Review of Transformations of the Graphs of Functions:

Vertical Shift

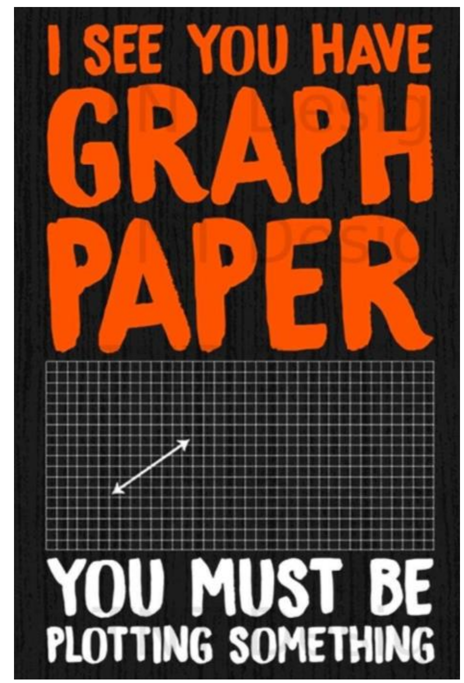
Horizontal Shift

Reflection about the *x*-axis

Reflection about the *y*-axis

Vertical Stretch/Compress

Horizontal Stretch/Compress



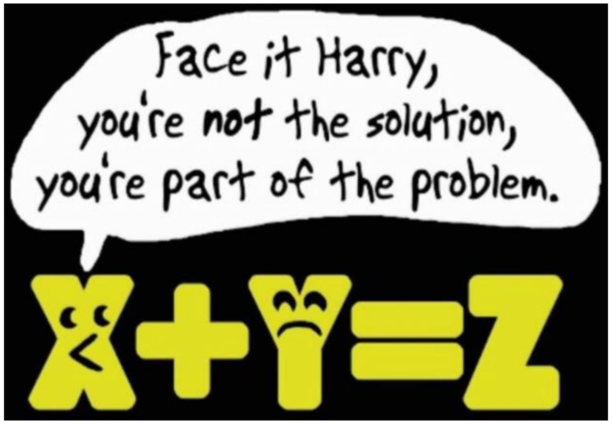
Vertical Shift:

For $c > 0$**,**

The graph of $g(x) = f(x) + c$**, is the graph of** $f(x)$ **shifted** ***c*** **units up.**

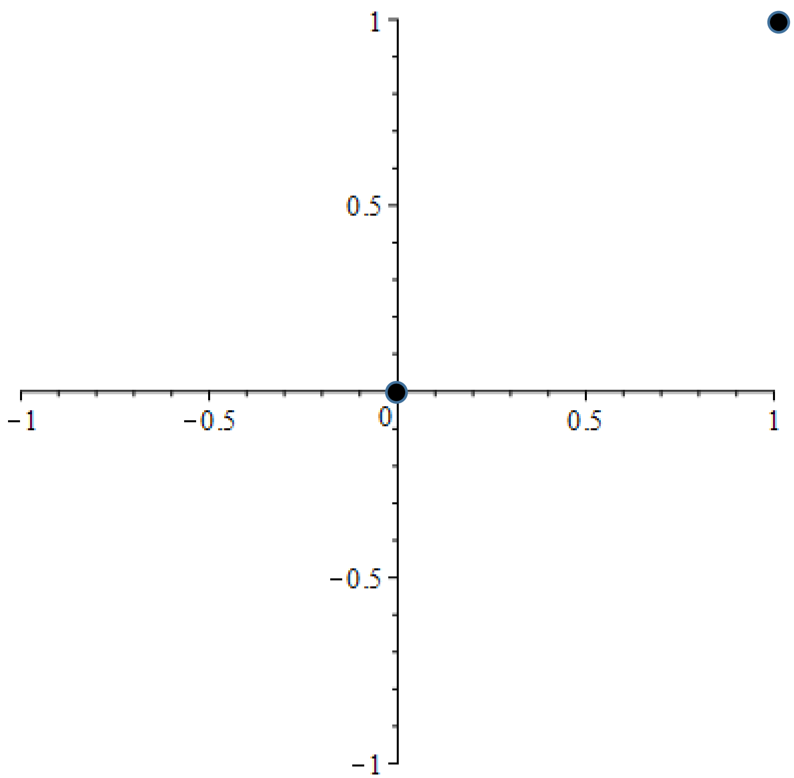
The graph of $g(x) = f(x) - c$**, is the graph of** $f(x)$ **shifted** ***c*** **units down.**

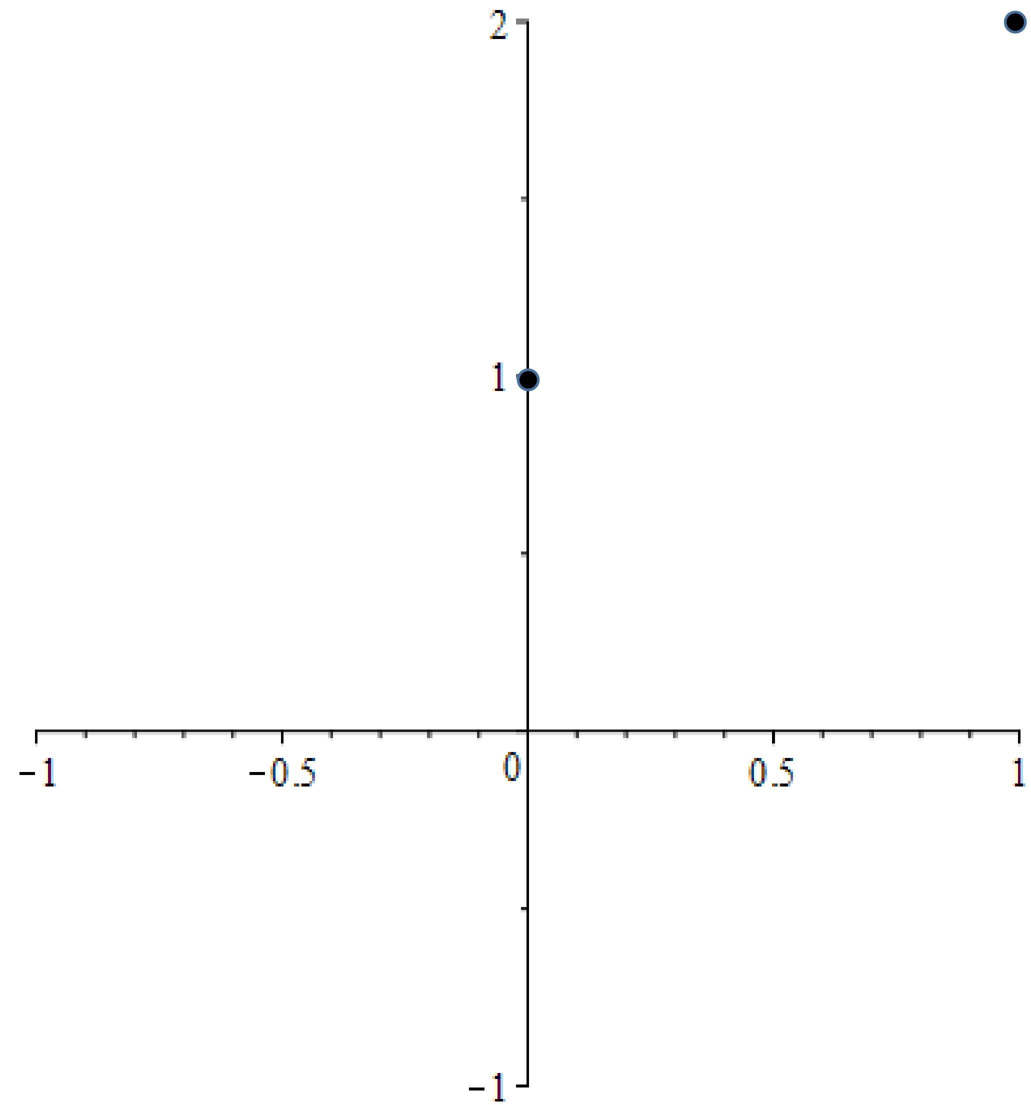
For a vertical shift, the ***y*****-coordinates change, but the** ***x*****-coordinates remain the same.**

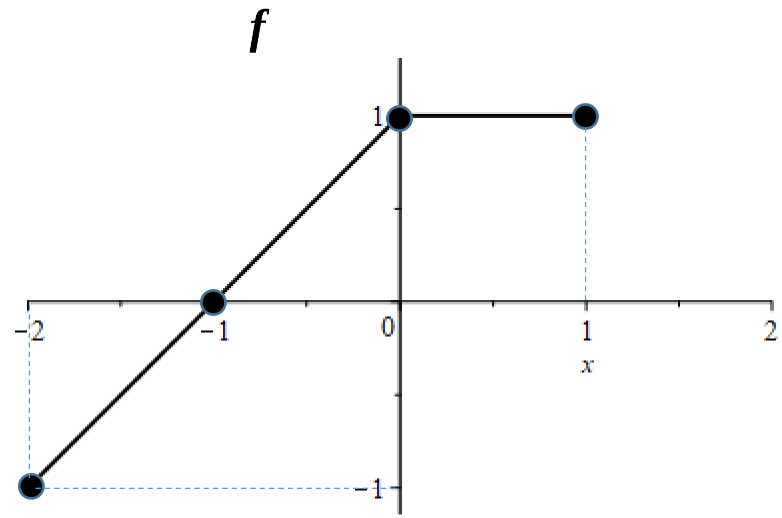


$f=\{(0,0),(1,1)\}$

$g(x)=f(x)+1,\ g=\{(0,\square),(1,\square)\}$

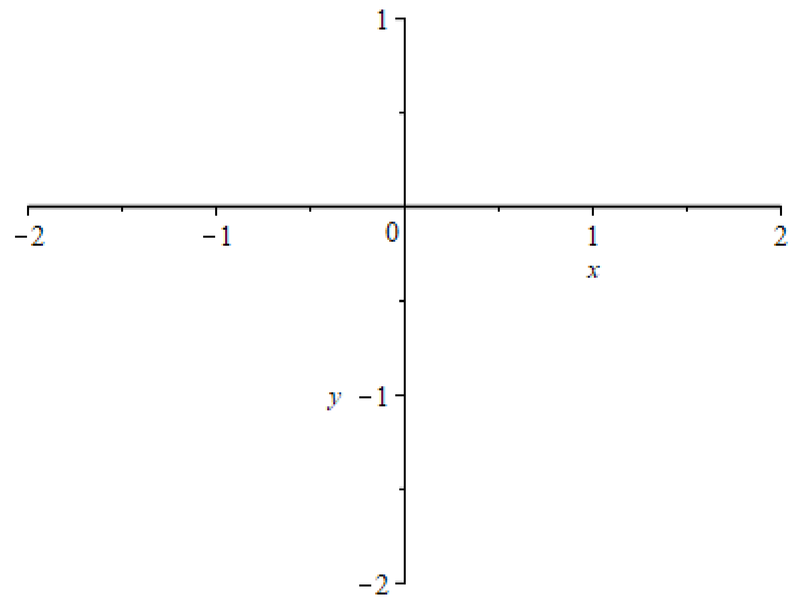






$g(x) = f(x) + 2$

$h(x) = f(x) - 1$



Horizontal Shift:

For $c > 0$,

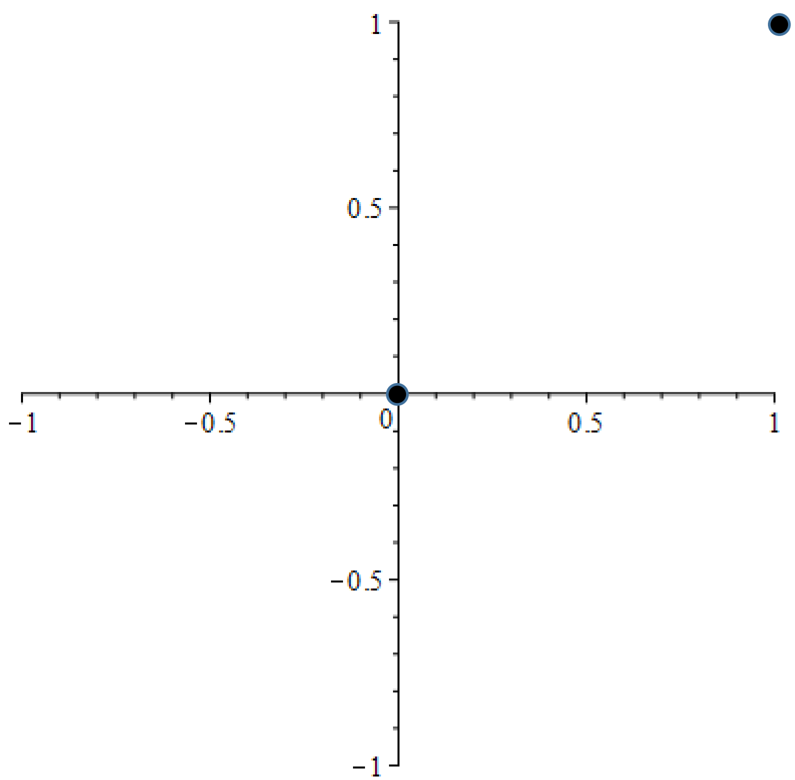
The graph of $g(x) = f(x - c)$, is the graph of $f(x)$ shifted *c* units to the right.

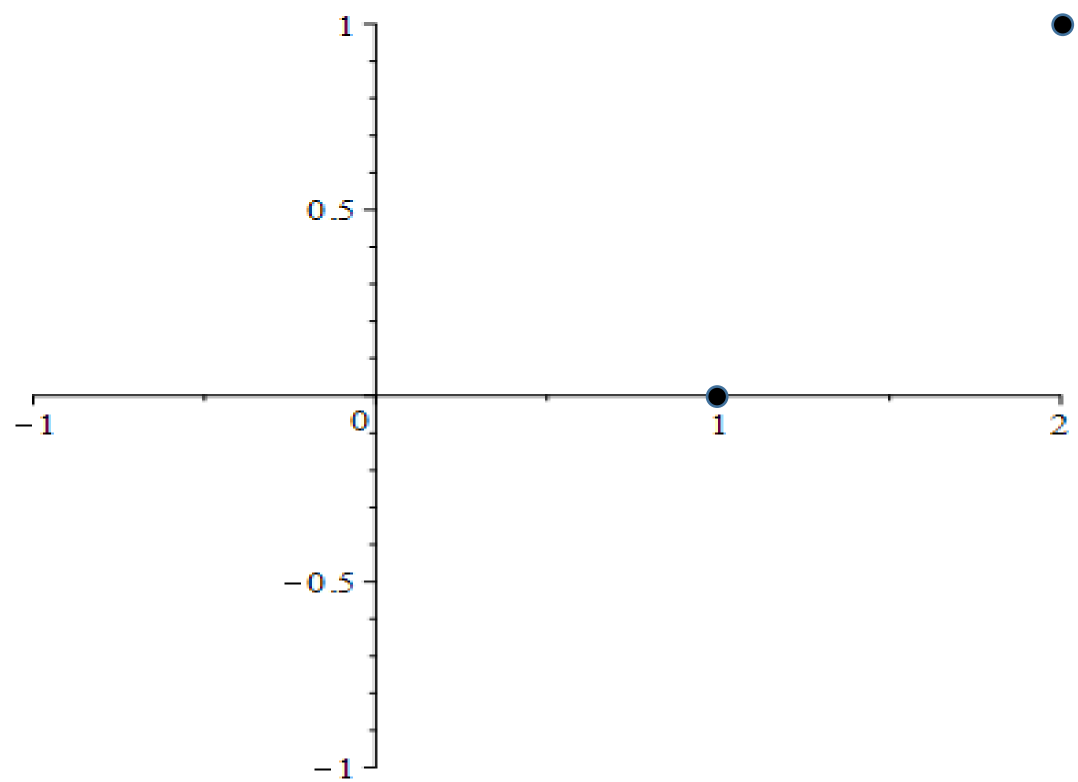
The graph of $g(x) = f(x + c)$, is the graph of $f(x)$ shifted *c* units to the left.

For a horizontal shift, the *x*-coordinates change, but the *y*-coordinates remain the same.

$f = \{(0,0),(1,1)\}$

$g(x) = f(x-1),\ g = \{(\square,0),(\square,1)\}$





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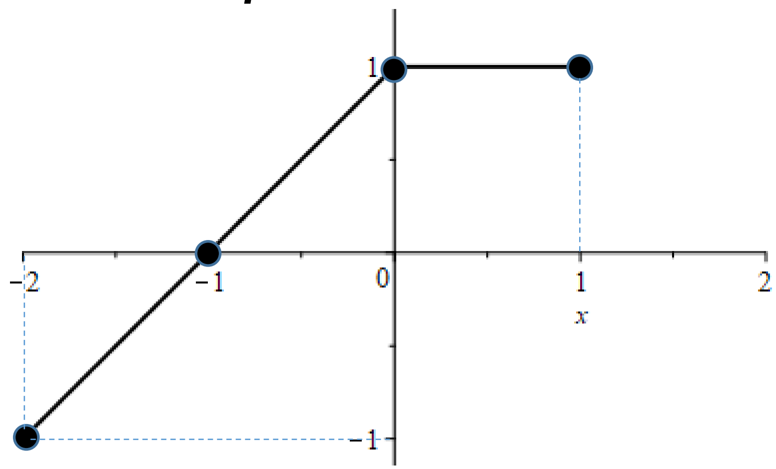
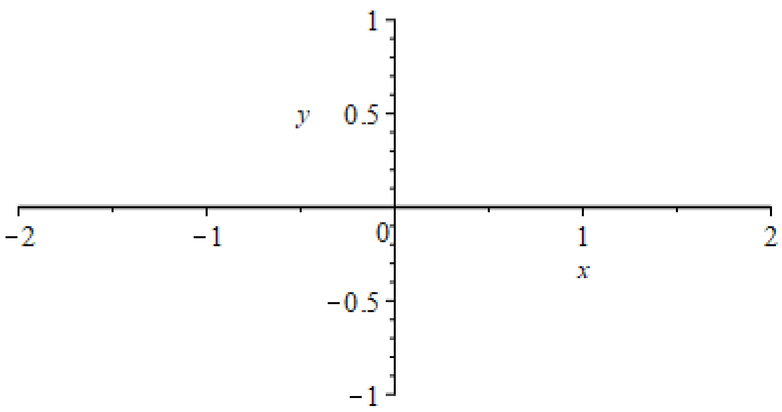
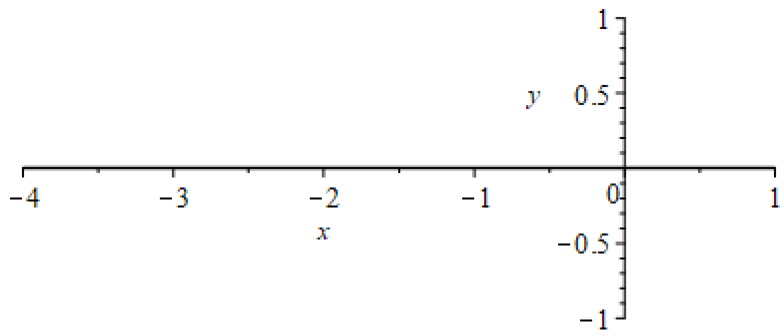


$g(x) = f(x-1)$

$h(x) = f(x+2)$

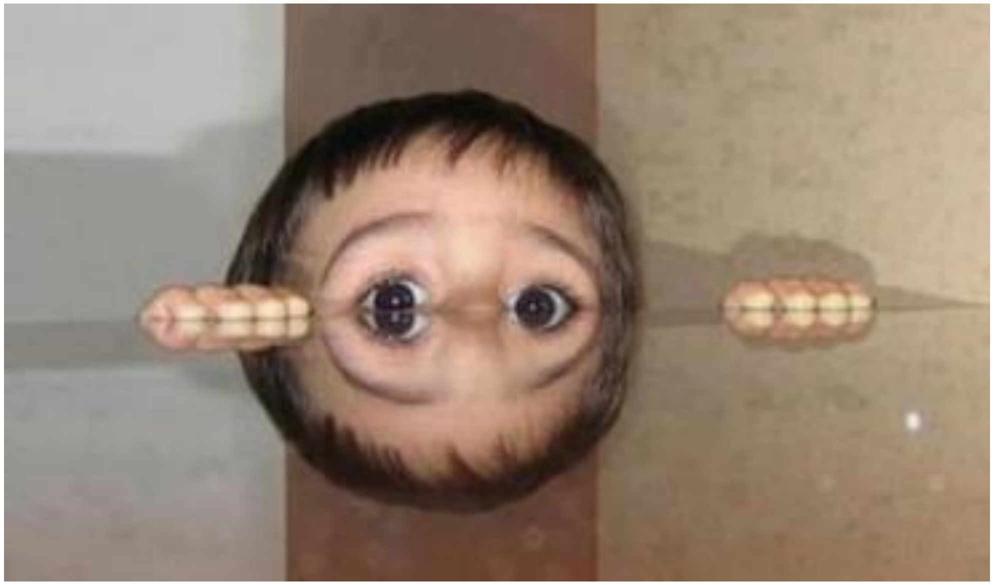


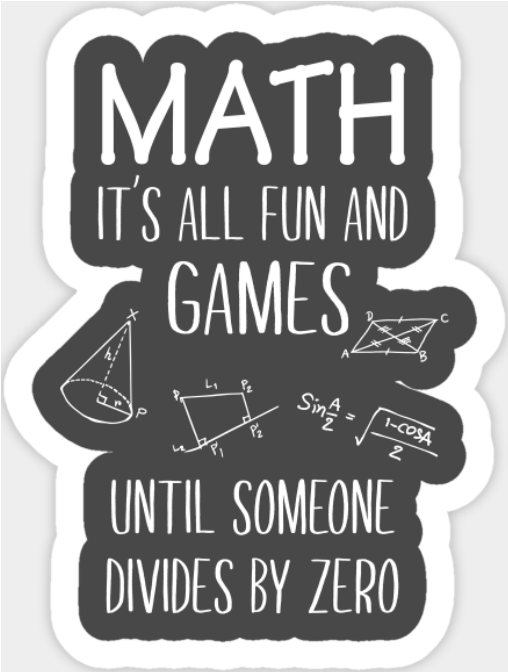


Reflection about the x-axis:

The graph of $g(x) = -f(x)$ is the graph of $f(x)$ reflected about the *x*-axis.

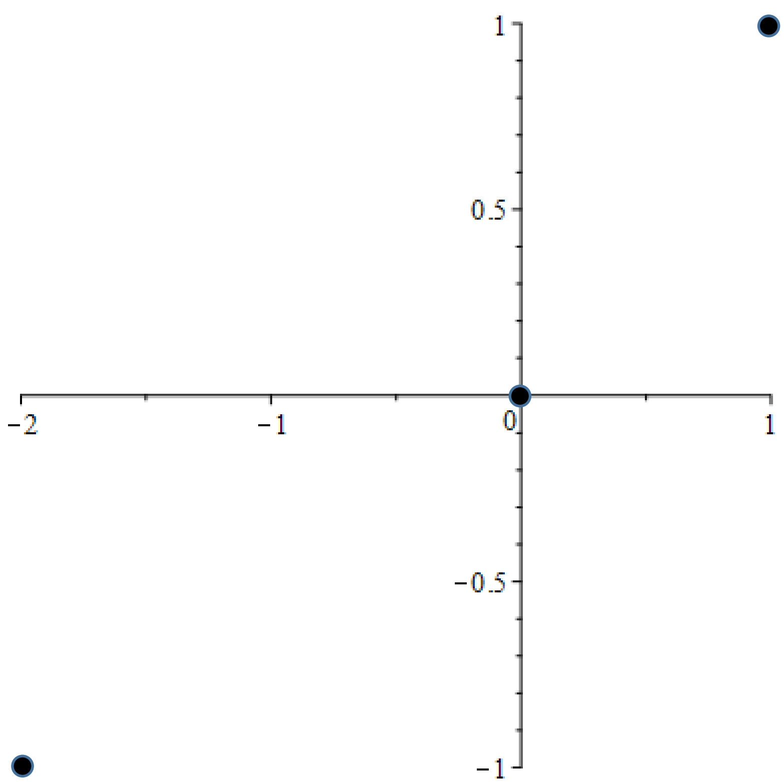
For reflection about the *x*-axis, the non-zero *y*-coordinates change, but the *x*-coordinates remain the same.

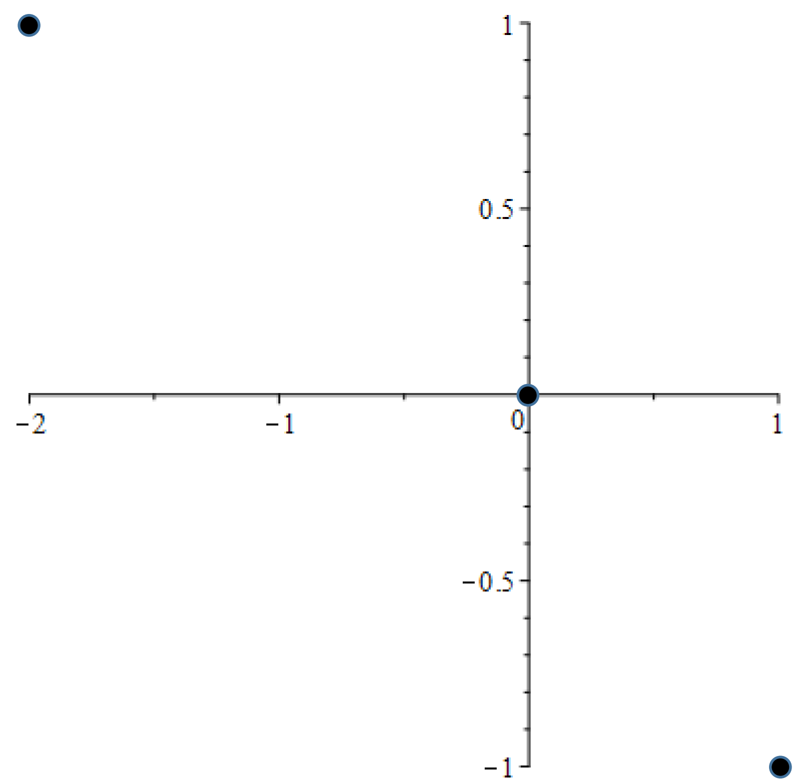




$$f = \{(0,0),(1,1),(-2,-1)\}$$

$$g(x) = -f(x),\ g = \{(0,\square),(1,\square),(-2,\square)\}$$





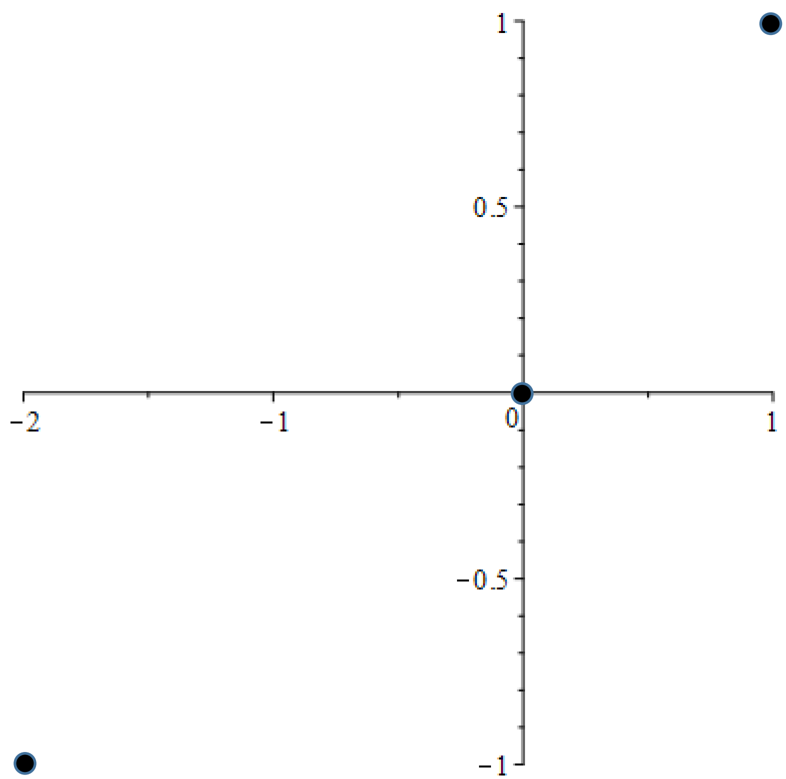
Reflection about the y-axis:

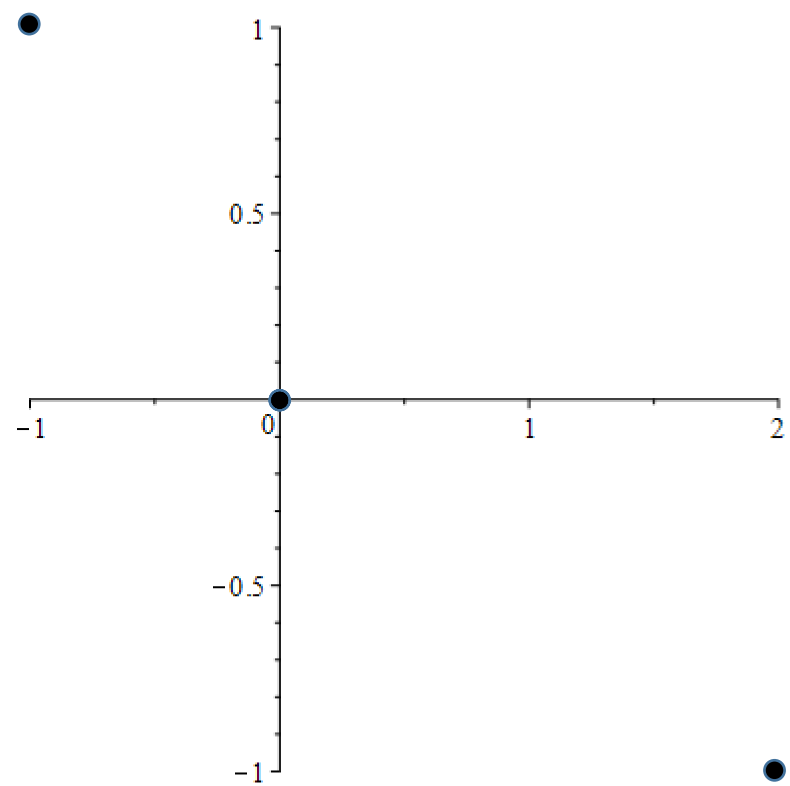
The graph of $g(x) = f(-x)$ **is the graph of** $f(x)$ **reflected about the *y*-axis.**

For reflection about the *y*-axis, the non-zero *x*-coordinates change, but the *y*-coordinates remain the same.

$f = \{(0,0),(1,1),(-2,-1)\}$

$g(x) = f(-x),\ g = \{(\square,0),(\square,1),(\square,-1)\}$





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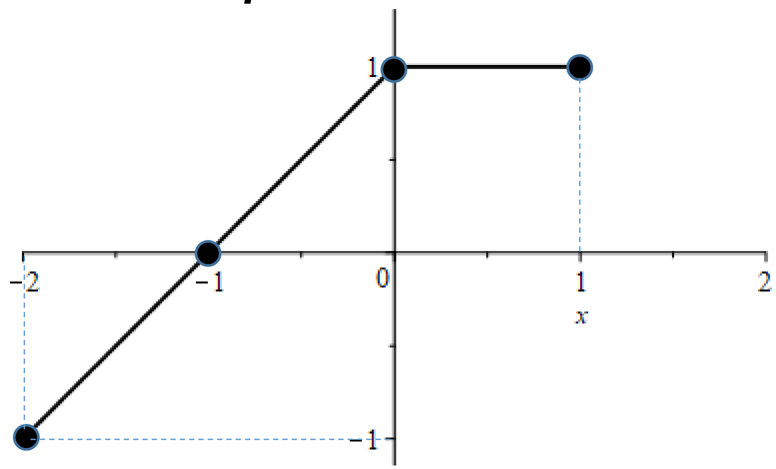
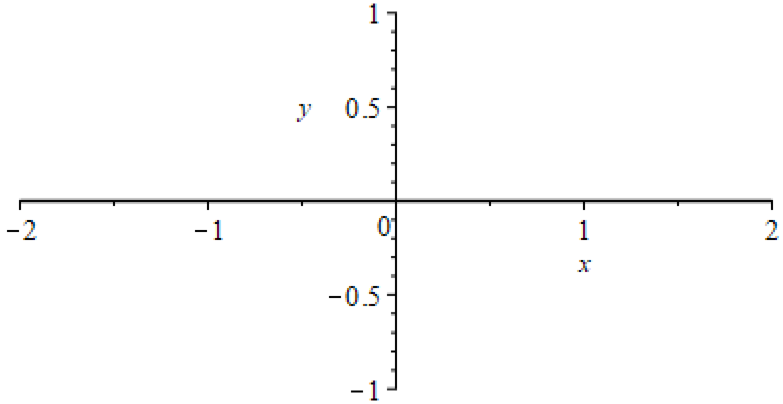
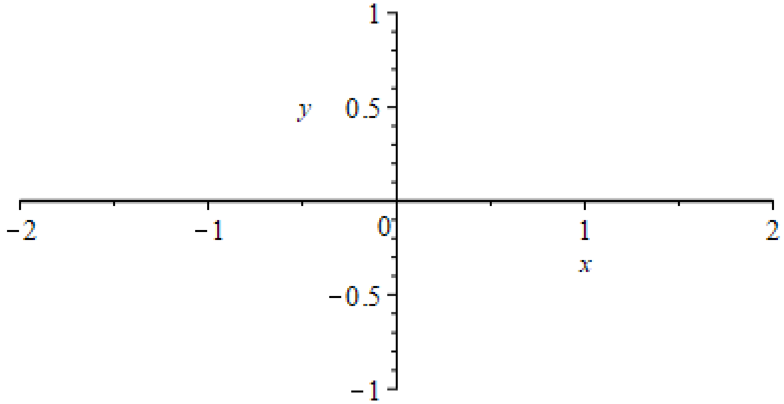


$g(x) = -f(x)$

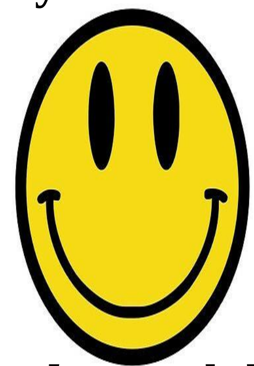


$h(x) = f(-x)$

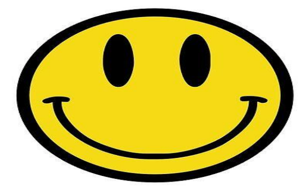


Vertical Stretch/Compress:

For $c > 1$, the graph of $g(x) = cf(x)$ is the graph of $f(x)$ *stretched* away from the *x*-axis by a factor of *c*.

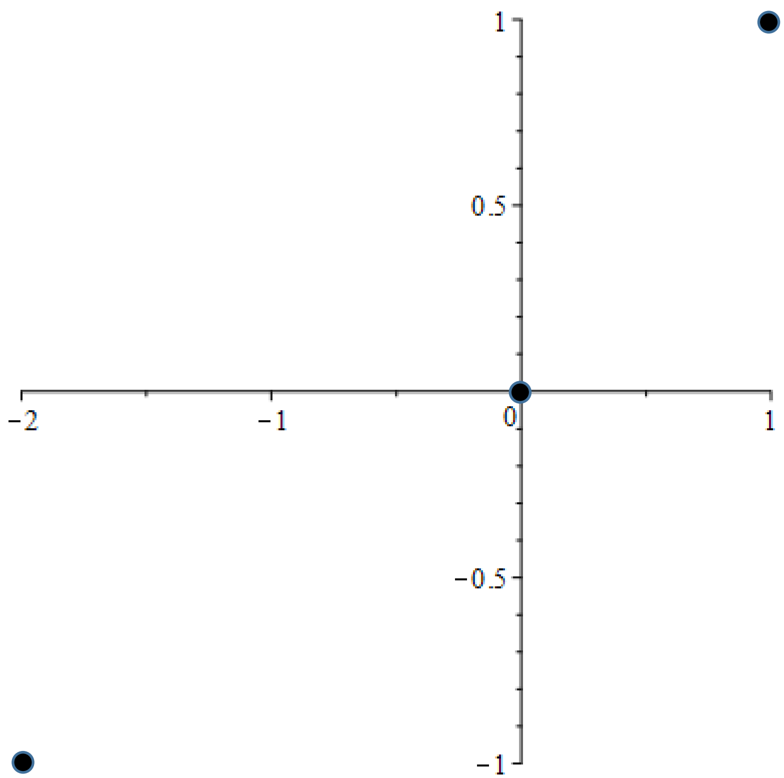


For $0 < c < 1$, the graph of $g(x) = cf(x)$ is the graph of $f(x)$ *compressed* toward the *x*-axis by a factor of *c*.

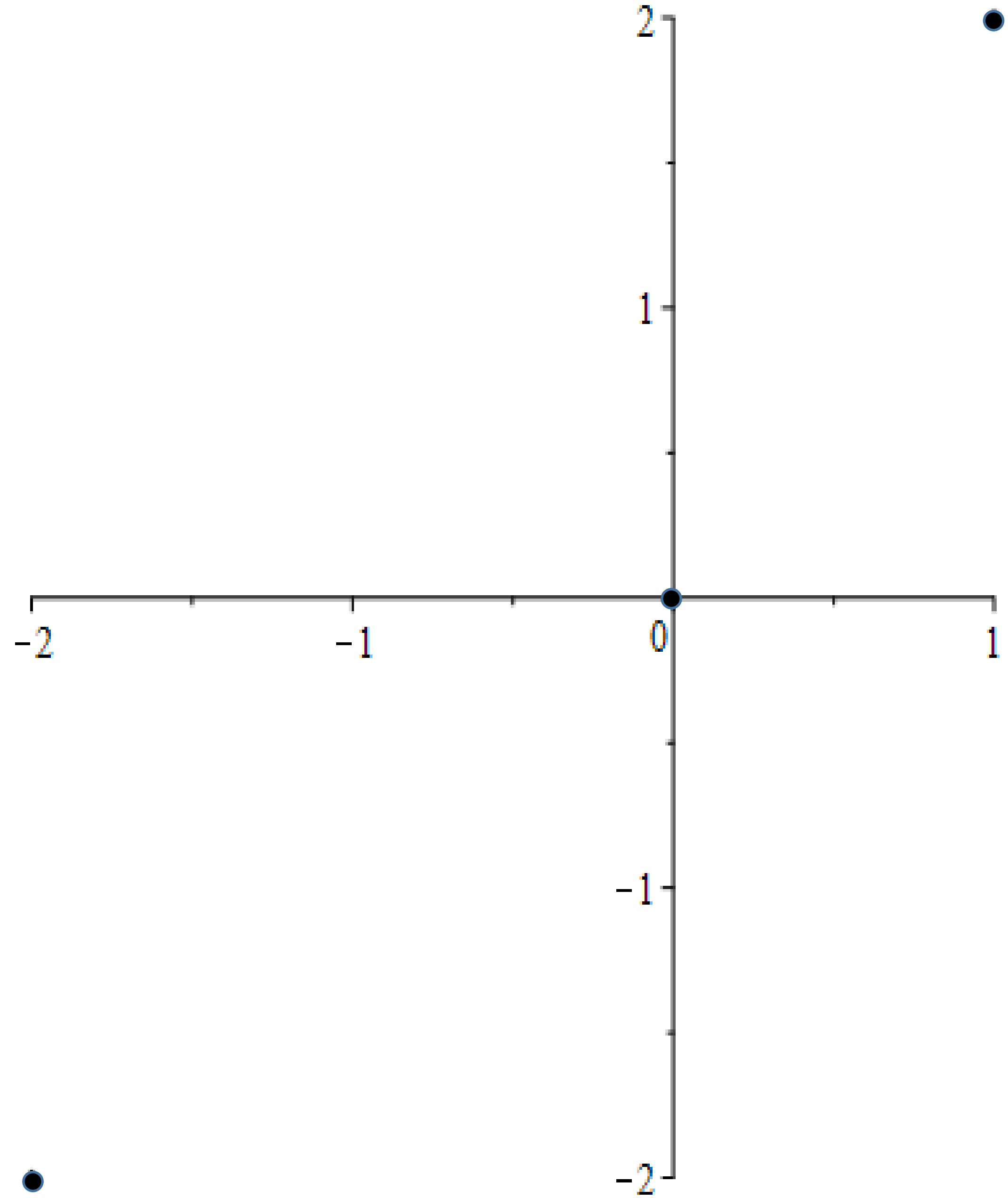


For a vertical stretch/compress, the non-zero *y*-coordinates change, but the *x*-coordinates remain the same.

$f = \{(0,0),(1,1),(-2,-1)\}$



$g(x) = 2f(x),\ g = \{(0,\square),(1,\square),(-2,\square)\}$



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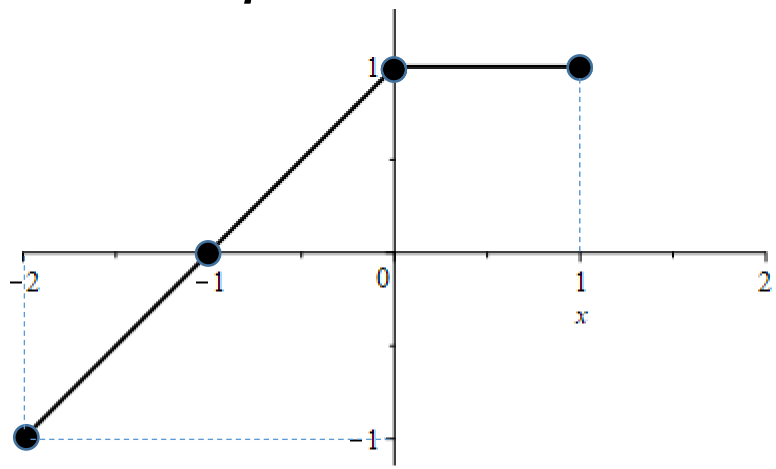
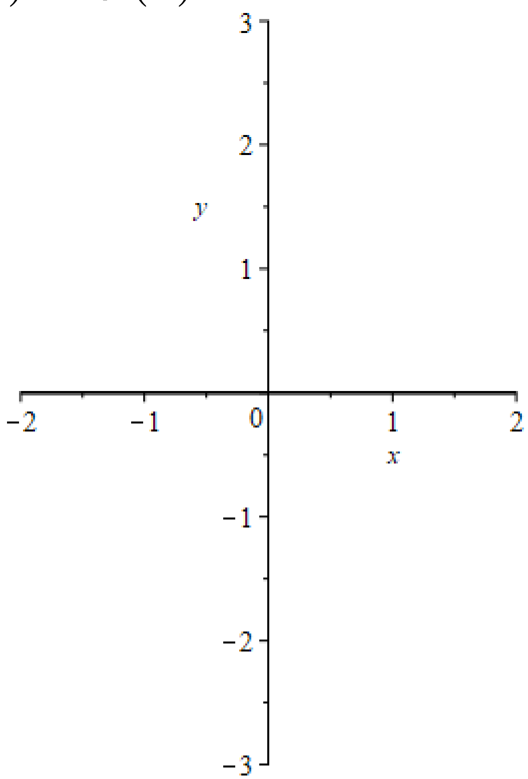
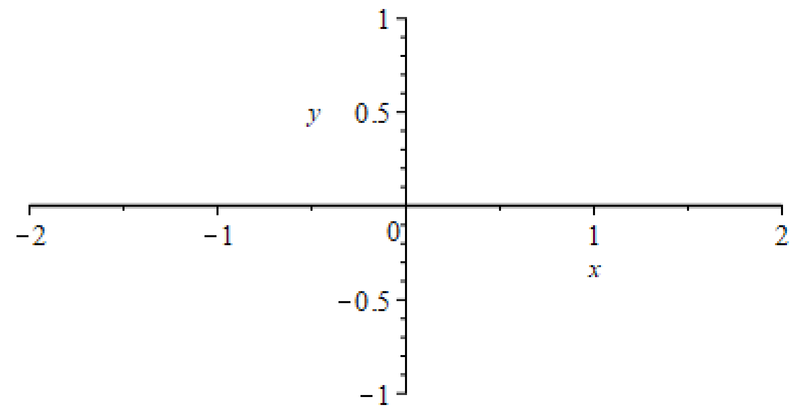


$g(x) = 3f(x)$

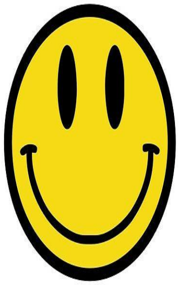


$h(x) = \frac{1}{2}f(x)$

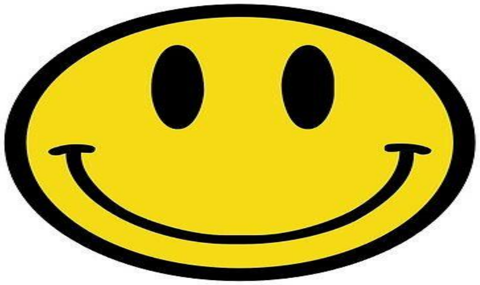


Horizontal Stretch/Compress:

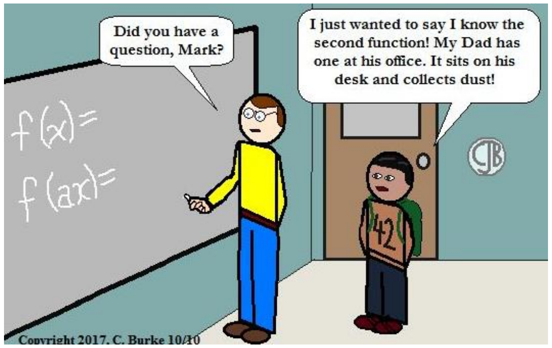
For $c > 1$, the graph of $g(x) = f(cx)$ is the graph of $f(x)$ *compressed* toward the *y*-axis by a factor of $\frac{1}{c}$.



For $0 < c < 1$, the graph of $g(x) = f(cx)$ is the graph of $f(x)$ *stretched* away from the *y*-axis by a factor of $\frac{1}{c}$.

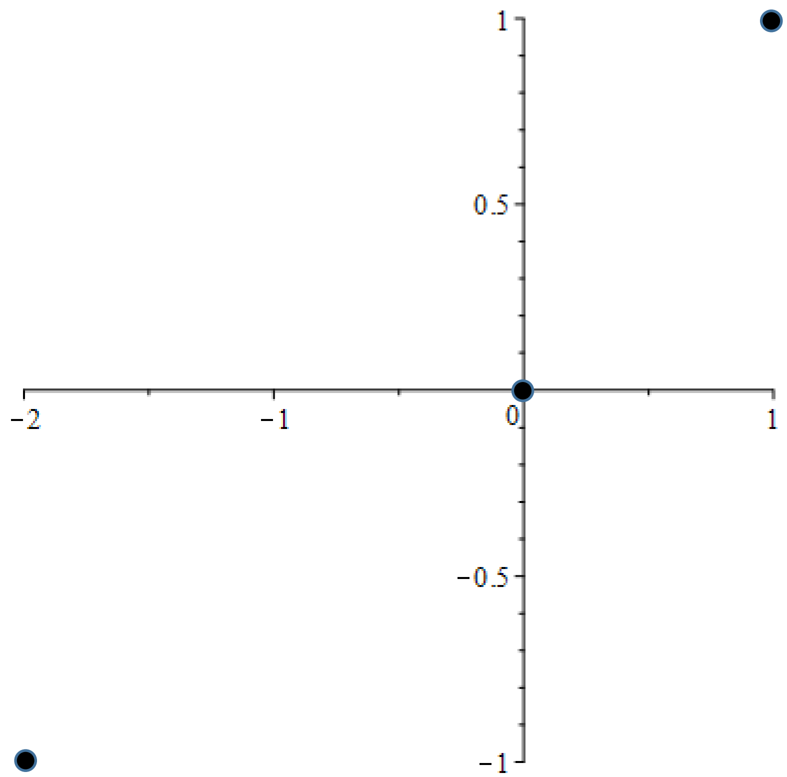


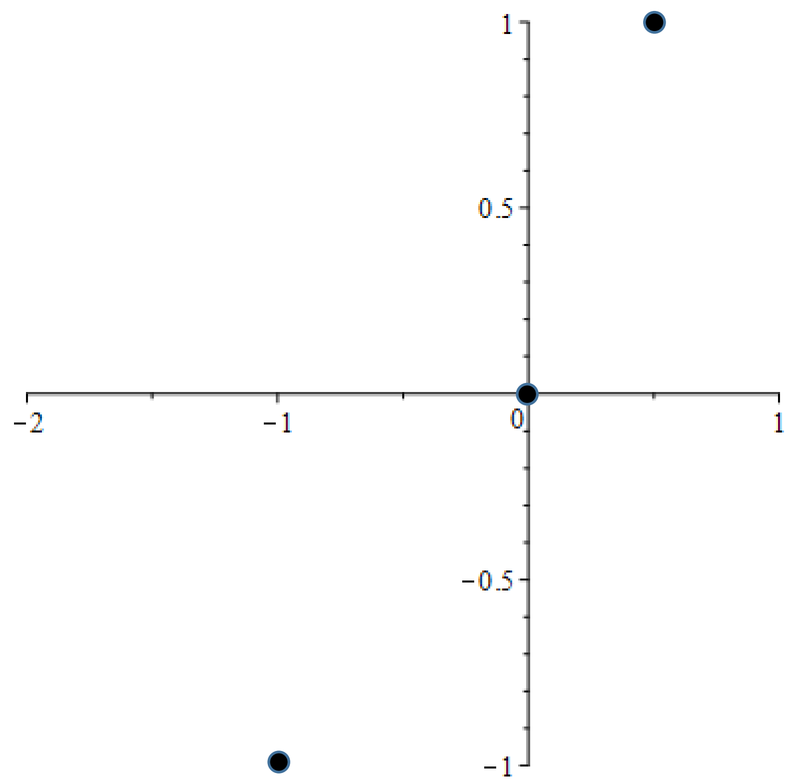
For a horizontal stretch/compress, the non-zero *x*-coordinates change, but the *y*-coordinates remain the same.



$f = \{(0,0),(1,1),(-2,-1)\}$

$g(x) = f(2x),\ g = \{(\square,0),(\square,1),(\square,-1)\}$





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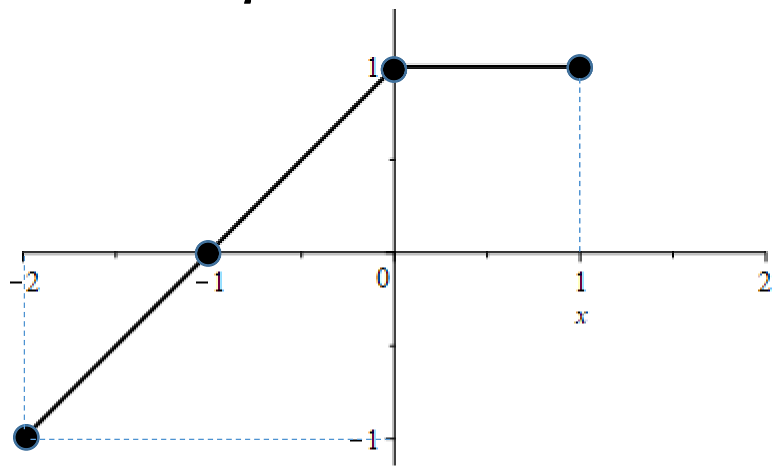
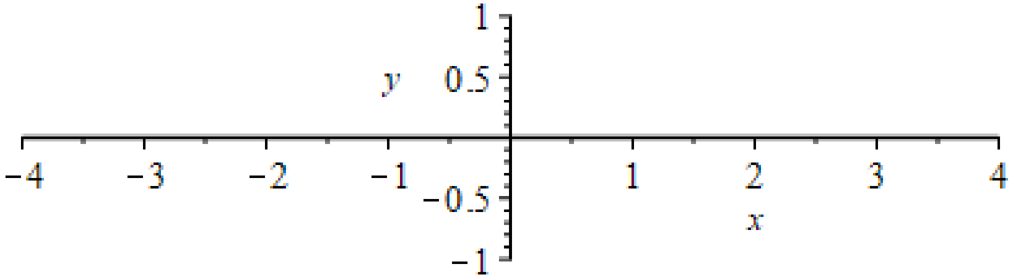
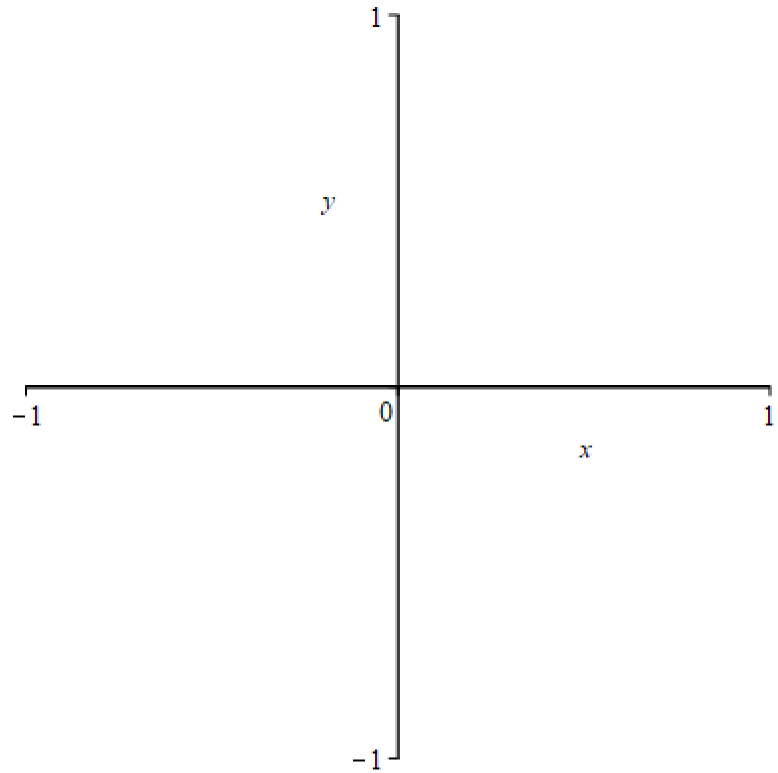


$g(x) = f\left(\frac{1}{2}x\right)$



$h(x) = f(3x)$



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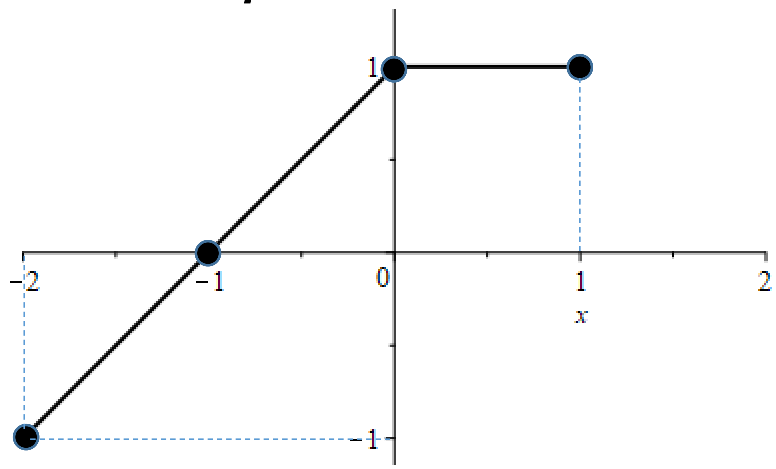
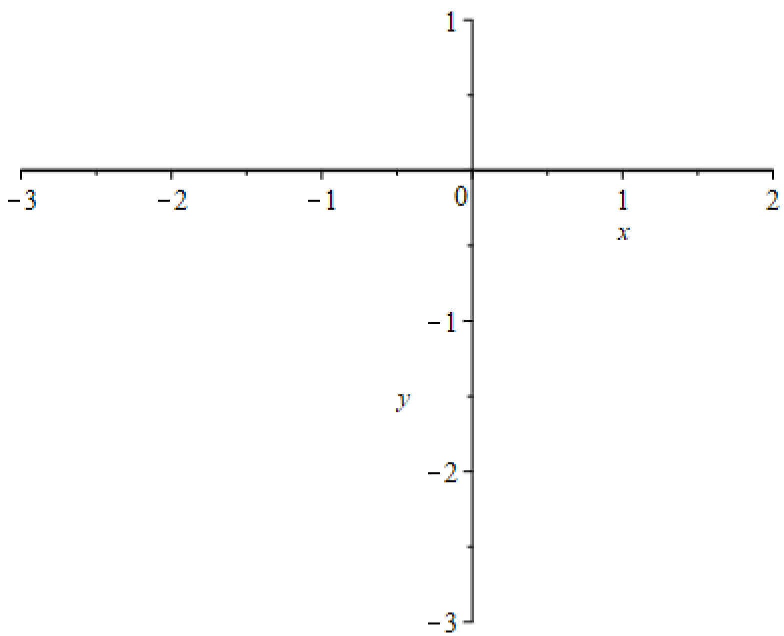
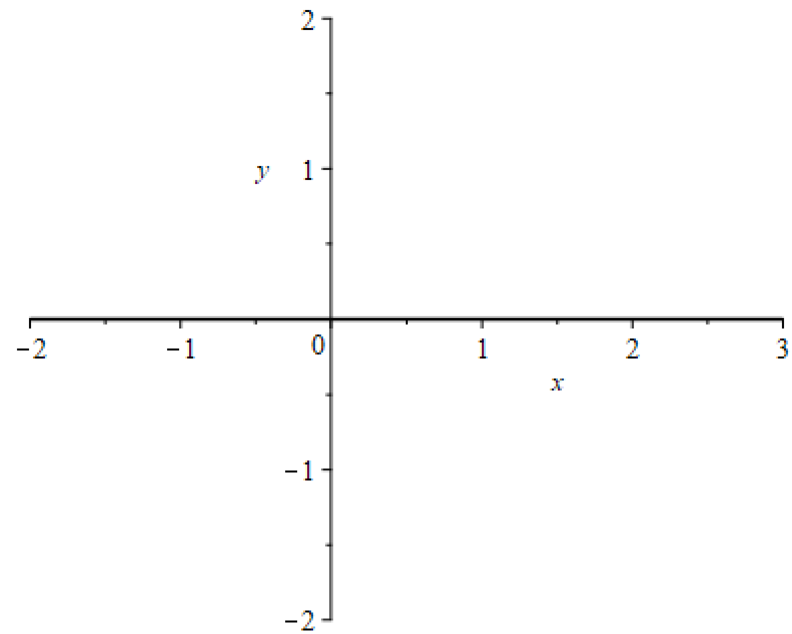
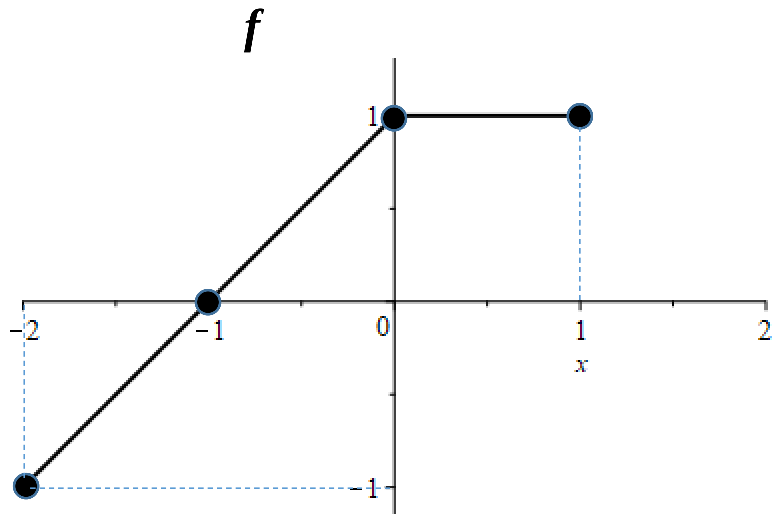


$g(x) = f(x+1) - 2$

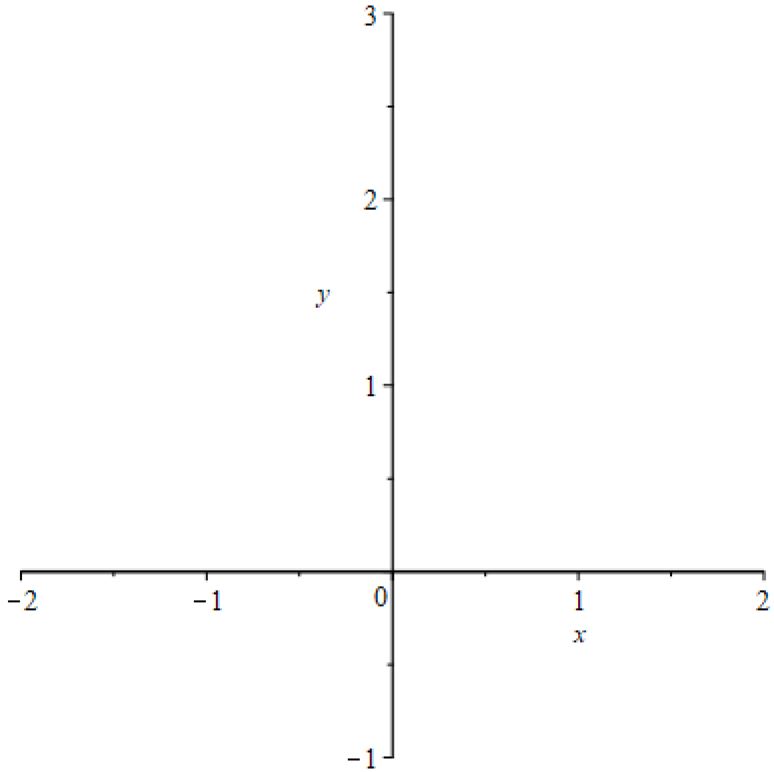


$h(x) = 2f(x-1)$

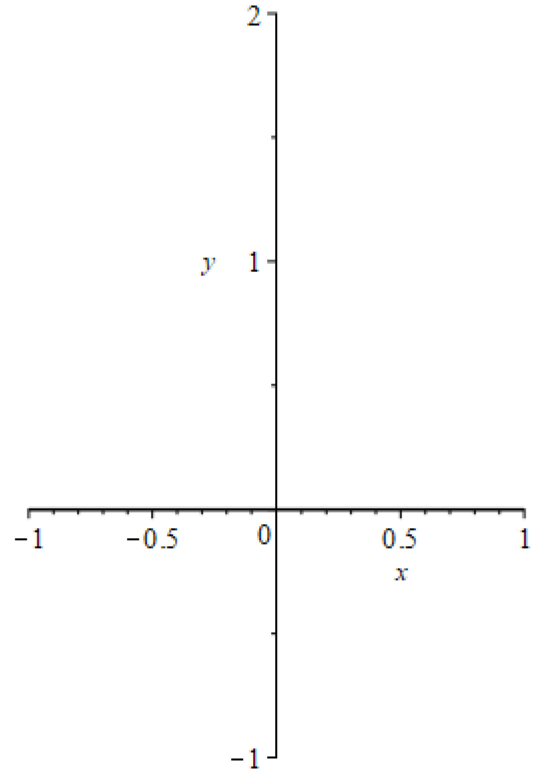




$g(x) = f(-x) + 2$



$h(x) = f(2x) + 1$



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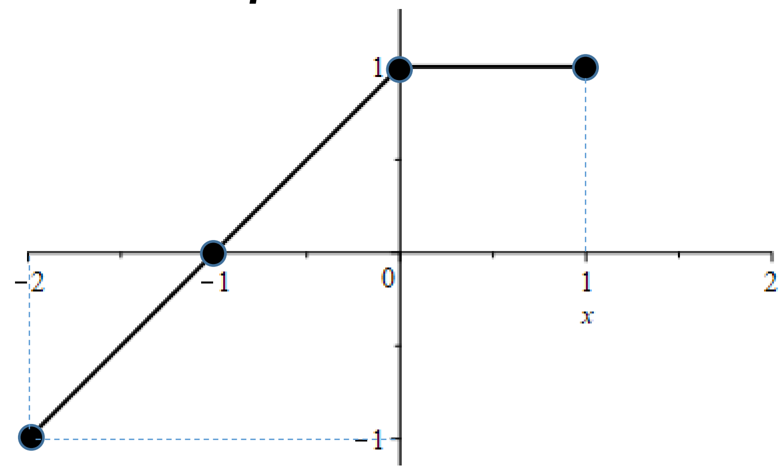
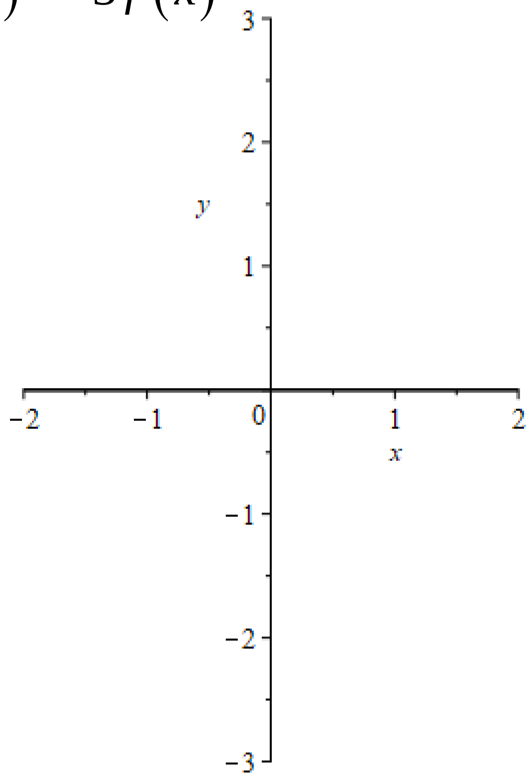
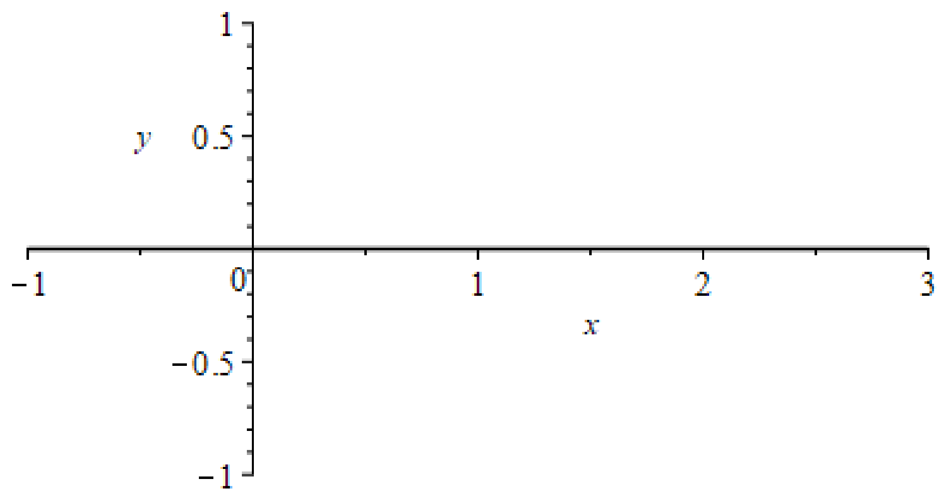
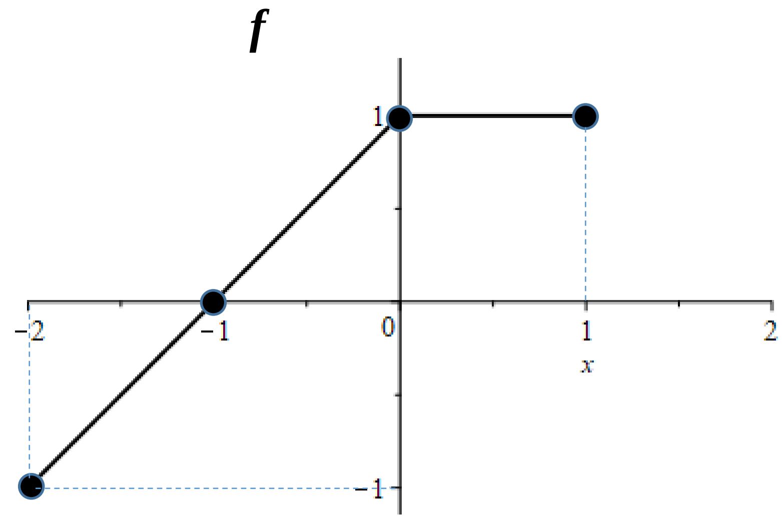


$g(x) = -3f(x)$



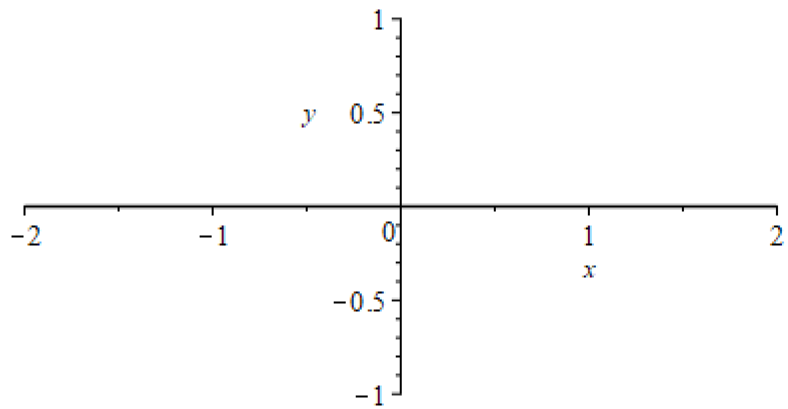
$h(x) = f(1-x)$

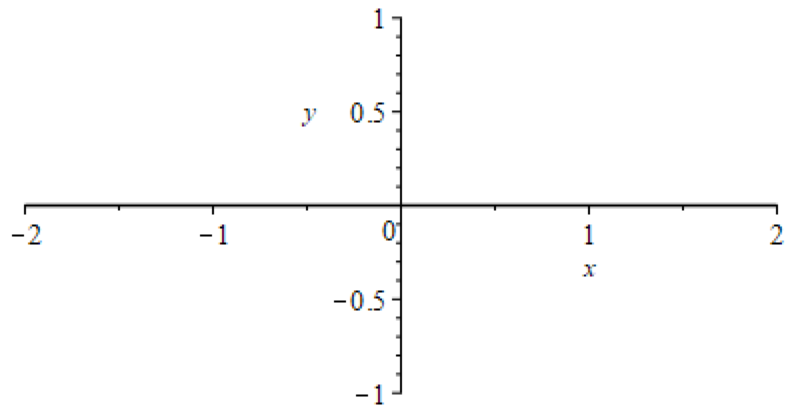


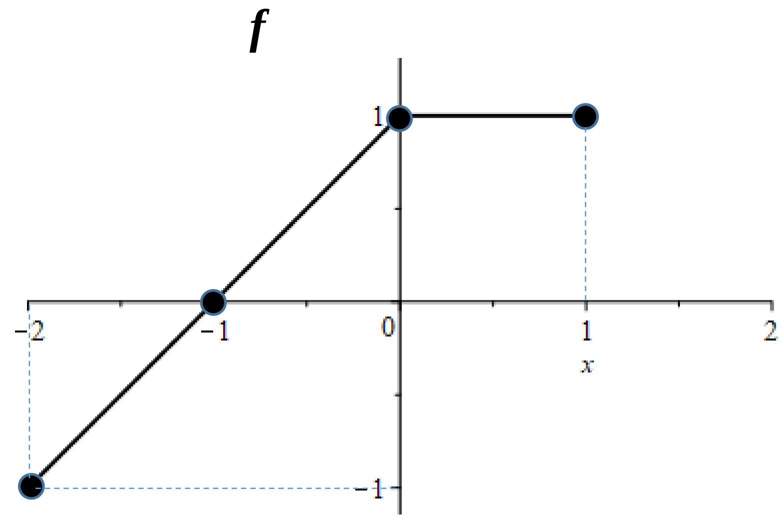


$g(x) = |f(x)|$

$h(x) = f(|x|)$

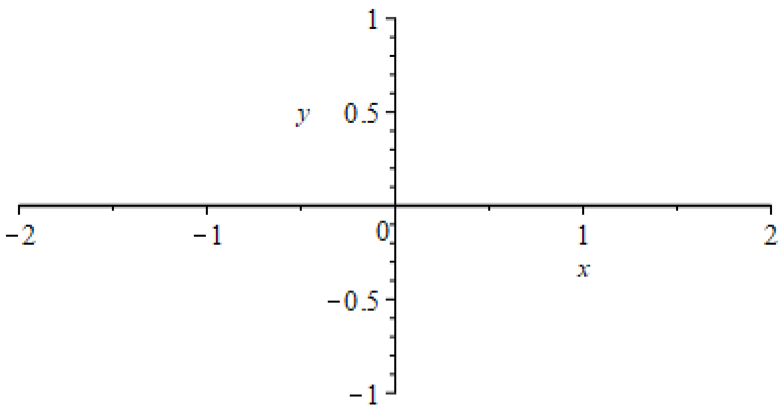


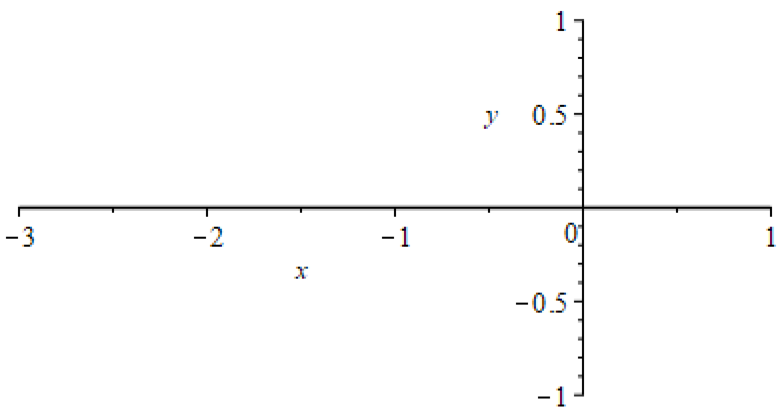




$g(x) = f(-|x|)$

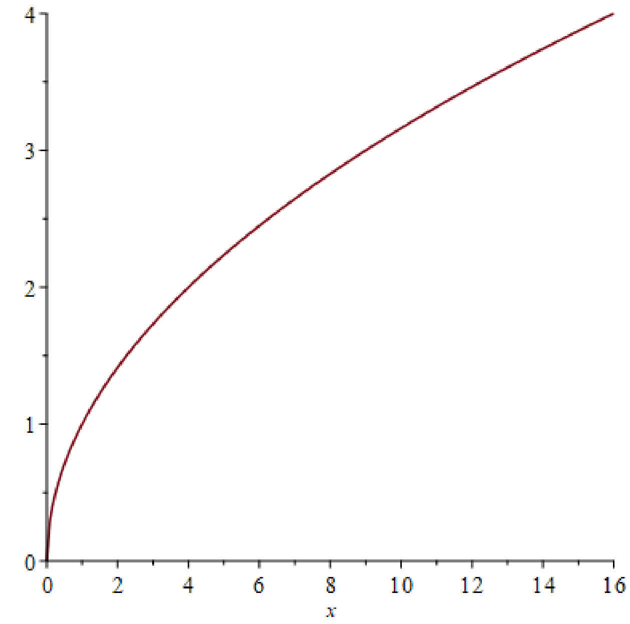
$h(x) = f(2x+4)$

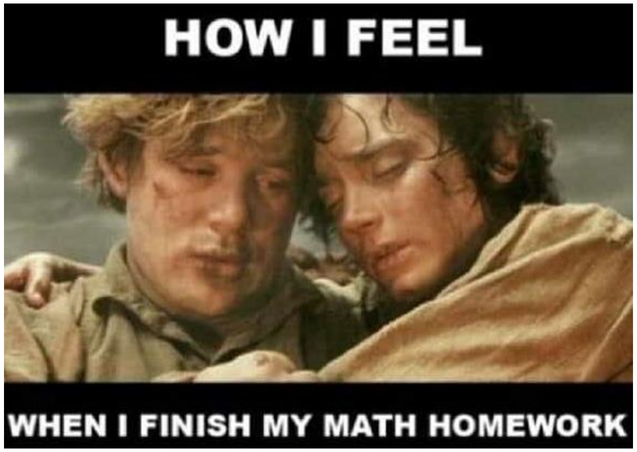




$f(x) = \sqrt{x+2}$

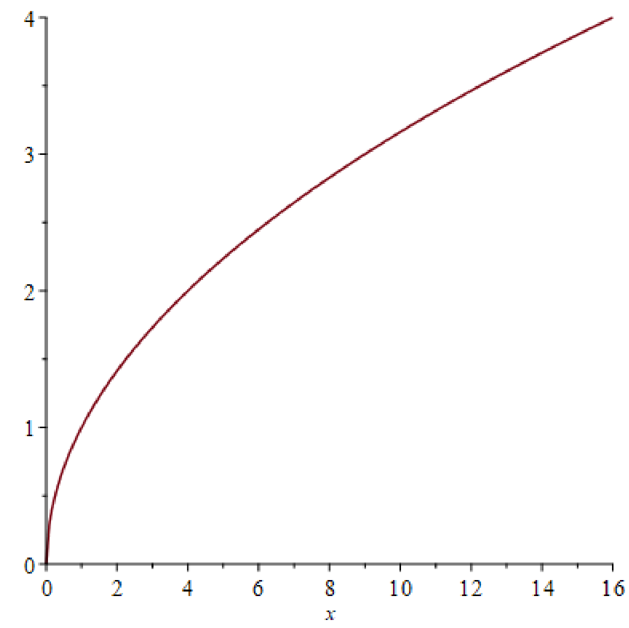
Start with the graph of the square-root function.





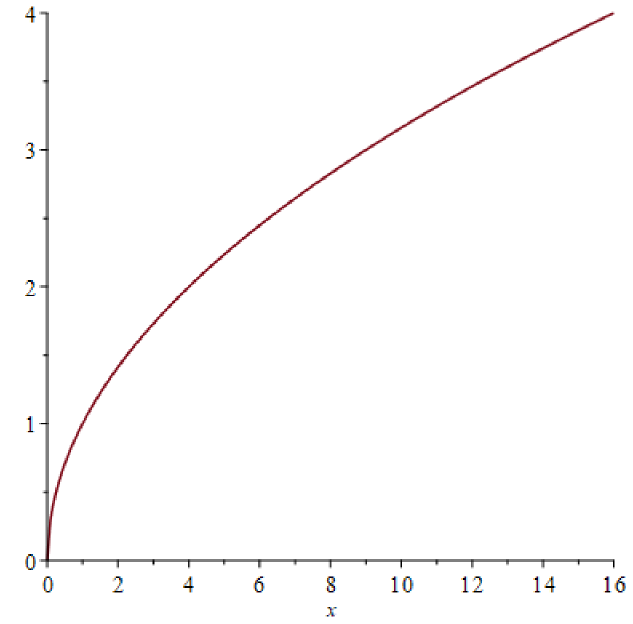
$f(x) = \sqrt{2 - x}$

Start with the graph of the square-root function.



$f(x) = \sqrt{|x|}$

Start with the graph of the square-root function.



$f(x) = \left| \sqrt[3]{x} \right|$

Start with the graph of the cube-root function.