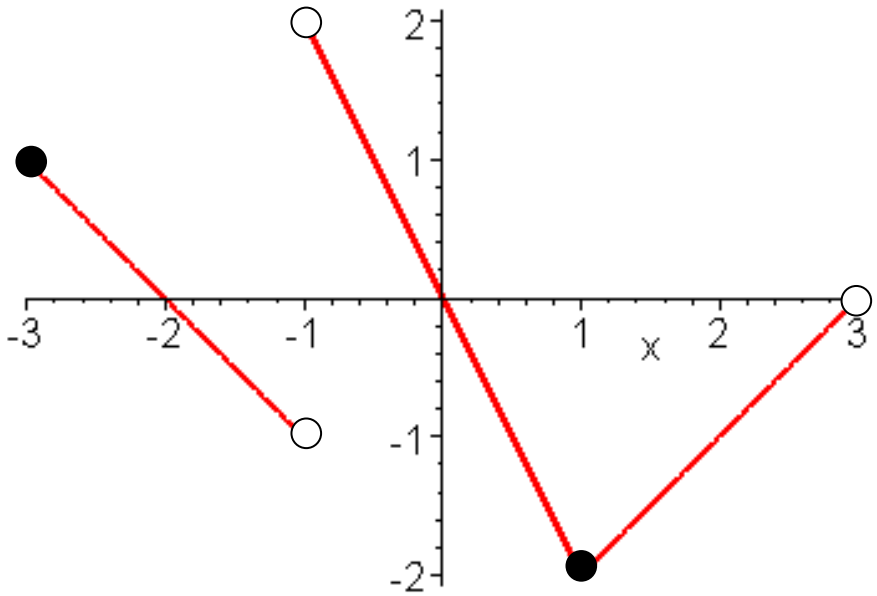
If the graph of a function has a lowest point, then it corresponds to an absolute minimum. The absolute minimum can occur at more than one point, but its value is unique.



What is the absolute maximum value of the function *f*?

What is the absolute minimum value of the function *f*?

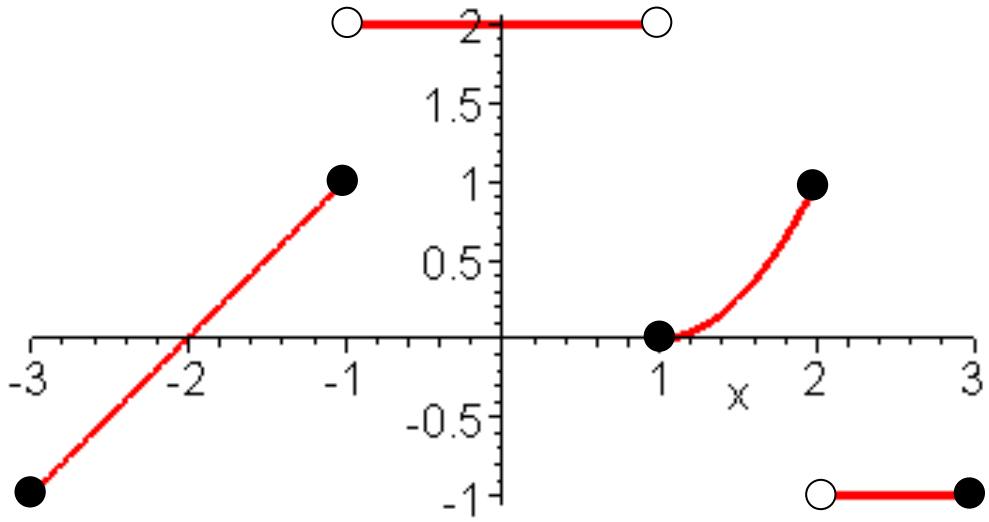
g



What is the absolute maximum value of the function *g*?

What is the absolute minimum value of the function *g*?

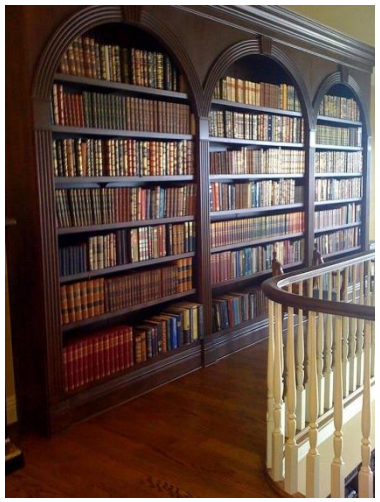
h



What is the absolute maximum value of the function *h*?

What is the absolute minimum value of the function *h*?

Library of Common Functions:



1. Linear Function:

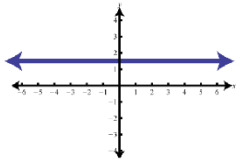
$f(x) = mx + b$

Graph? **Domain/Range?**

Increasing/decreasing/constant? **Local/Absolute extrema?**

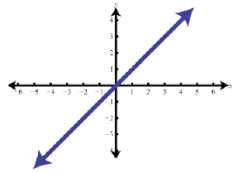
Constant Function

$f(x) = c$



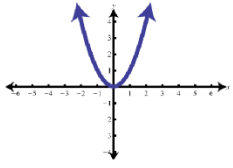
Identity Function

$f(x) = x$



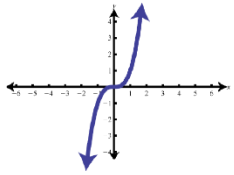
Squaring Function

$f(x) = x^2$



Cubing Function

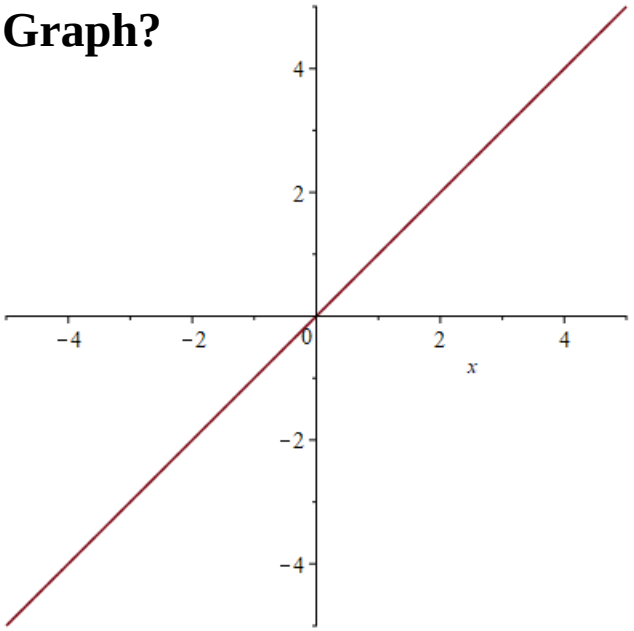
$f(x) = x^3$



2. Identity Function:

$f(x) = x$

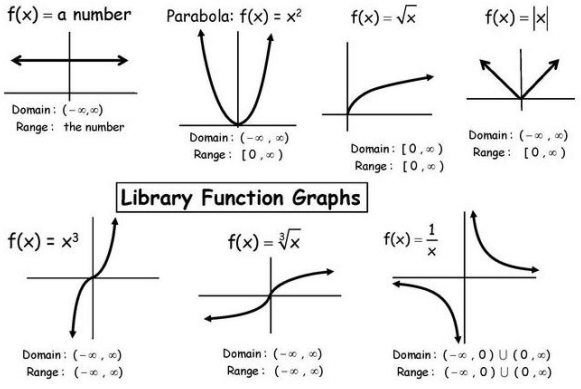
Graph?



Domain/Range?

Increasing/decreasing?

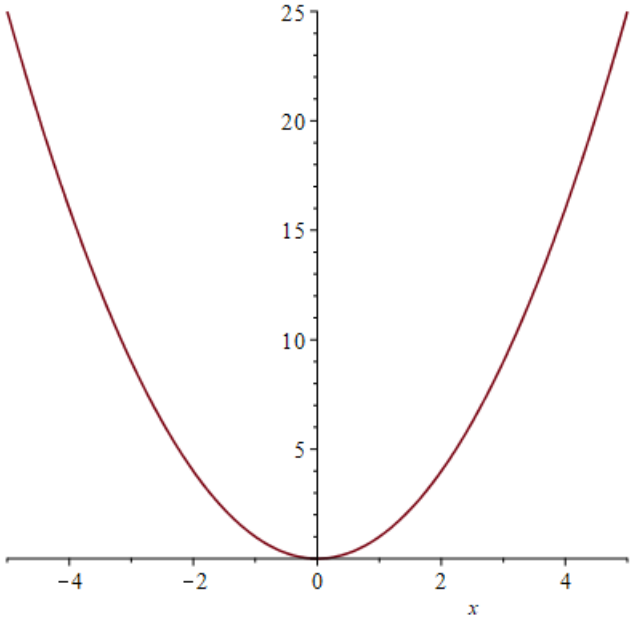
Local/Absolute extrema?



3. Squaring Function:

$f(x) = x^2$

Graph?



Domain/Range?

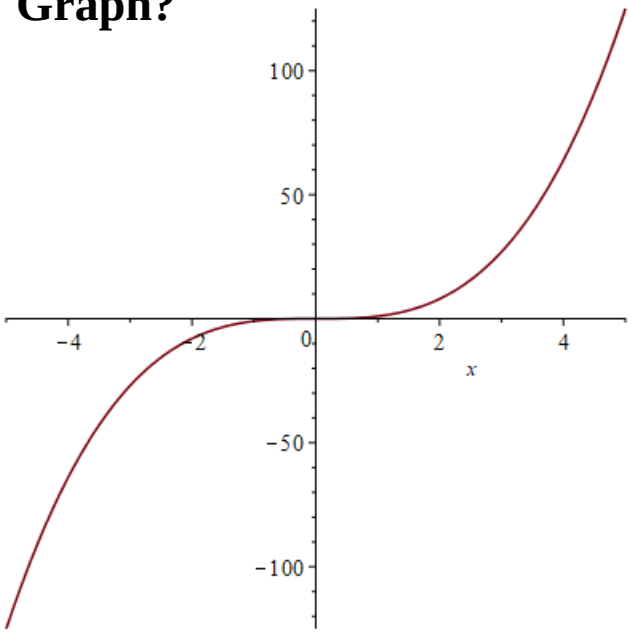
Increasing/decreasing?

Local/Absolute extrema?

4. Cubing Function:

$f(x) = x^3$

Graph?



Domain/Range?

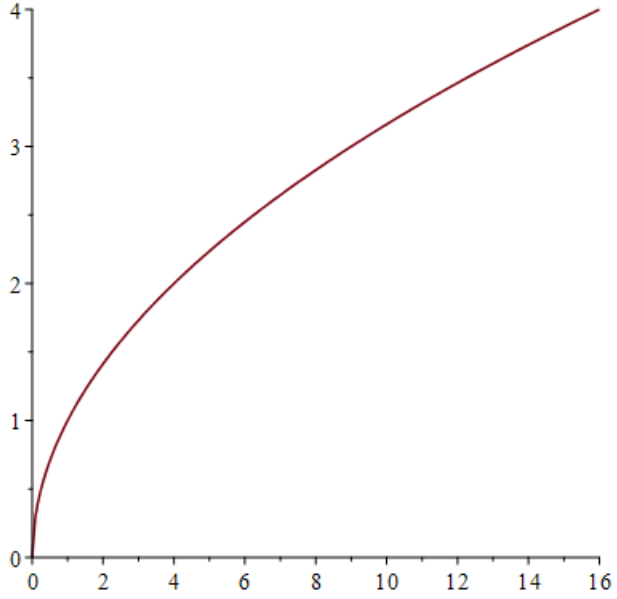
Increasing/decreasing?

Local/Absolute extrema?

5. Square Root Function:

$f(x) = \sqrt{x}$

Graph?



Domain/Range?

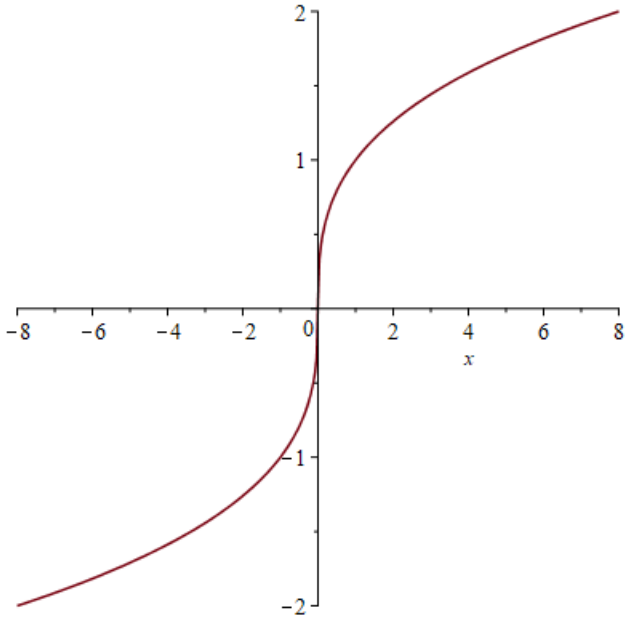
Increasing/decreasing?

Local/Absolute extrema?

6. Cube Root Function:

$$f(x) = \sqrt[3]{x}$$

Graph?



Domain/Range?

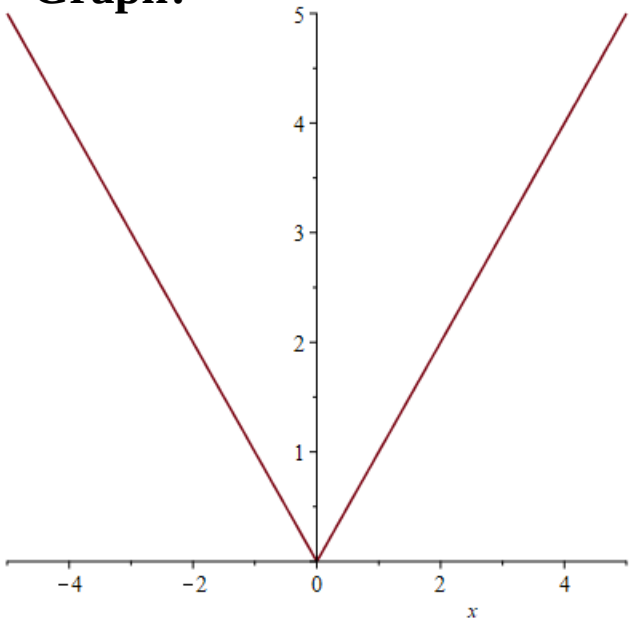
Increasing/decreasing?

Local/Absolute extrema?

7. Absolute Value Function:

$f(x) = |x|$

Graph?



Domain/Range?

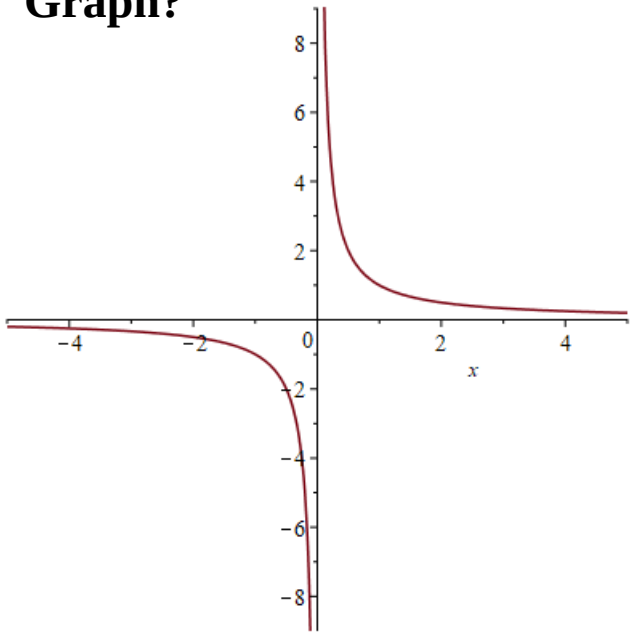
Increasing/decreasing?

Local/Absolute extrema?

8. Reciprocal Function:

$$f(x)=\frac{1}{x}$$

Graph?



Domain/Range?

Increasing/decreasing?

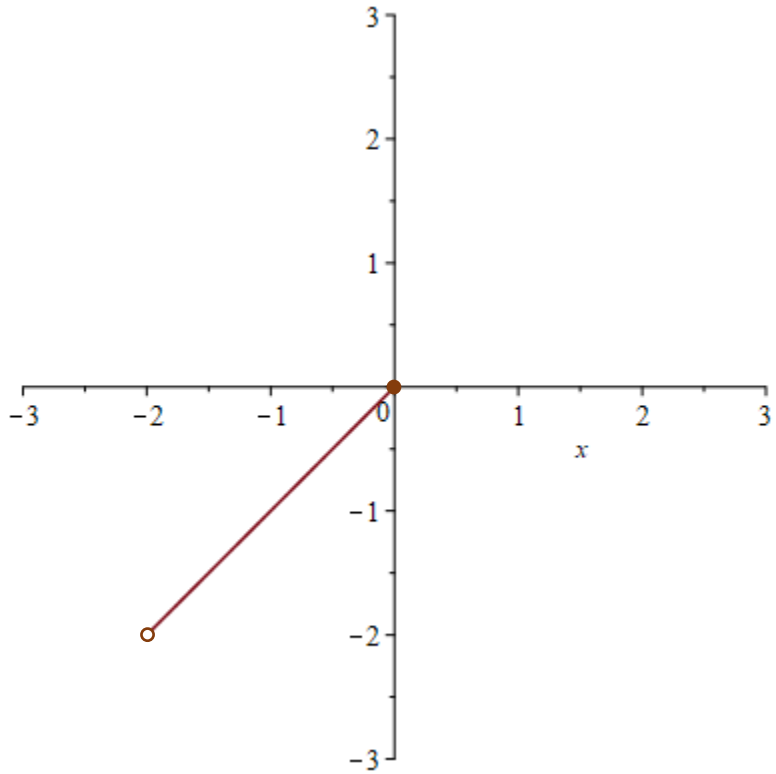
Local/Absolute extrema?

Graphing Piecewise-defined Functions Constructed From the Library Functions.

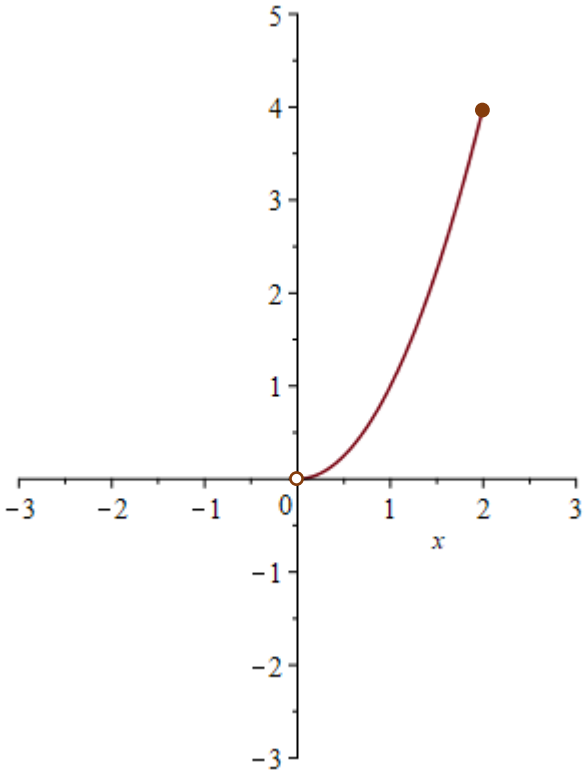
1. $f(x)=\begin{cases} x; -2 < x \le 0 \\ x^2; 0 < x \le 2 \end{cases}$

Graph?

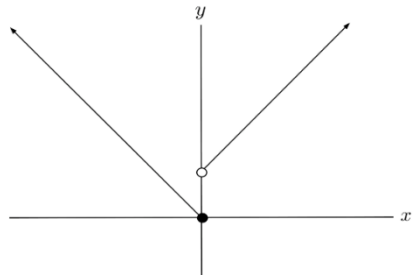
$x; -2 < x \le 0$



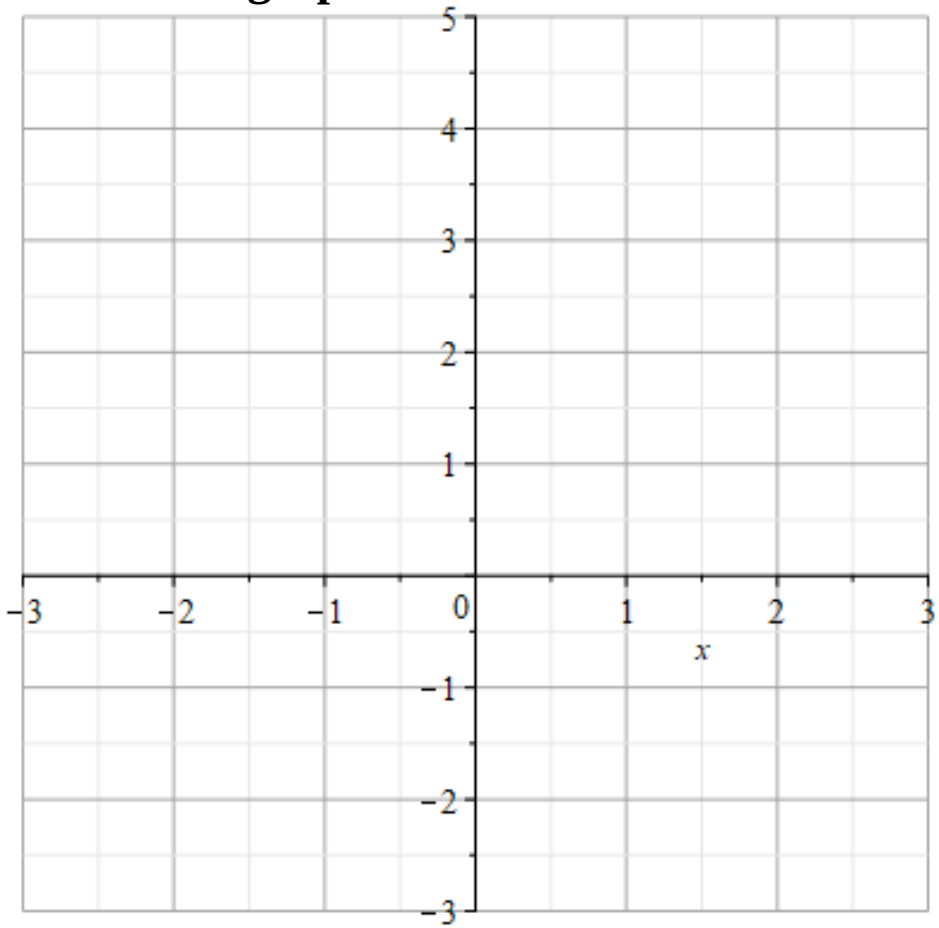
$x^2; 0 < x \le 2$



$$f(x)=\begin{cases} x+3 & \text{if } x > 0 \\ -x & \text{if } x \le 0 \end{cases}$$



Now put them together into one graph.



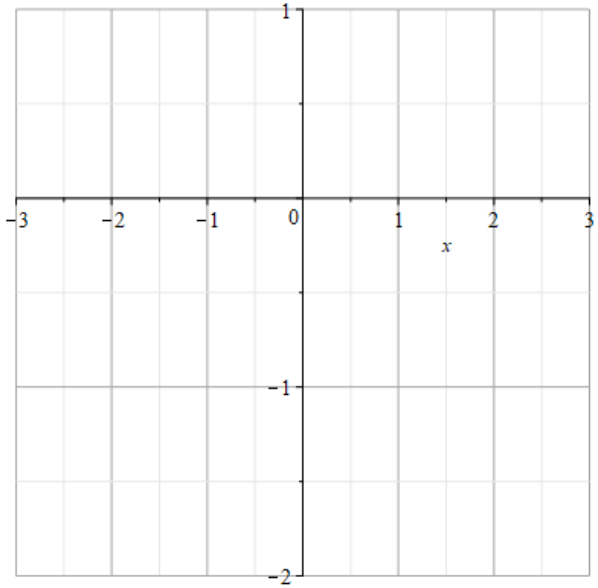
Domain? **Range?** **Increasing?**

Decreasing? **Local extrema?** **Absolute extrema?**

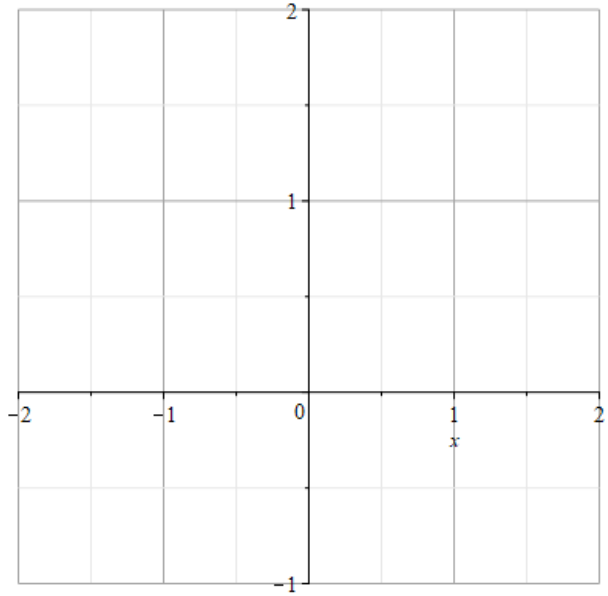
2. $g(x)=\begin{cases} x+1; -2 \le x < -1 \\ |x| \; ; -1 \le x < 1 \\ \sqrt{x} \; ; 1 \le x < 4 \end{cases}$

Graph?

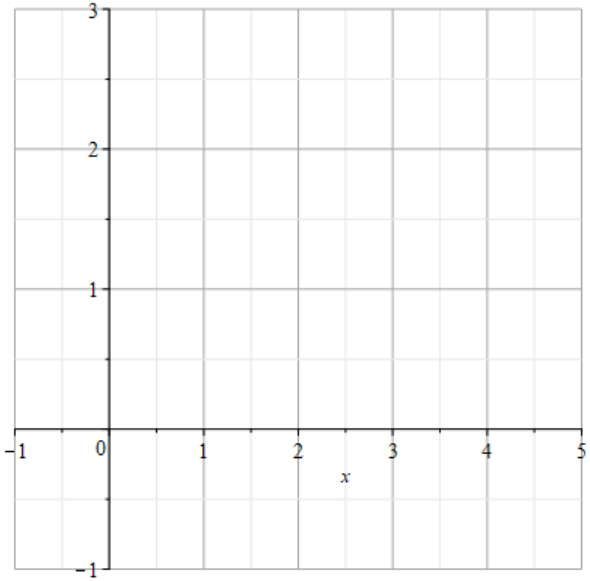
$x+1; -2 \le x < -1$



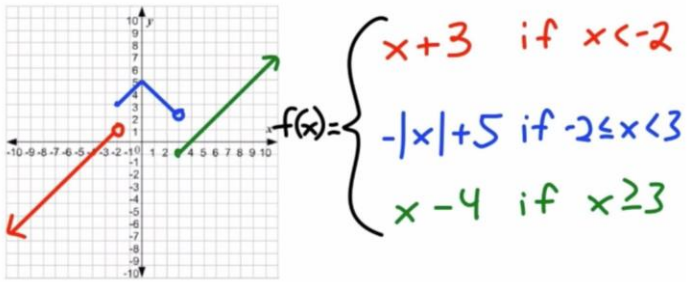
$|x|; -1 \le x < 1$



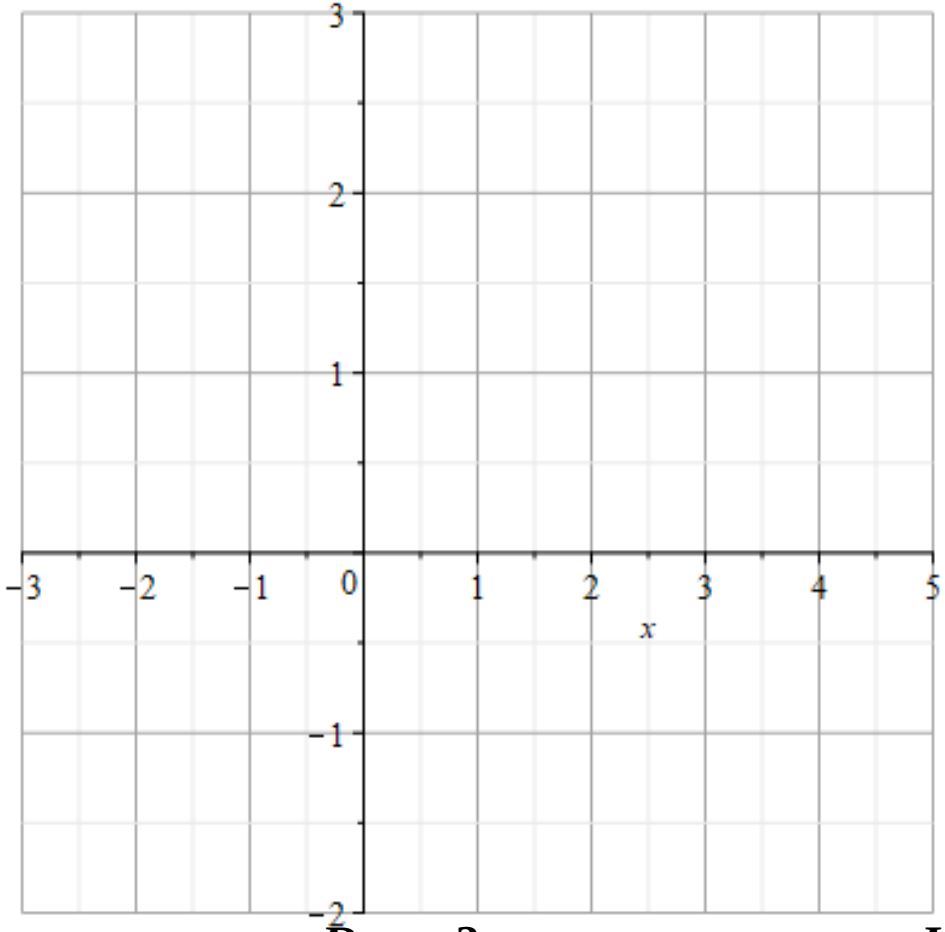
$\sqrt{x}; 1 \le x < 4$



Piecewise Functions



Now put them together into one graph.



Domain? **Range?** **Increasing?**

Decreasing? **Local extrema?** **Absolute extrema?**