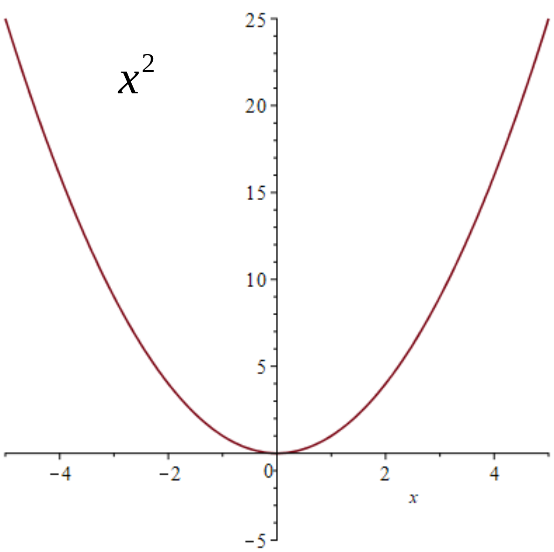
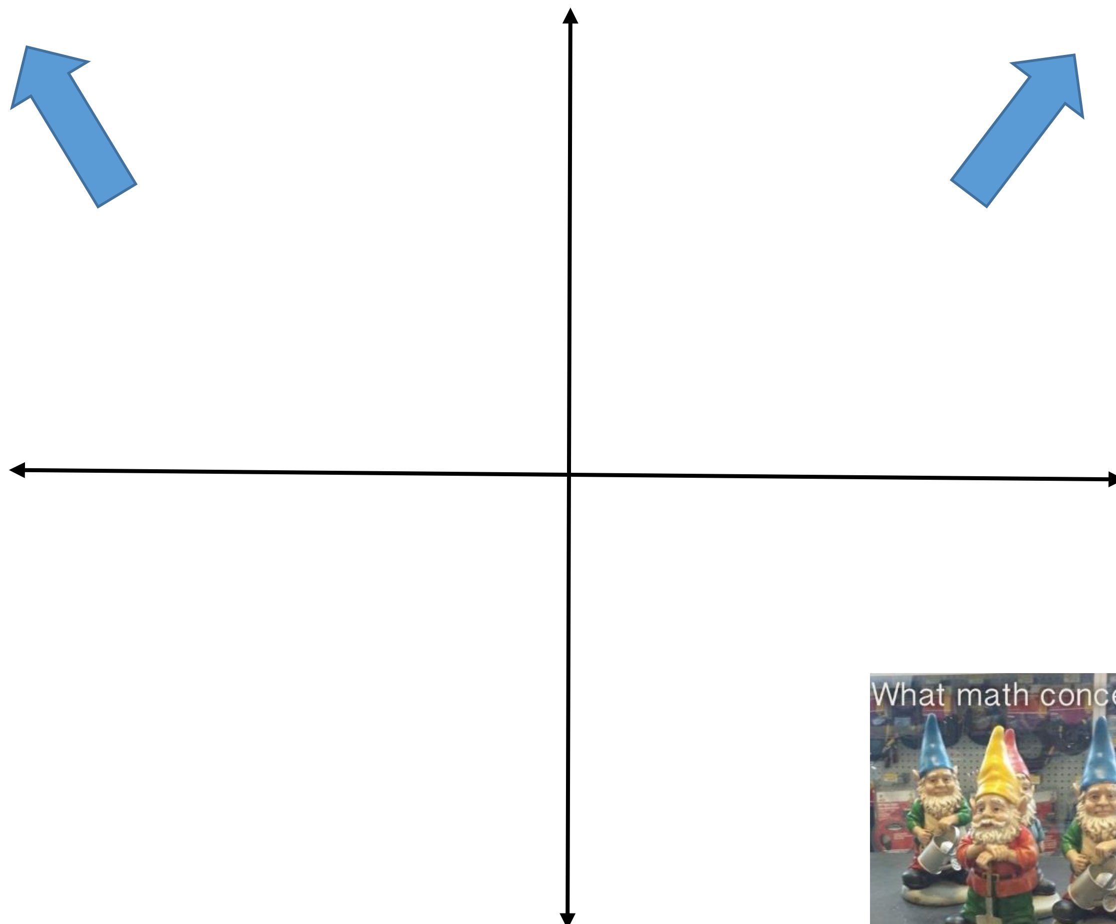
<u>Review of Graphing Polynomial Functions:</u>

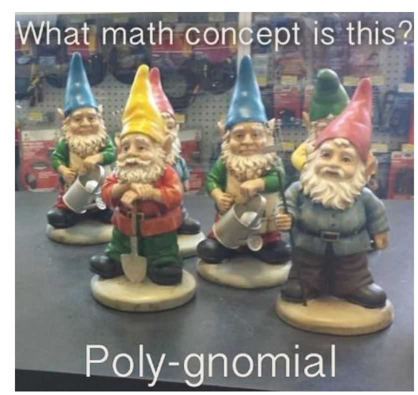
<u>The Leading Coefficient Test and End Behavior:</u>

For an n^{th} –degree polynomial function $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ **with** $a_n \neq 0$**,**

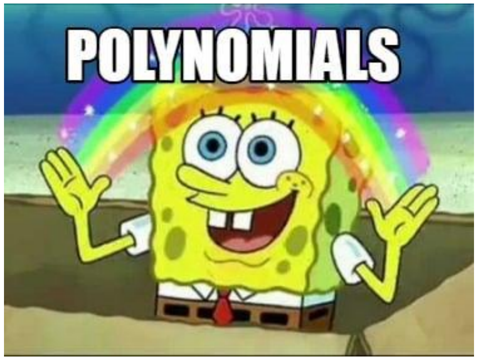
If *n* is even and $a_n > 0$**, then**

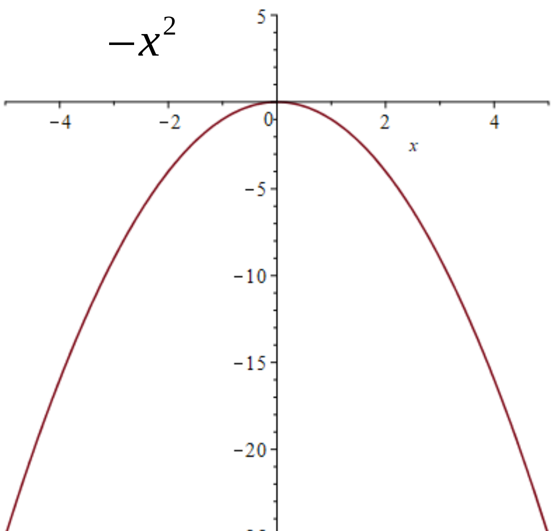


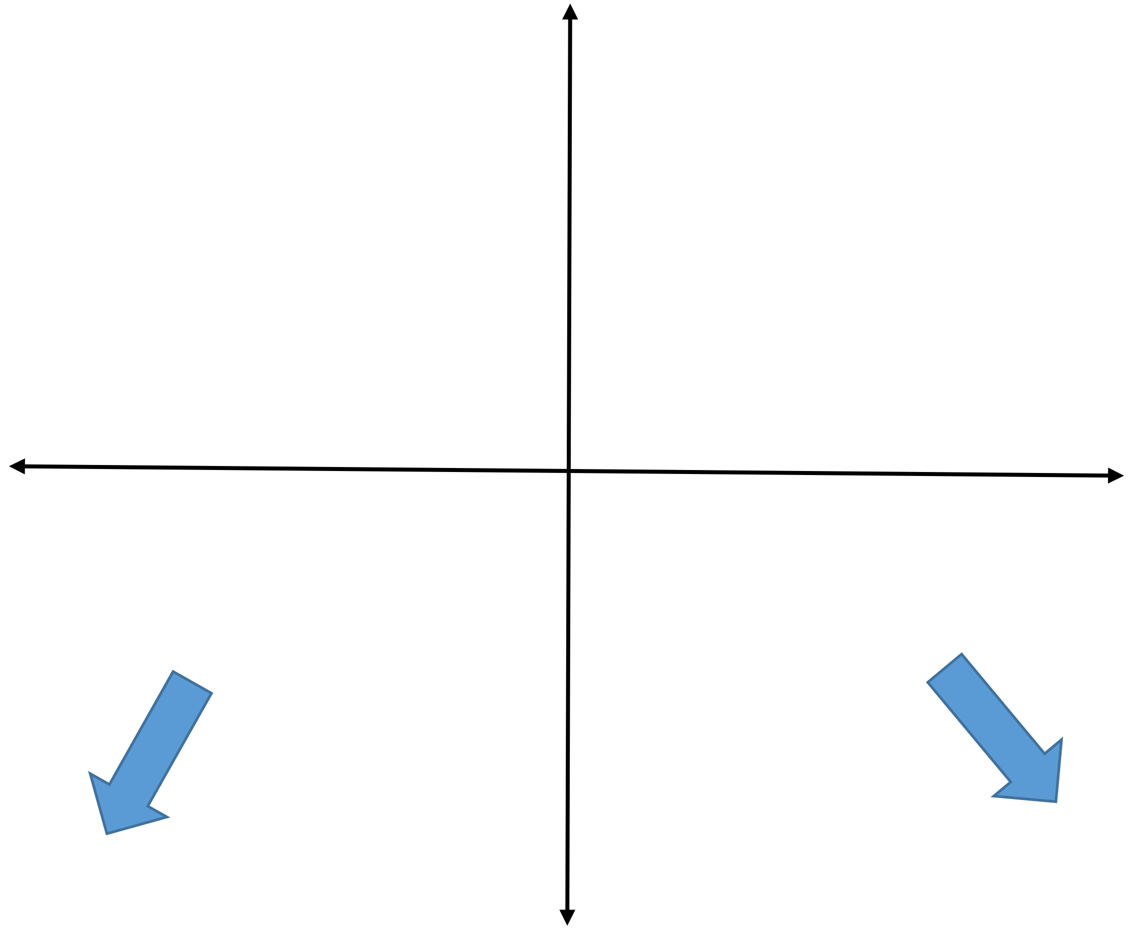




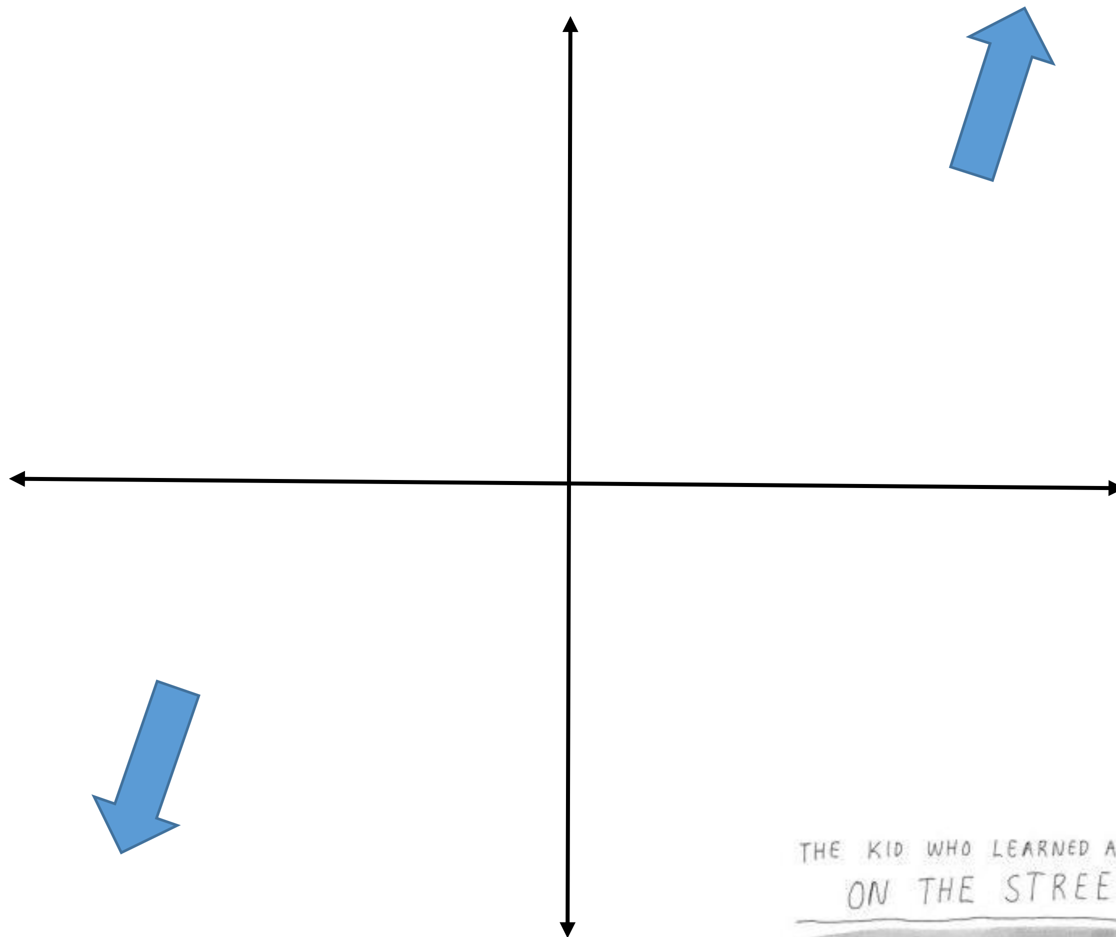
If *n* is even and $a_n < 0$, then



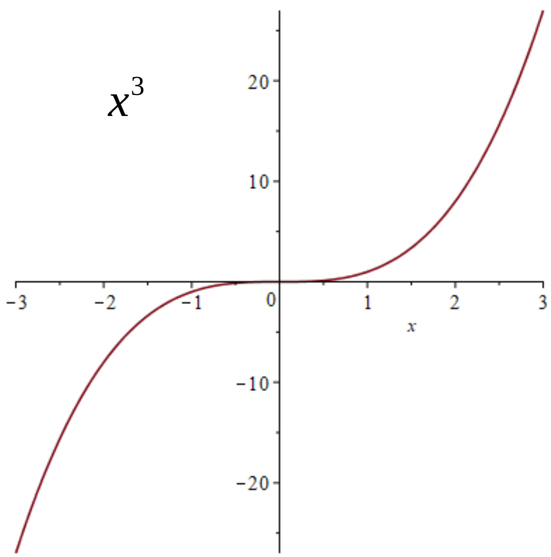




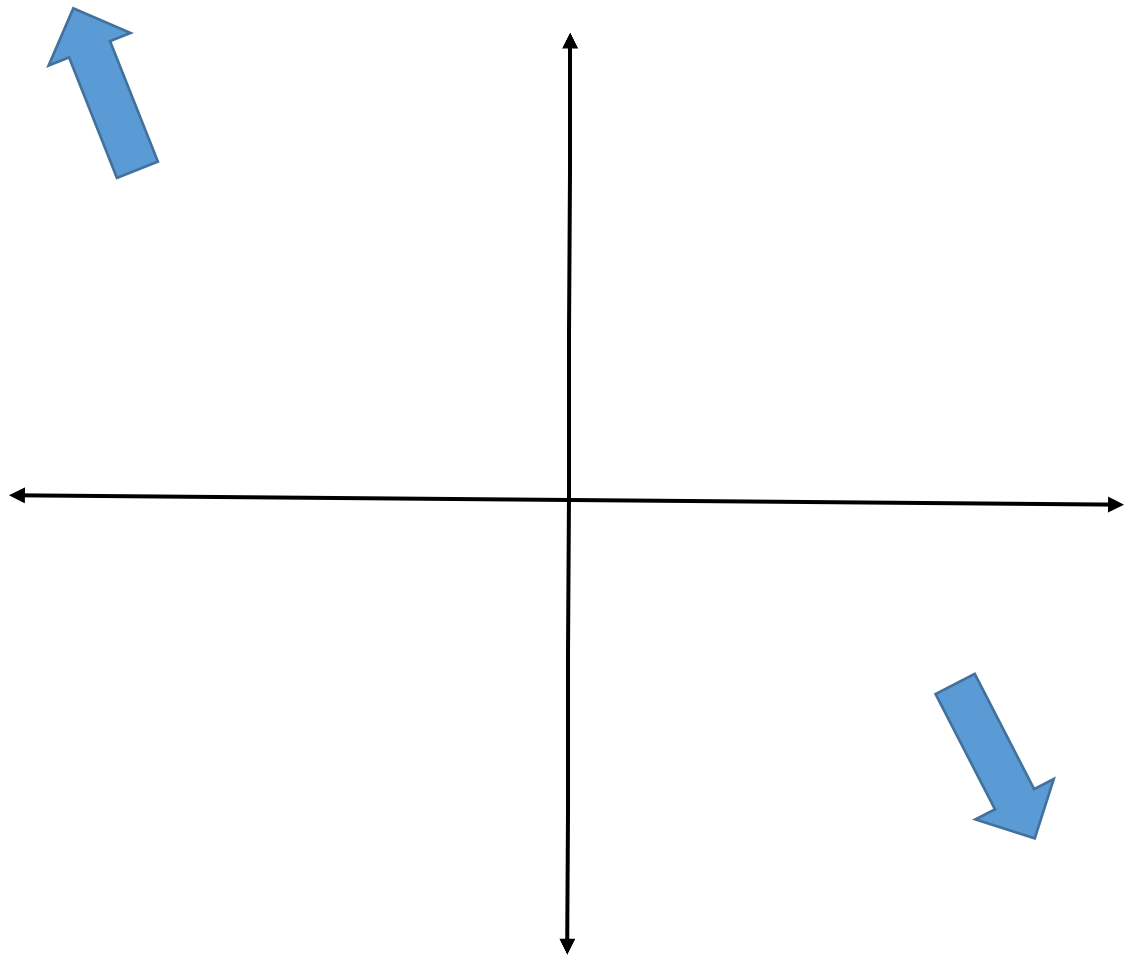
If *n* is odd and $a_n > 0$, then

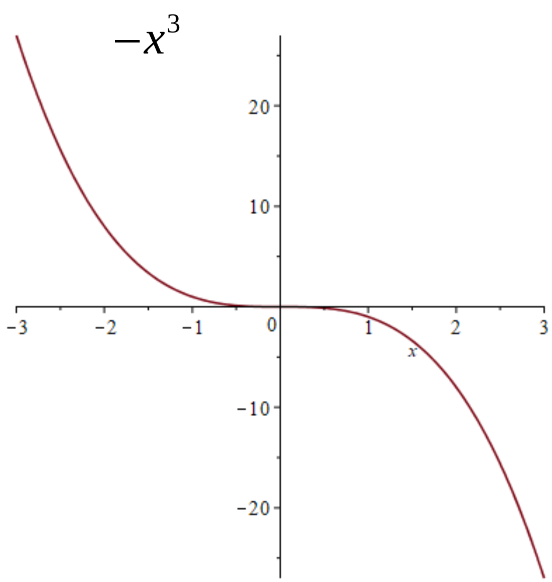


x^3



If *n* is odd and $a_n < 0$, then





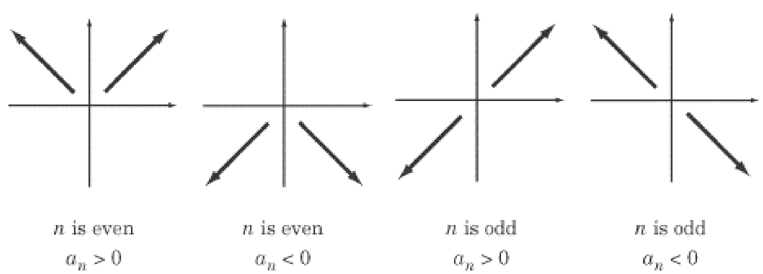
Determine the end behavior of the following polynomial functions.

1. $f(x) = 4x - x^3$ **Left:** **Right:**

2. $f(x) = 2x^4 + 12x - 4$ **Left:** **Right:**

3. $f(x) = x^3 + 2x^2 - 8x$ **Left:** **Right:**

4. $f(x) = 4x - x^6$ **Left:** **Right:**

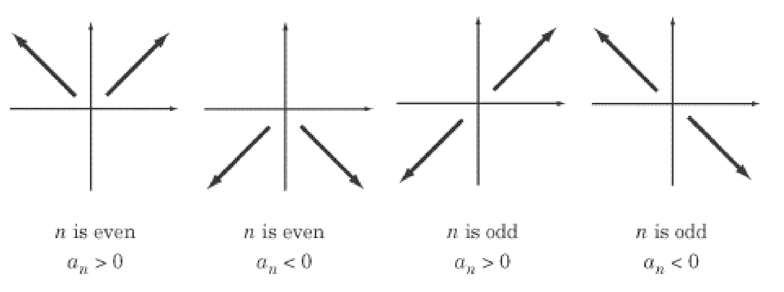


5. $f(x) = x^2(x-3)$ **Left:** **Right:**

6. $f(x) = -2(x+2)(x-2)^3$ **Left:** **Right:**

7. $f(x) = (x+1)^2(x-2)^2$ **Left:** **Right:**

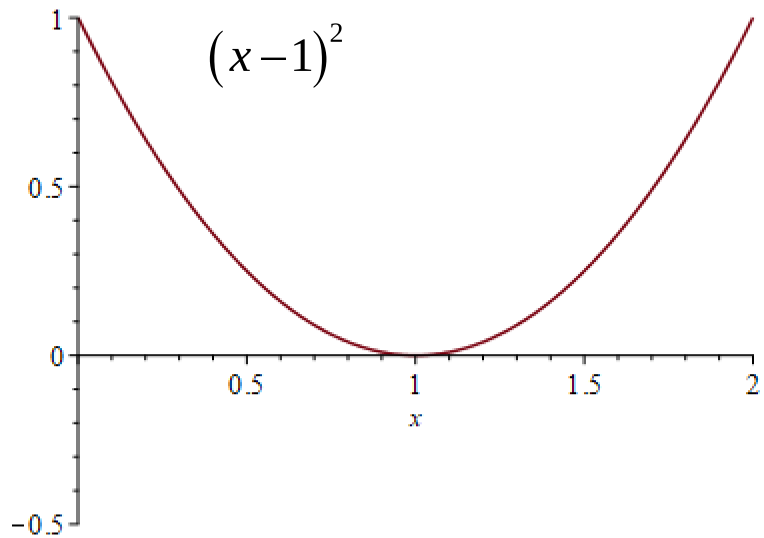
8. $f(x) = -2(x+2)^2(x-2)^3$ **Left:** **Right:**

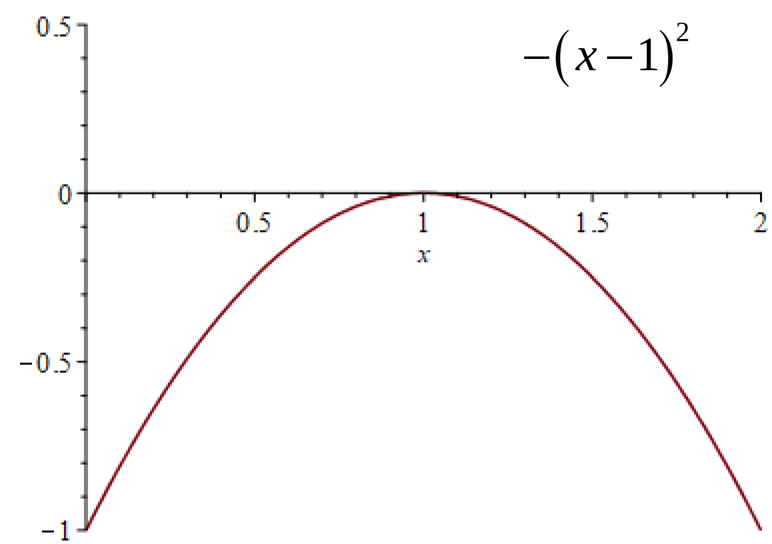


Behavior at the x-intercepts:

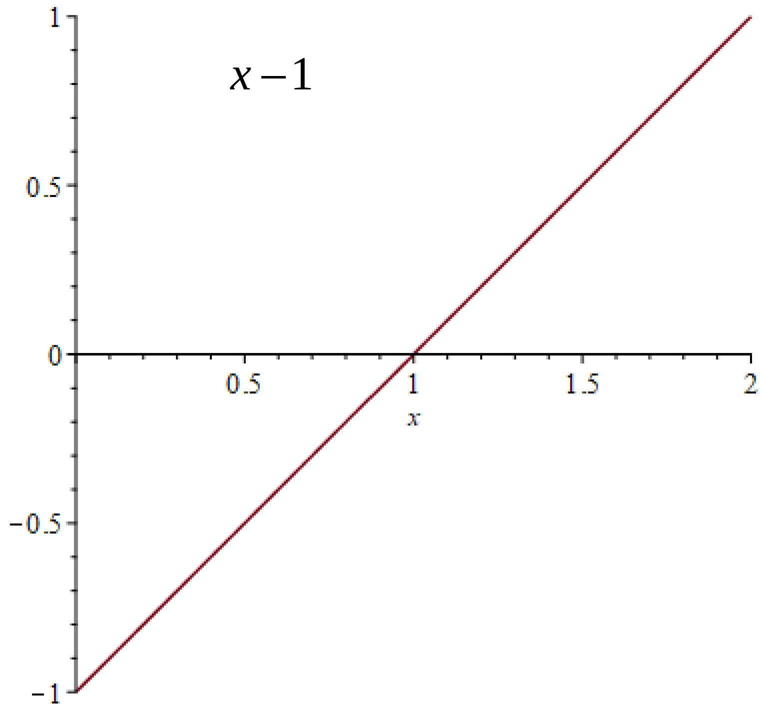
If $(x-c)^k$ is the highest power of $(x-c)$ that is a factor of $f(x)$, with *c* a real number, then

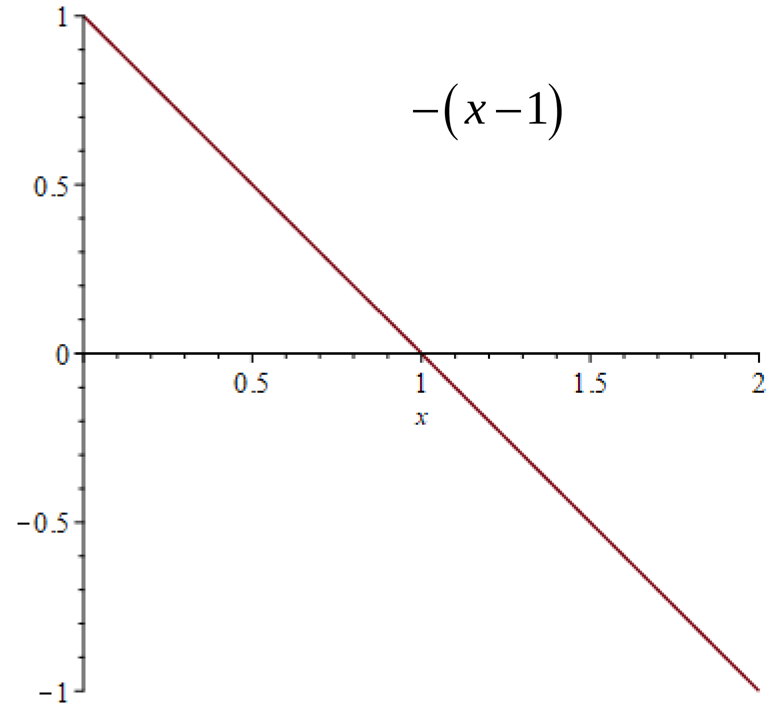
If *k* is even, then the graph touches the *x*-axis at *c* but doesn't cross the axis.



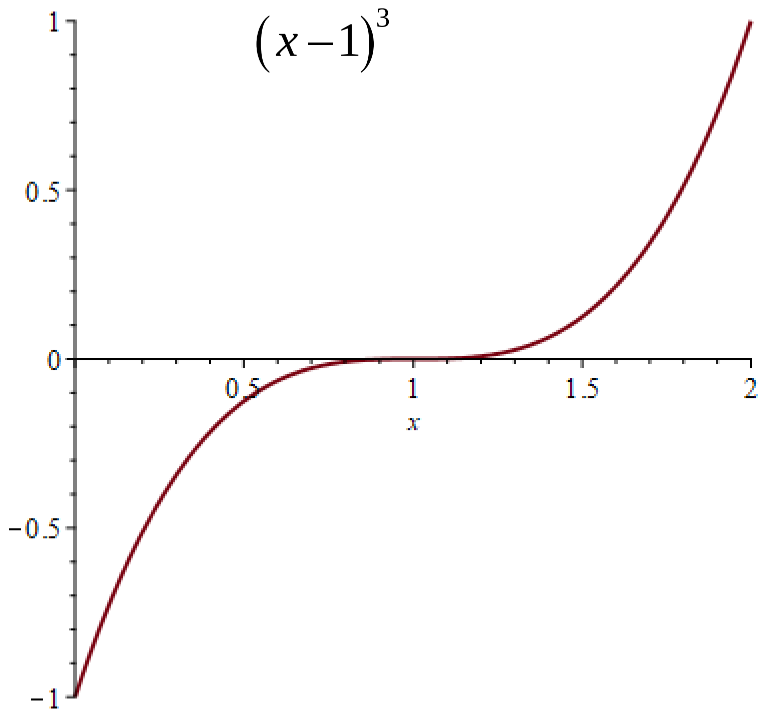


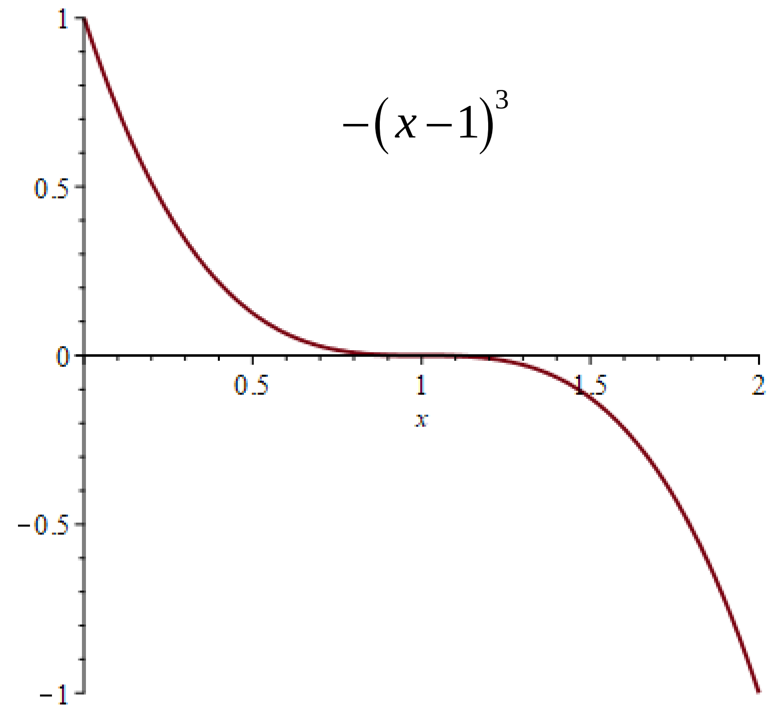
If *k* is 1, then the graph crosses the *x*-axis at *c* with a non-zero angle.





If *k* is odd and greater than 1, then the graph crosses the *x*-axis at *c* with a zero angle(flat).





<u>Steps for sketching graphs of polynomial functions:</u>

1. Determine the end behavior, and indicate it on the graph with arrows.

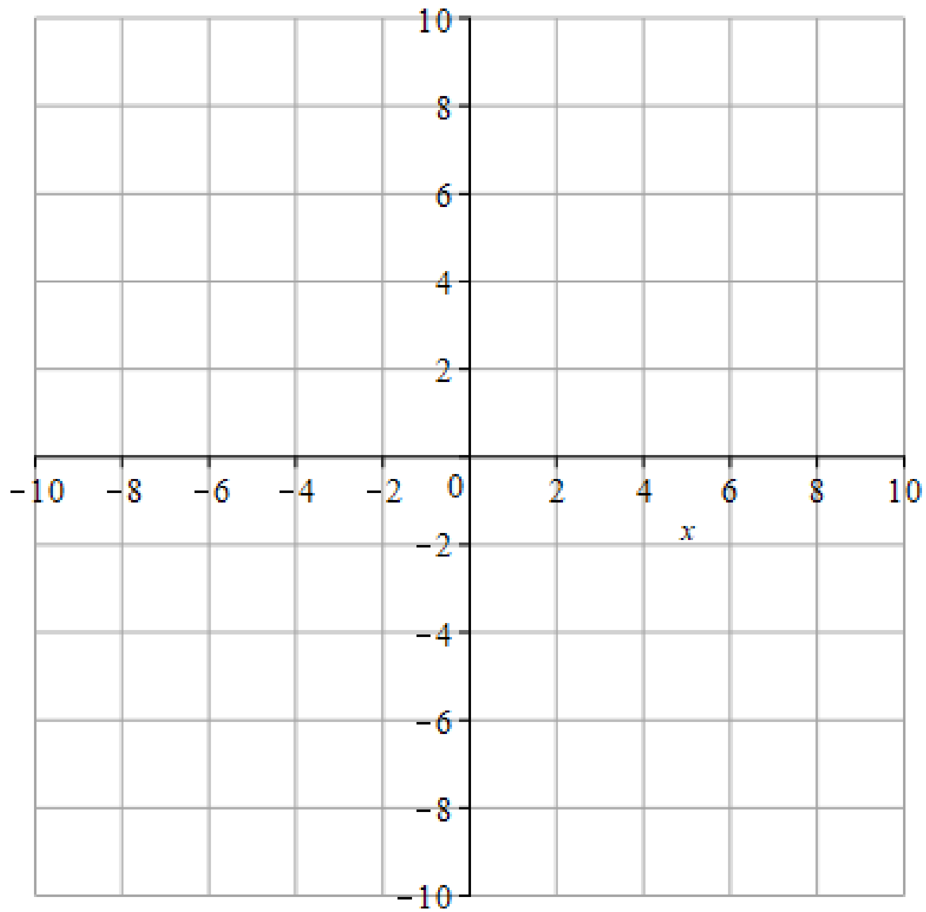
2. Find all the real zeros(*x*-intercepts) of $f(x)$, and indicate them on the graph with points.

3. Find the *y*-intercept(by setting *x* to zero), and indicate it on the graph with a point.

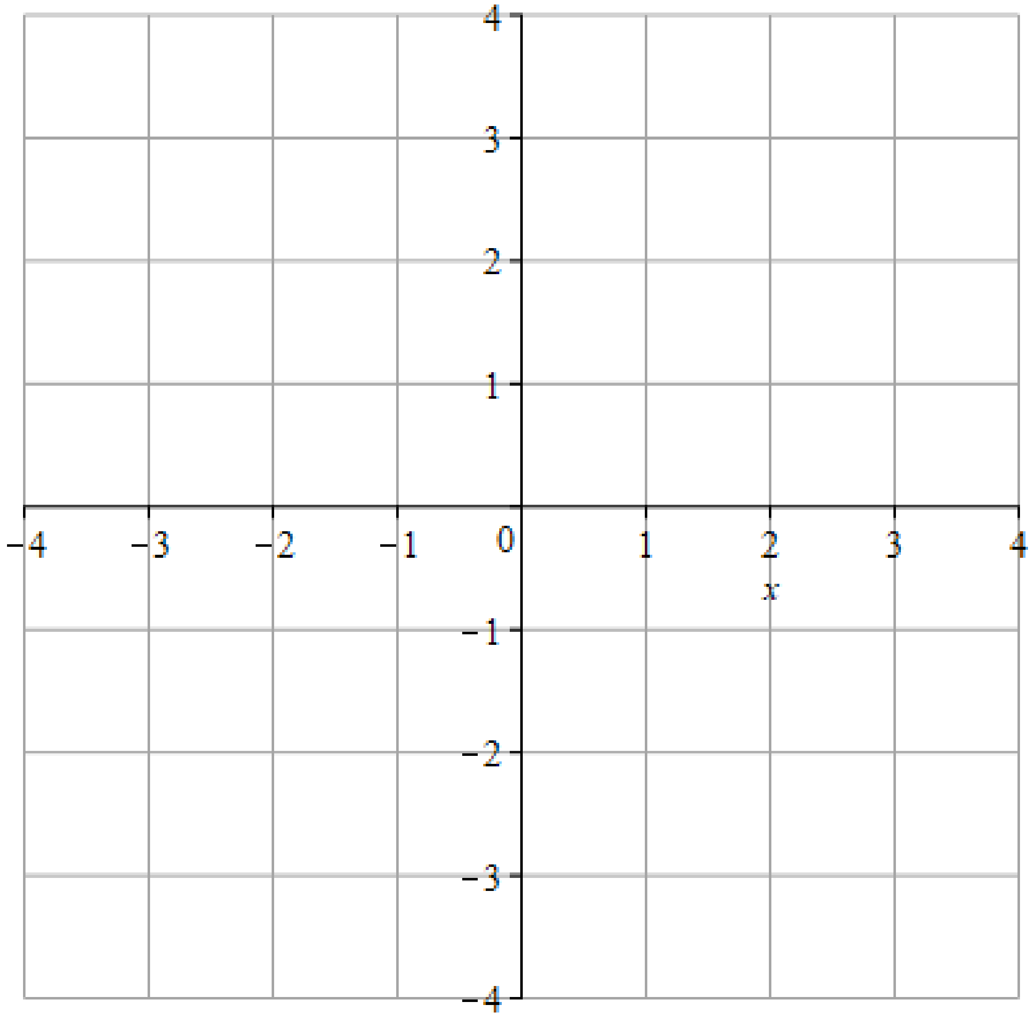
4. Use the end behavior and *x*-intercept behavior to connect the previous points and arrows into a reasonable graph.

<u>Sketch the graphs of the following polynomial functions.</u>

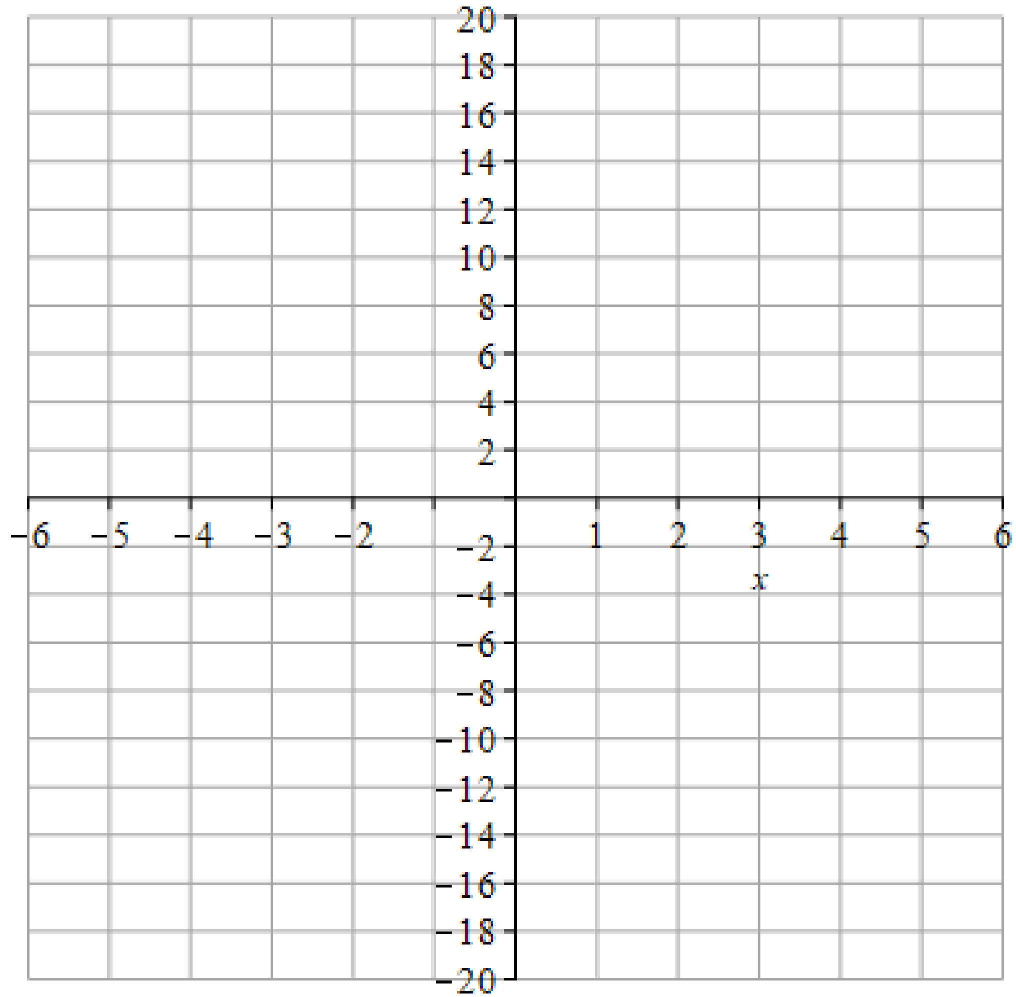
1. $f(x)=\frac{1}{27}(x+4)(x-3)^3$

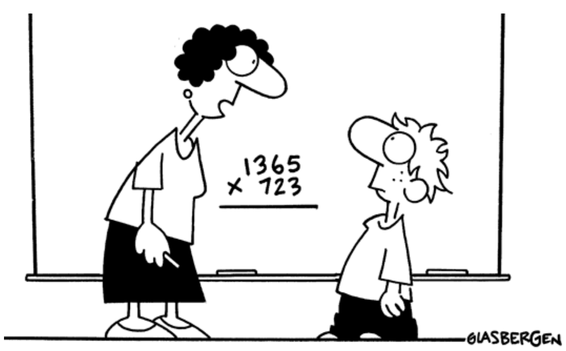


2. $f\left(x\right)=x^{2}\left(x-2\right)$



3. $f(x) = -(x+2)(x-2)^3$

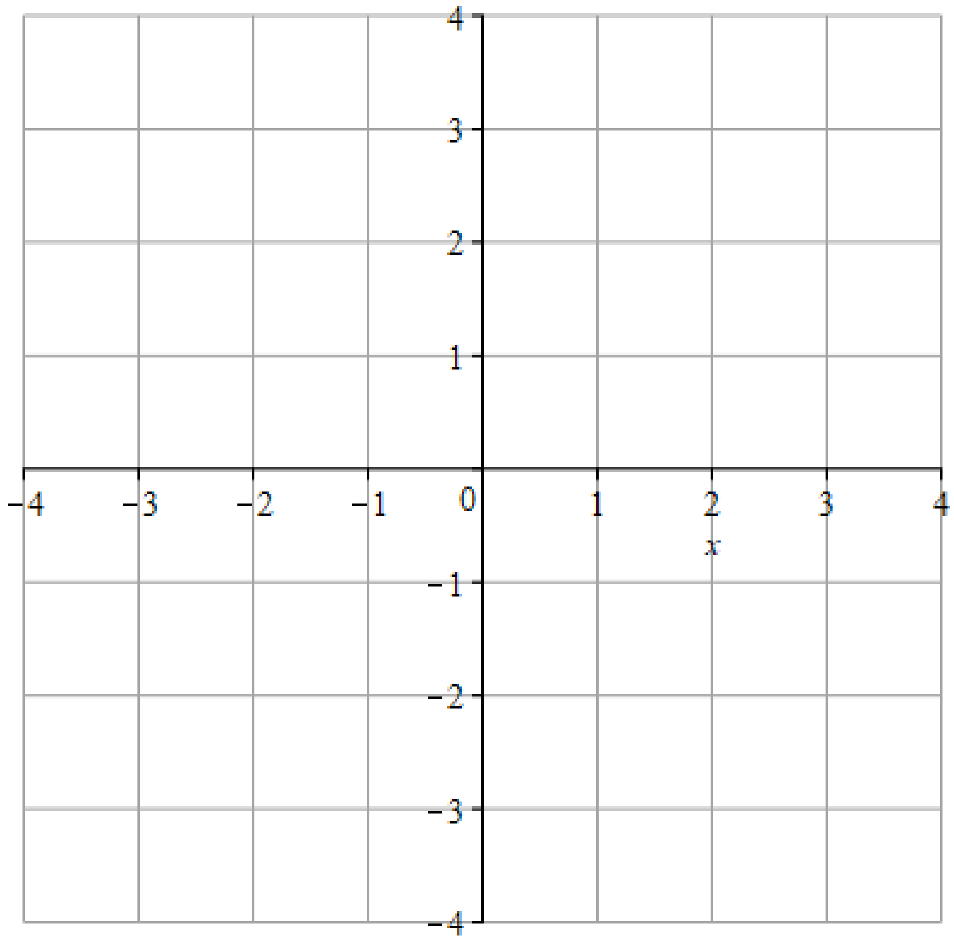


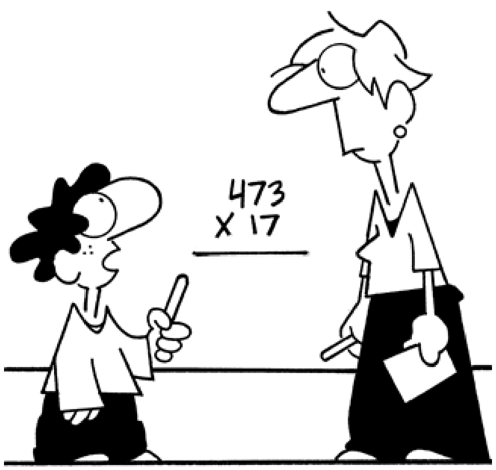


"Pretend you're starring in a reality show about a kid who can make his dreams come true if he works hard and gets good grades."

4. $f(x) = -\left(x^2 - 2\right)x^3$

{Factor first.}

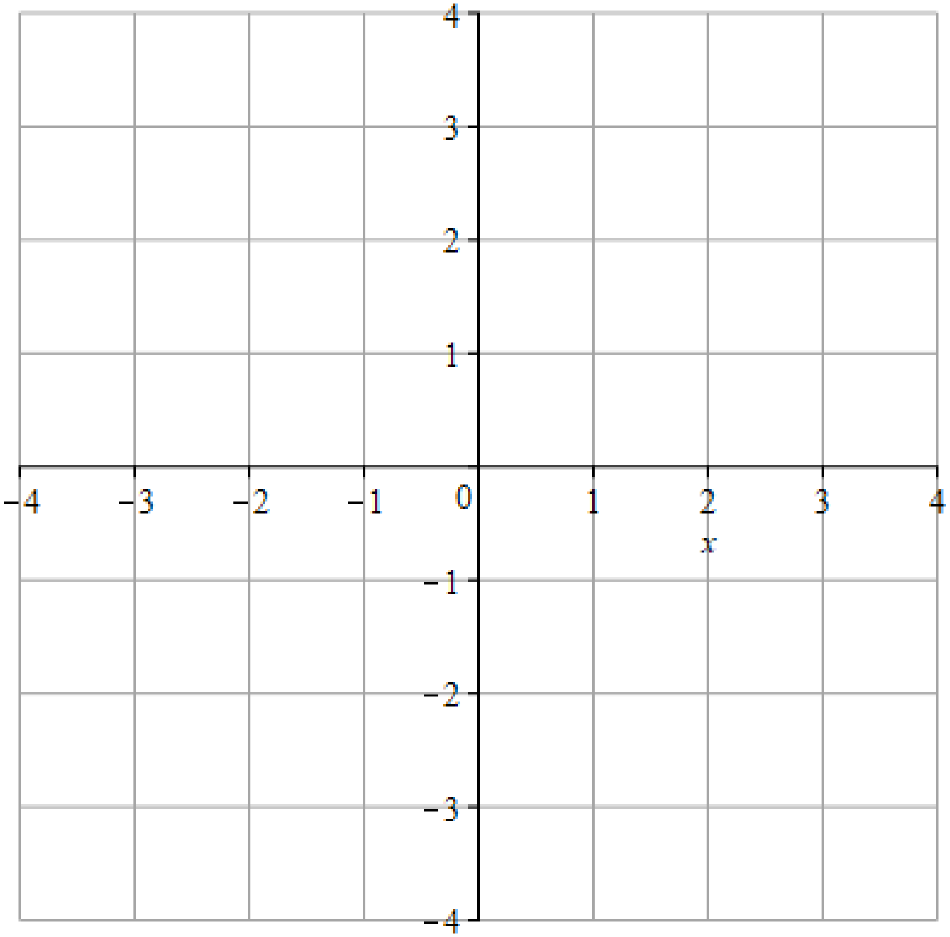


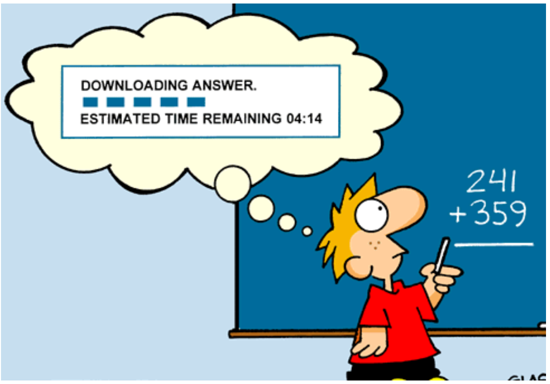


"If we learn from our mistakes, shouldn't I make as many mistakes as possible?"

5. $f\left(x\right)=x-x^{3}$

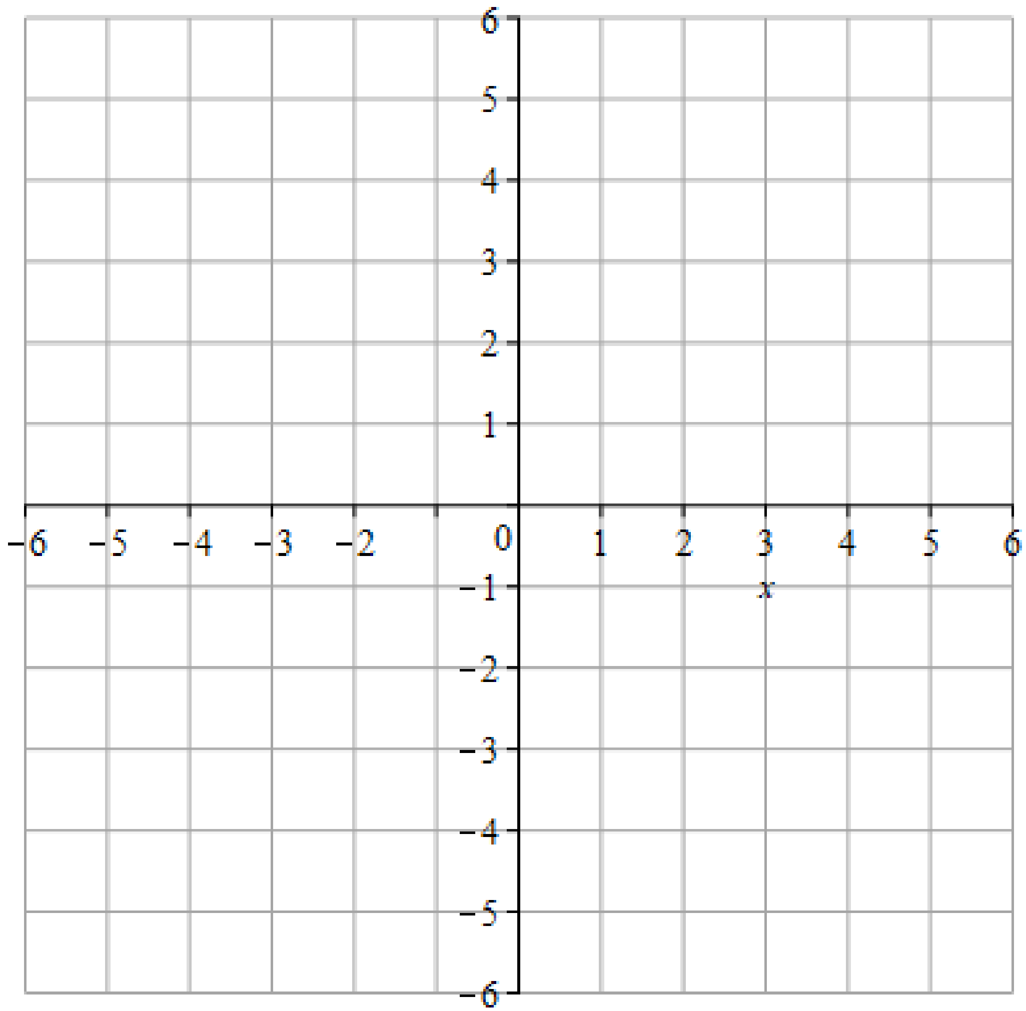
{Factor first.}

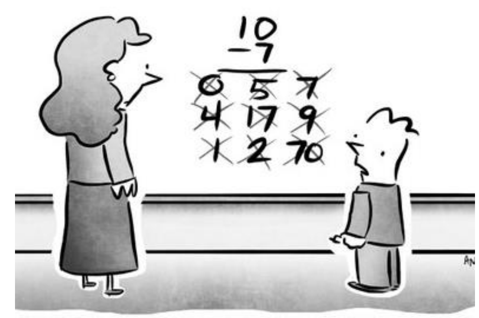




6. $f\left(x\right)=x^{3}+2x^{2}-8x$

{Factor first.}

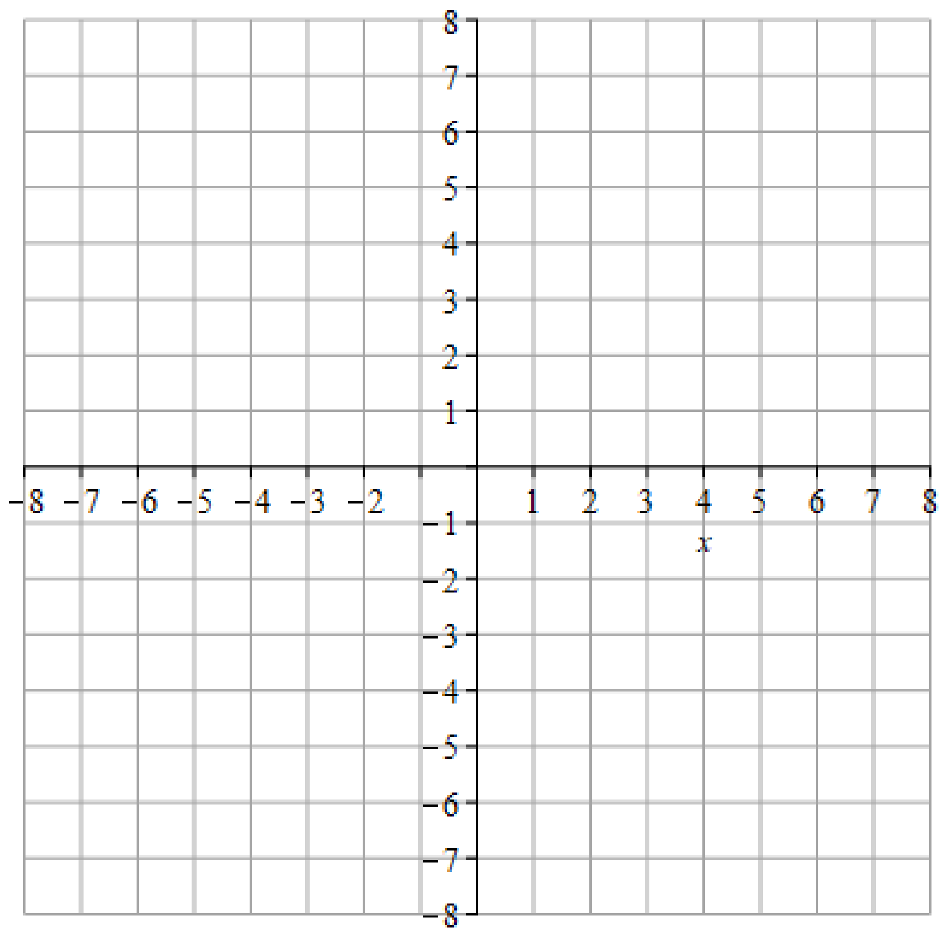


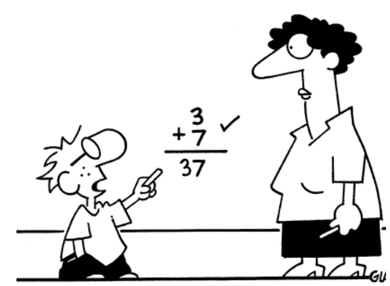


"OK, the good news is we've ruled these out."

7. $f(x) = 2x^4 + 12x^3 - 8x^2 - 48x$

{Factor first.}

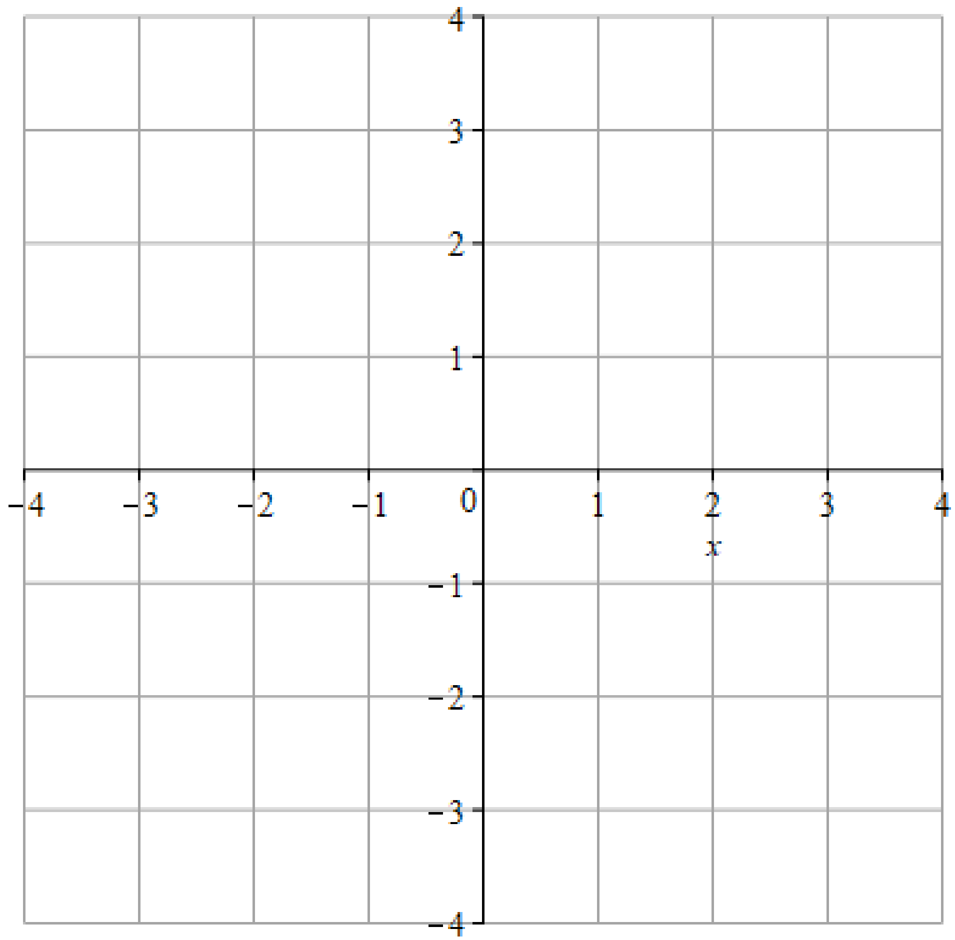




"In the corporate world they pay you big bucks for thinking outside of the box!"

8. $f(x) = x^2 - x^4$

{Factor first.}



things i haven't learned in school

how to:

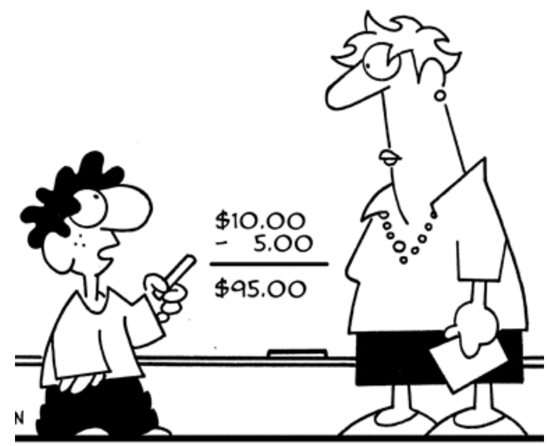
pay bills
buy a house
apply for college

but thank jesus i can graph a polynomial function

Finding zeros of polynomials is not always an easy task. For quadratic polynomials, you have the quadratic formula, but sometimes it gives results like $\frac{3 \pm \sqrt{2}}{4}$, and you still have to approximate the zeros because of the radical, $\sqrt{2}$.

There's also a cubic formula for cubic polynomials, and a quartic formula for quartic polynomials, but they still can lead to results requiring approximations because of the presence of radicals.

It was proven in the 1800's that there are no equivalent formulas for the zeros of fifth degree and higher polynomials in terms of radicals of the coefficients.

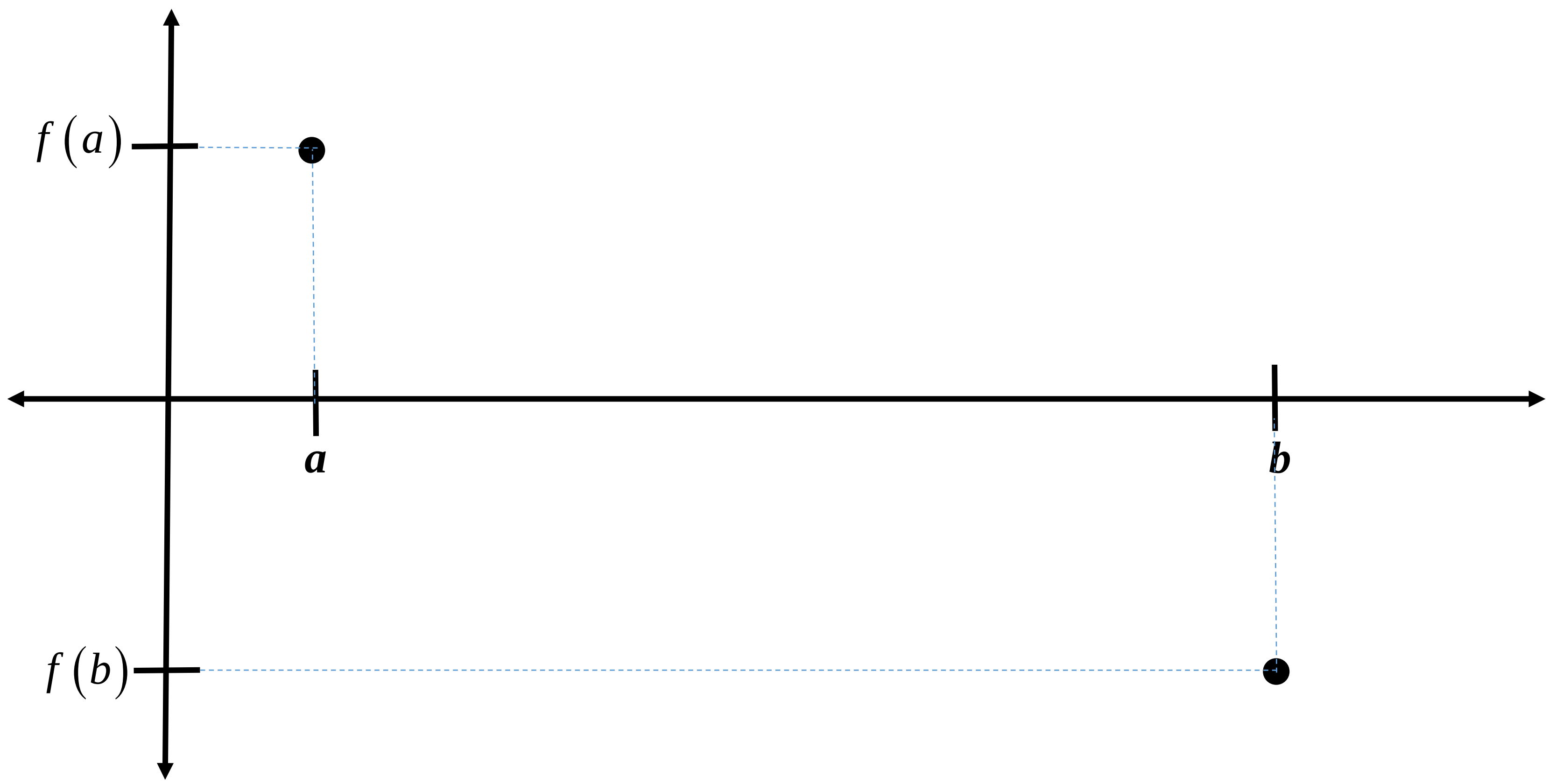


"I'm getting cash back with my debit card."

Approximating real zeros of polynomials:

Intermediate Value Theorem:

For $f(x)$ a polynomial function, if $f(a)$ and $f(b)$ have opposite signs, then there is at least one value *c* between *a* and *b* with $f(c)=0$.



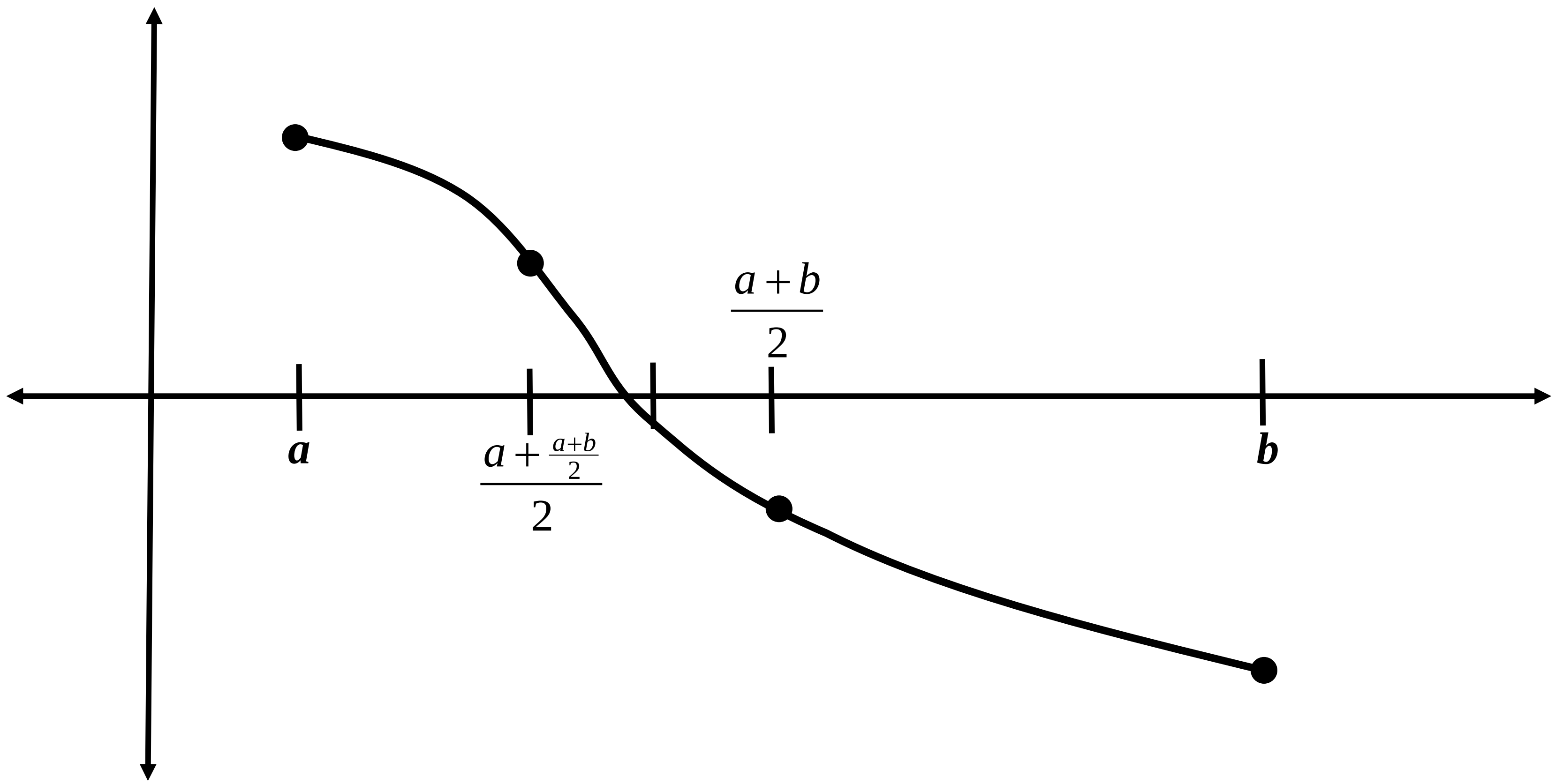
The Bisection Method:

If $f(a)$ **and** $f(b)$ **have opposite signs with** $a<b$**, then there is at least one zero in the interval** (a,b)**, so our first approximation is the midpoint,** $\frac{a+b}{2}$**. The error of our first approximation is less than half the width of the interval,** $\frac{b-a}{2}$**.**

Next, examine the sign at the midpoint $\frac{a+b}{2}$**.**

If $f(a)$ **and** $f\left(\frac{a+b}{2}\right)$ **have opposite signs, then there is at least one zero in the interval** $\left(a,\frac{a+b}{2}\right)$**, and our next approximation is the midpoint of this interval,** $\frac{a+\frac{a+b}{2}}{2}$**, with an error bound of** $\frac{b-a}{4}$**.**

If $f\left(\frac{a+b}{2}\right)$ **and** $f(b)$ **have opposite signs, then there is at least one zero in the interval** $\left(\frac{a+b}{2},b\right)$**, and our next approximation is the midpoint of this interval,** $\frac{\frac{a+b}{2}+b}{2}$**, with an error bound of** $\frac{b-a}{4}$**. The process continues.**



Left Endpoint(sign)	Midpoint(sign)	Right Endpoint(sign)	Error Bound
$a(+)$	$\frac{a+b}{2}(-)$	$b(-)$	$\frac{b-a}{2}$
$a(+)$	$\frac{a+\frac{a+b}{2}}{2}(+)$	$\frac{a+b}{2}(-)$	$\frac{b-a}{4}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Example:

1. Use the Bisection Method to approximate the zero of $f(x) = x^3 - 3x + 1$ **between 0 and 1.**

Left Endpoint(sign)	**Midpoint(sign)**	**Right Endpoint(sign)**	**Error Bound**
0(+)	$\frac{1}{2}(-)$	**1(-)**	$\frac{1}{2}$
0(+)	$\frac{1}{4}(\ )$	$\frac{1}{2}(-)$	$\frac{1}{4}$

See the link Bisection Worksheet to get a fast expansion of the table.

How many steps of bisection would we need to do to approximate the zero to within .05?

The formula for the error bound is $\frac{1}{2^n}$**, so we'd want**

$\frac{1}{2^n} \le .05 \Rightarrow 2^n \ge \frac{1}{.05} \Rightarrow 2^n \ge 20 \Rightarrow n \ge 5$. **So we'd need to do at least 5 bisections.**

2. Use the Bisection Method to approximate the zero of $f\left(x\right)=x^3-x-1$ **between 1 and 2.**

Left Endpoint(sign)	**Midpoint(sign)**	**Right Endpoint(sign)**	**Error Bound**
1(-)	$\frac{3}{2}\left(+\right)$	**2(+)**	$\frac{1}{2}$
1(-)	$\frac{5}{4}\left(\ \right)$	$\frac{3}{2}\left(+\right)$	$\frac{1}{4}$

See the link Bisection XL to get a fast expansion of the table.