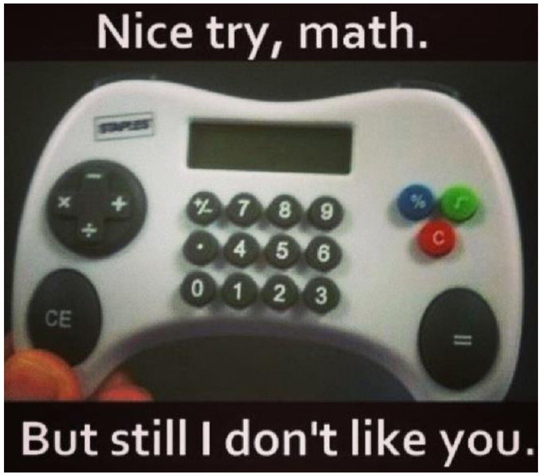


<u>Review of Properties of Logarithms:</u>

For *M* and *N* positive numbers and *r* a real number,

Product Rule:

Expansion

$$log_b(MN) = log_b M + log_b N$$

Compression

Why? $b^{log_b(MN)} =$ **and** $b^{(log_b M + log_b N)} = b^{log_b M} \cdot b^{log_b N} =$

Expand and simplify:

$$log_5(25x)$$

Compress and simplify:

$$log_6 9 + log_6 4$$

Quotient Rule:

Expansion

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

Compression

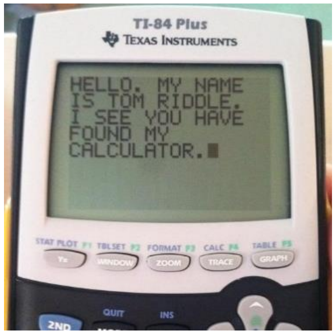
Why? $b^{\log_b\left(\frac{M}{N}\right)} =$ **and** $b^{(\log_b M - \log_b N)} = \frac{b^{\log_b M}}{b^{\log_b N}} =$

Expand and simplify:

$$\log_3\left(\frac{x}{9}\right)$$

Compress and simplify:

$$\log_3 2 - \log_3 6$$



Power Rule:

Expansion

$$\log_b\left(M^r\right) = r\log_b M$$

Compression

Why? $b^{\log_b\left(M^r\right)} =$ **and** $b^{r\log_b M} = \left(b^{\log_b M}\right)^r =$

Expand and simplify:

$$\log_7\left(7x^5\right)$$

Compress:

$$2\log_3 x - 4\log_3 y$$

$$\ln(ab) = \ln(a) + \ln(b)$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$\ln(a^b) = b\ln(a)$$

Expand: $log_2\left[\dfrac{x^3(x+2)}{(x+3)^2}\right]$

$$\log_b x + \log_b y = \log_b(x \cdot y)$$

$$\log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$$

$$z \cdot \log_b x = \log_b(x^z)$$

Compress: $3log_5(3x+1) - 2log_5(2x-1) - log_5 x$

Change of Base Formula:

Suppose that $y = log_b x$**. Then** $b^y = x$ **and therefore** $log_a\left(b^y\right) = log_a x$**. From the Power Rule, you get** $y\,log_a b = log_a x$**, and solving for *y* yields** $y = \frac{log_a x}{log_a b}$**. So**

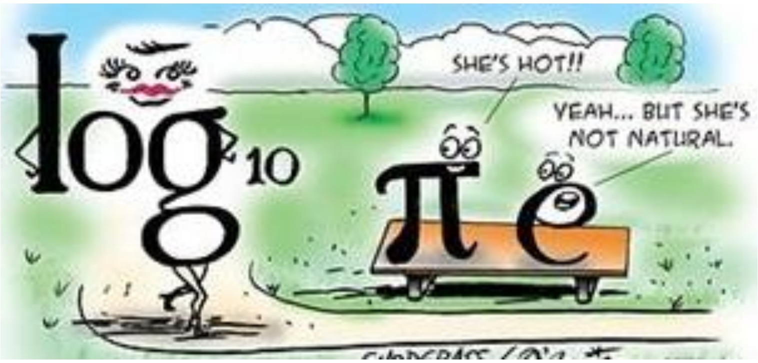
$$log_b x = \frac{log_a x}{log_a b}.$$

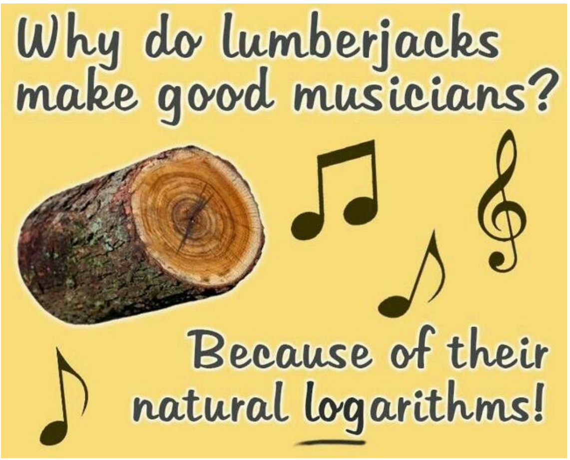
Calculators have a logarithm key for base 10, log, called the common logarithm. They also have a logarithm key for base *e*, ln, called the natural logarithm. $e = 2.7182818...$

$$log_b x = \frac{log\, x}{log\, b}$$

Or

$$log_b x = \frac{ln\, x}{ln\, b}$$





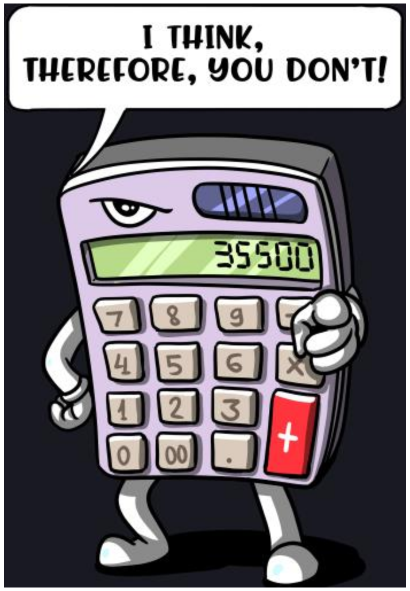
Example:

Calculate $log_3 5$ **to 3 decimal places.**

$$log_3 5 = \frac{log\,5}{log\,3}$$

Or

$$log_3 5 = \frac{ln5}{ln3}$$



Exponential and Logarithmic Equations:

The goal in solving exponential and logarithmic equations is to remove the exponential and logarithmic parts, eventually.

$$b^x = b^y \Rightarrow x = y$$

$$log_b x = log_b y \Rightarrow x = y$$

$$log_b \left(b^x \right) = x; \text{for all } x$$

$$b^{log_b x} = x; \text{for } x > 0$$

Examples:

1. $log(x+6)=1$

2. $log_4(x+2)=log_4 8$

3. $log_4(x+2)=log_4(2x+7)$ **{Be careful!}**

4. $2\log_5 x = 3\log_5 4$

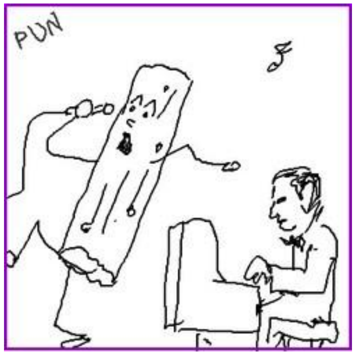
5. $\log_6(x+4)+\log_6(x+3)=1$

6. $\log_3 x - 2\log_3 5 = \log_3(x+1) - 2\log_3 10$

7. $3^{2x}+3^{x}-2=0$

8. $2^{2x}+2^{x+2}-12=0$

9. $3^{1-2x}=4^{x}$



10. $5^{2x} - 8 \cdot 5^{x} = -16$

11. $3^{x} - 14 \cdot 3^{-x} = 5$

Approximating Solutions of Equations Using the Method of Successive Approximation:

Step #1: Arrange the equation into the form $x = f(x)$**.**

Step #2: Choose a starting guess/approximation for a solution, x_1**.**

Step #3: Evaluate the function, *f*, at x_1 **to get the second approximation,** x_2**.**

Step #4: Continue this process of evaluating the function, *f*, at the current approximation, x_{n-1}**, to get the next approximation,** x_n**.**

$$x_n = f\left(x_{n-1}\right)$$

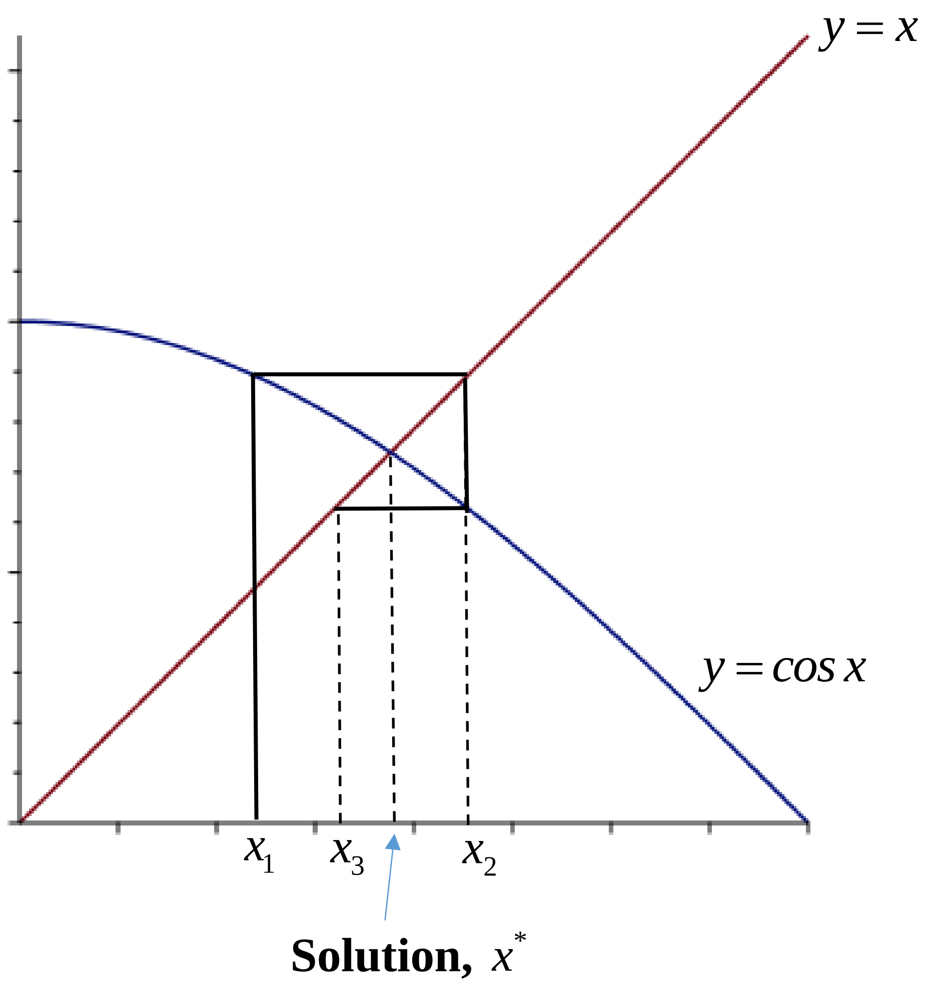
Example: **Approximate a solution of the equation $x = \cos x$ using the Method of Successive Approximation with a starting guess of $x_1 = .5$.**

x_1	**.5**
x_2	**.877582561**
x_3	**.639012494**
x_4	**.8026851**
x_5	**.694778026**
x_6	**.768195831**
x_7	**.719165445**
x_8	**.752355759**
x_9	**.730081063**
x_{10}	**.745120341**
x_{11}	**.735006309**
x_{12}	**.741826522**

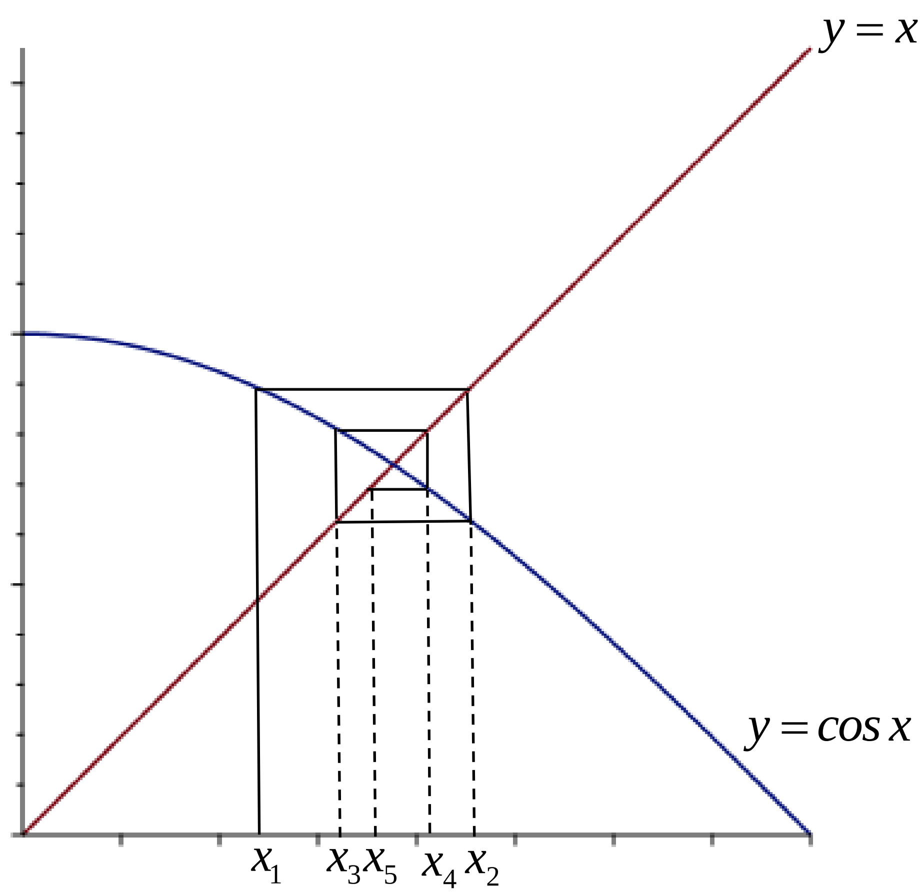
Check out the link Successive XL.

How do you know if the Method of Successive Approximation is producing approximations that are getting closer to an actual solution?

Do a graphical analysis called a *Cobweb Diagram*. Solutions correspond to points of intersection between the graph of $y = x$ and $y = f(x)$.



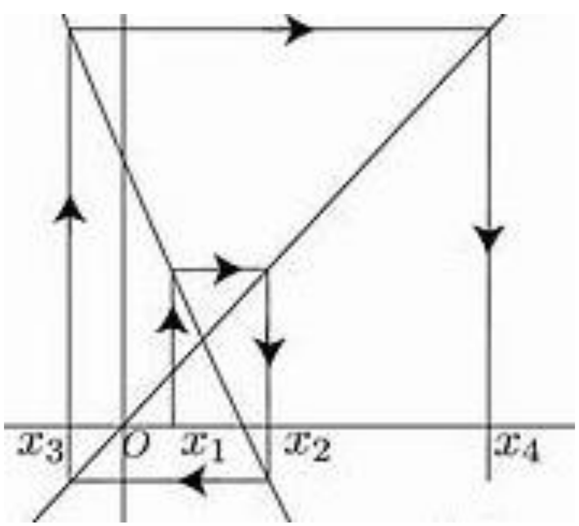
The order you follow to create the Cobweb Diagram is vertically to the curve($y = f(x)$), horizontally to the line($y = x$), vertically to the curve, horizontally to the line, …

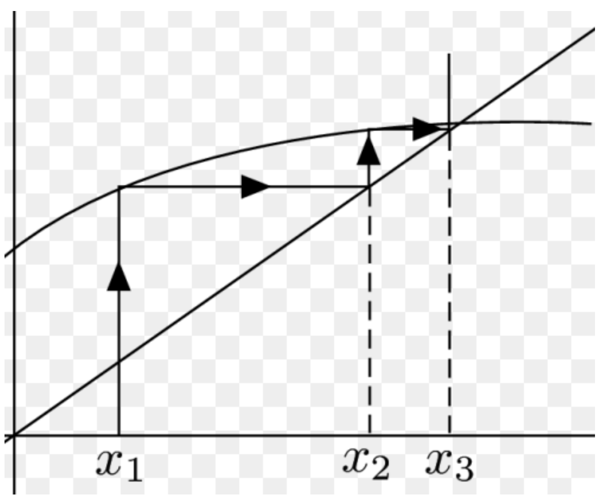


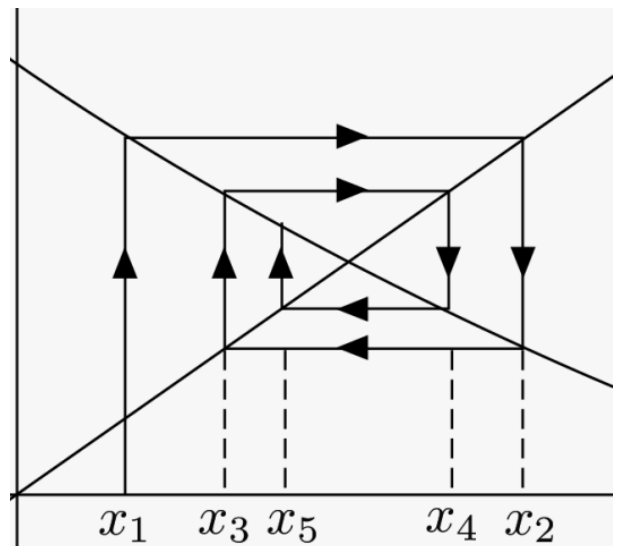
Solutions for which nearby starting guesses generate approximations that get closer to the solution are called *attracting solutions*.

Solutions for which nearby starting guesses generate approximations that are pushed away from the solution are called *repelling solutions*.

Repelling solutions are invisible to this method of approximation.



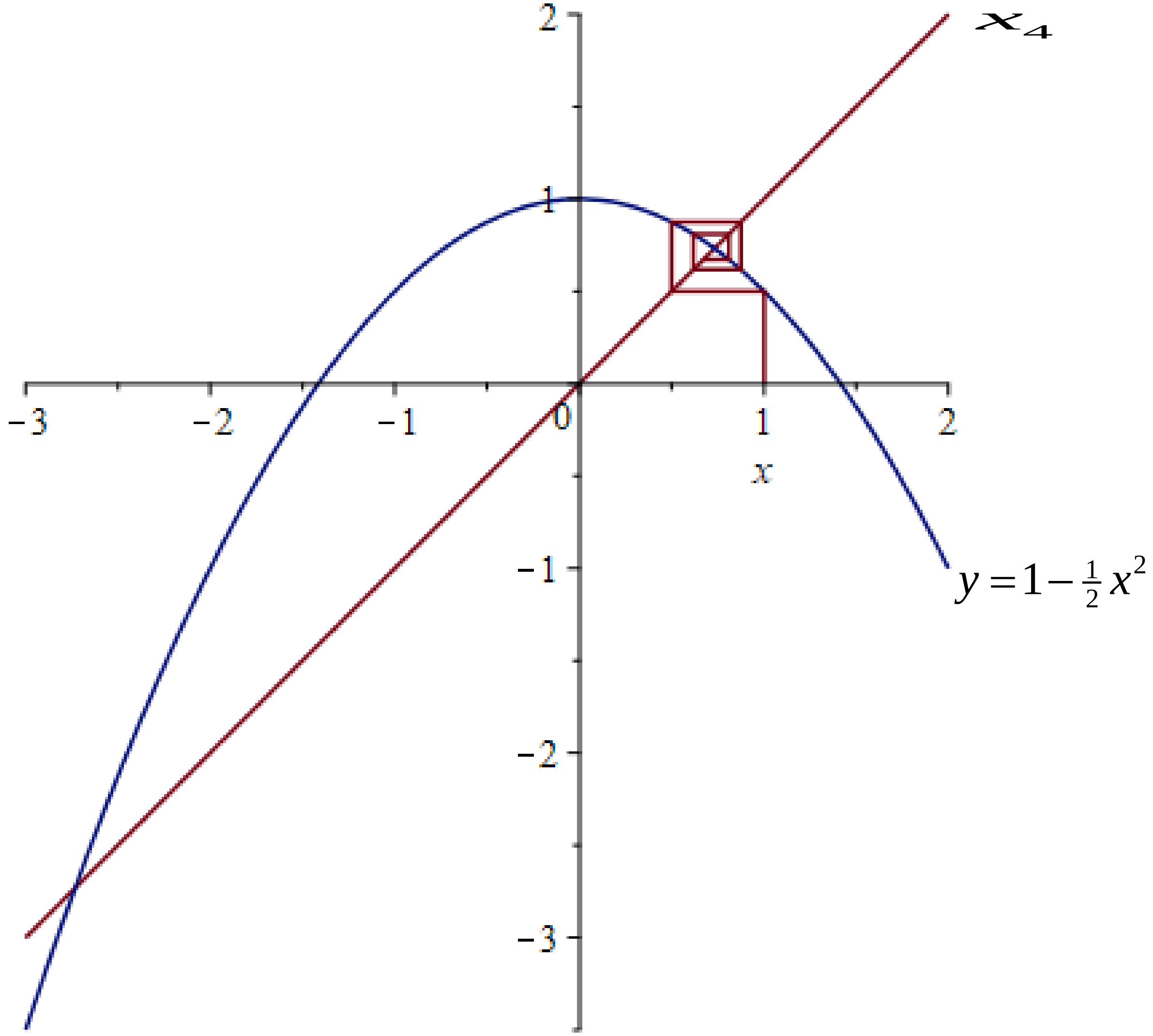




The equation $1-\frac{1}{2}x^2 = x$ has two solutions. Classify them as attracting or repelling. Demonstrate with the starting values of $x_1 = 1, 0, -2, -3$.

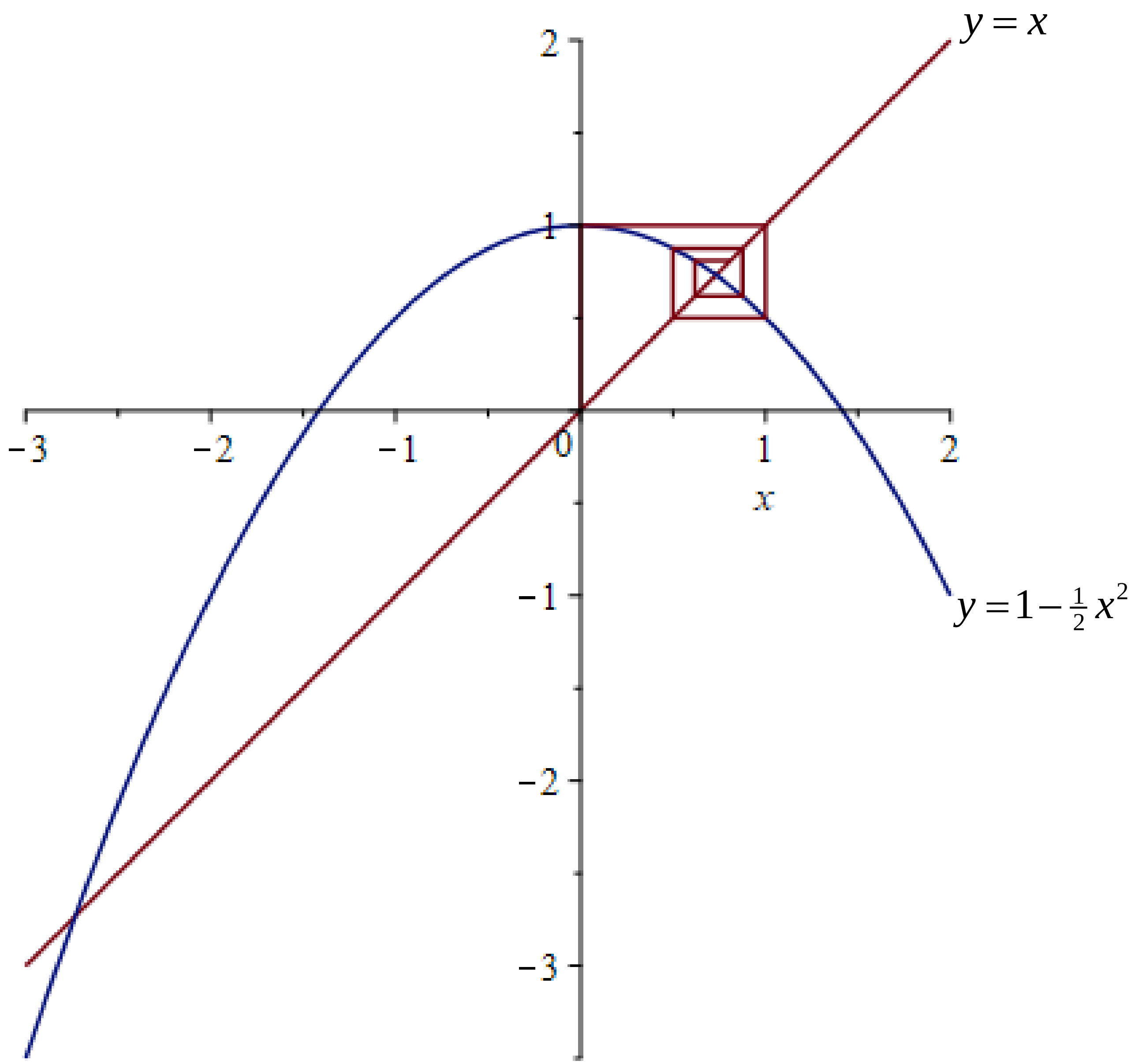
$x_1 = 1$

x_1	**1**
x_2	**0.5**
x_3	**0.875**
x_4	**0.6171875**
x_5	**0.809539795**
x_6	**0.67232266**
x_7	**0.77399112**
x_8	**0.700468873**
x_9	**0.754671679**
x_{10}	**0.715235328**
x_{11}	**0.744219213**
x_{12}	**0.723068882**
x_{13}	**0.738585696**
x_{14}	**0.727245585**
x_{15}	**0.73555693**
x_{16}	**0.729478002**
x_{17}	**0.733930923**
x_{18}	**0.7306727**
x_{19}	**0.733058702**
x_{20}	**0.731312469**



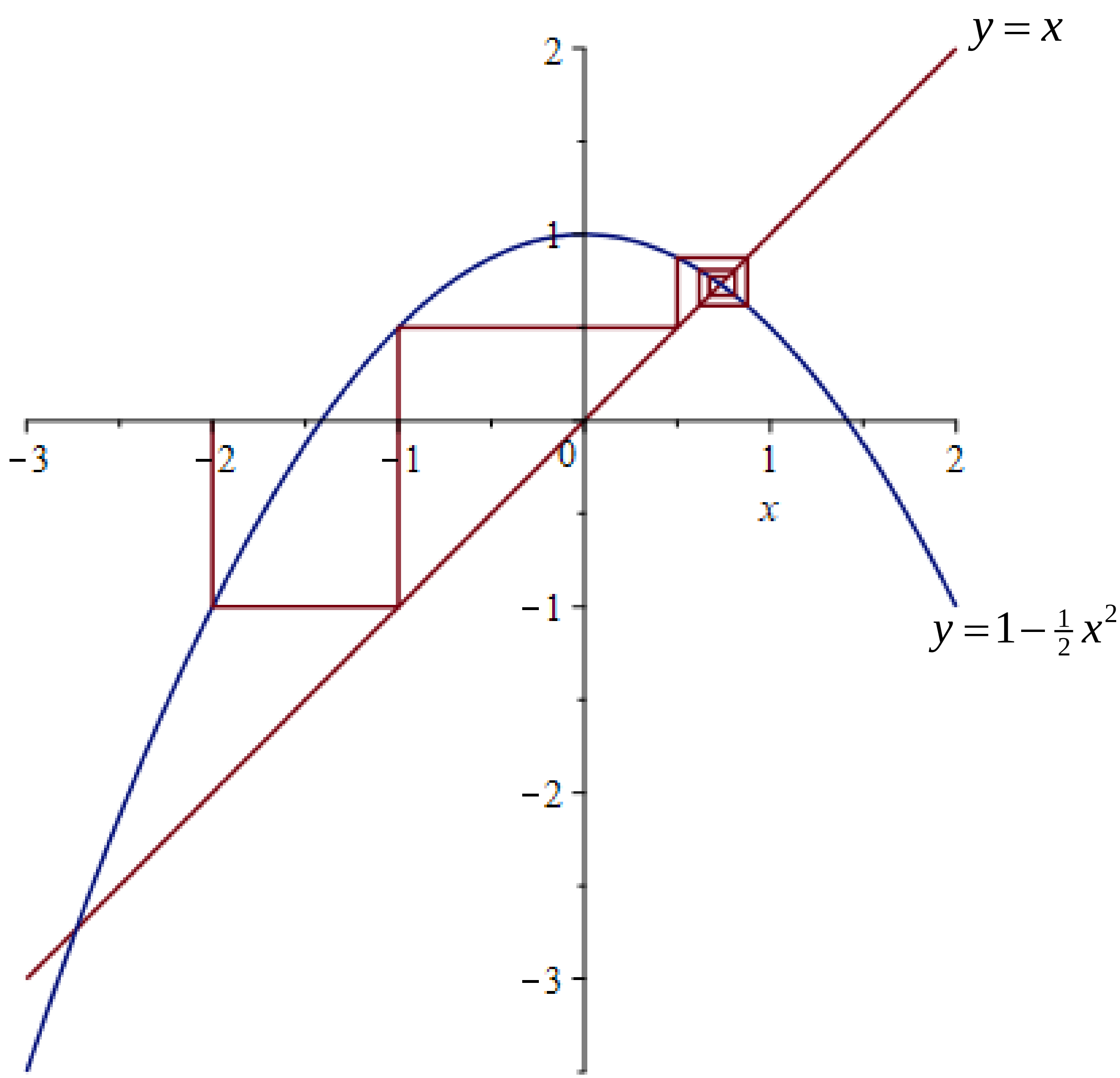
$x_1 = 0$

x_1	**0**
x_2	**1**
x_3	**0.5**
x_4	**0.875**
x_5	**0.6171875**
x_6	**0.809539795**
x_7	**0.67232266**
x_8	**0.77399112**
x_9	**0.700468873**
x_{10}	**0.754671679**
x_{11}	**0.715235328**
x_{12}	**0.744219213**
x_{13}	**0.723068882**
x_{14}	**0.738585696**
x_{15}	**0.727245585**
x_{16}	**0.73555693**
x_{17}	**0.729478002**
x_{18}	**0.733930923**
x_{19}	**0.7306727**
x_{20}	**0.733058702**
x_{21}	**0.731312469**



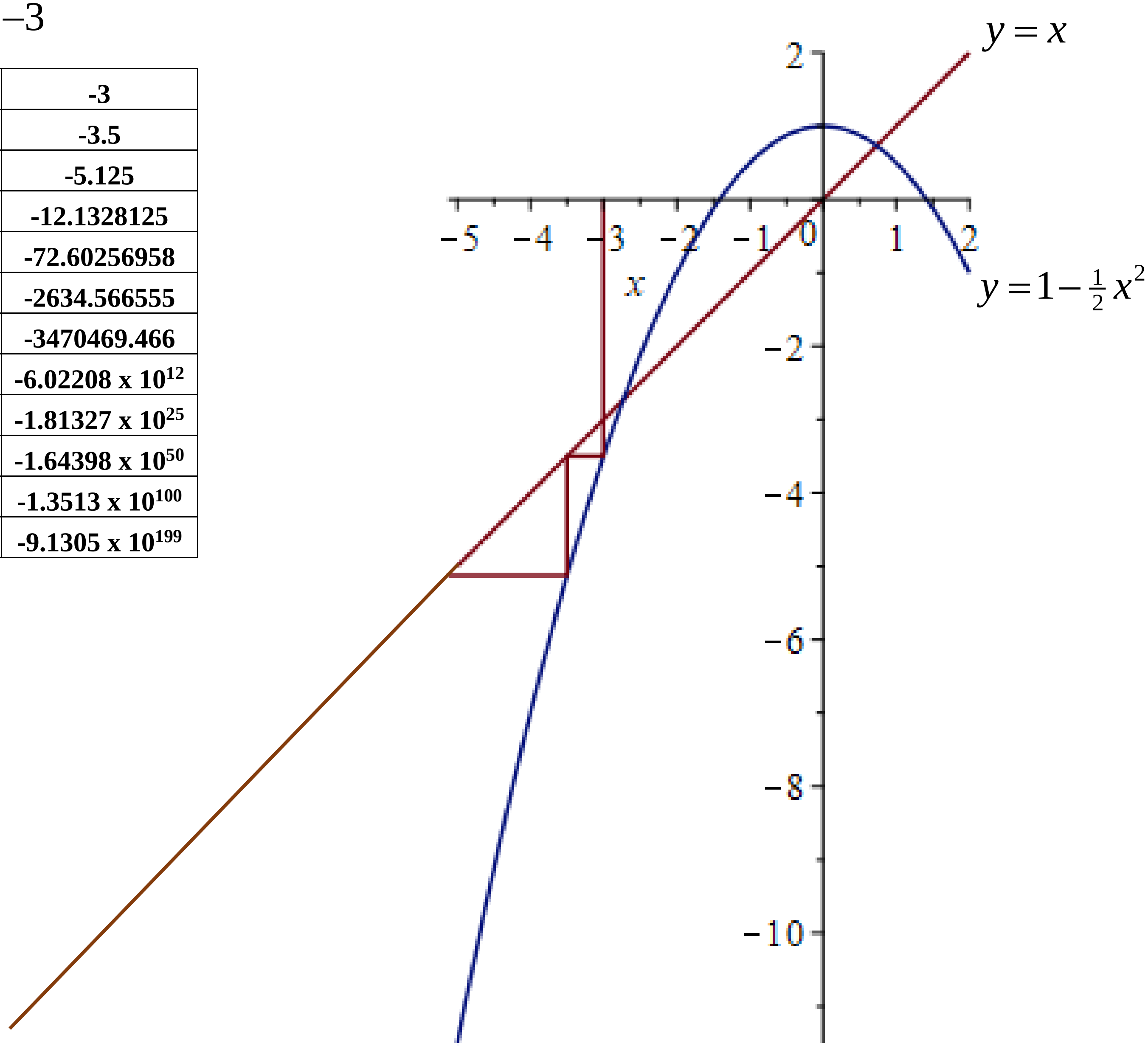
$x_1 = -2$

x_1	**-2**
x_2	**-1**
x_3	**0.5**
x_4	**0.875**
x_5	**0.6171875**
x_6	**0.809539795**
x_7	**0.67232266**
x_8	**0.77399112**
x_9	**0.700468873**
x_{10}	**0.754671679**
x_{11}	**0.715235328**
x_{12}	**0.744219213**
x_{13}	**0.723068882**
x_{14}	**0.738585696**
x_{15}	**0.727245585**
x_{16}	**0.73555693**
x_{17}	**0.729478002**
x_{18}	**0.733930923**
x_{19}	**0.7306727**
x_{20}	**0.733058702**
x_{21}	**0.731312469**



$x_1 = -3$

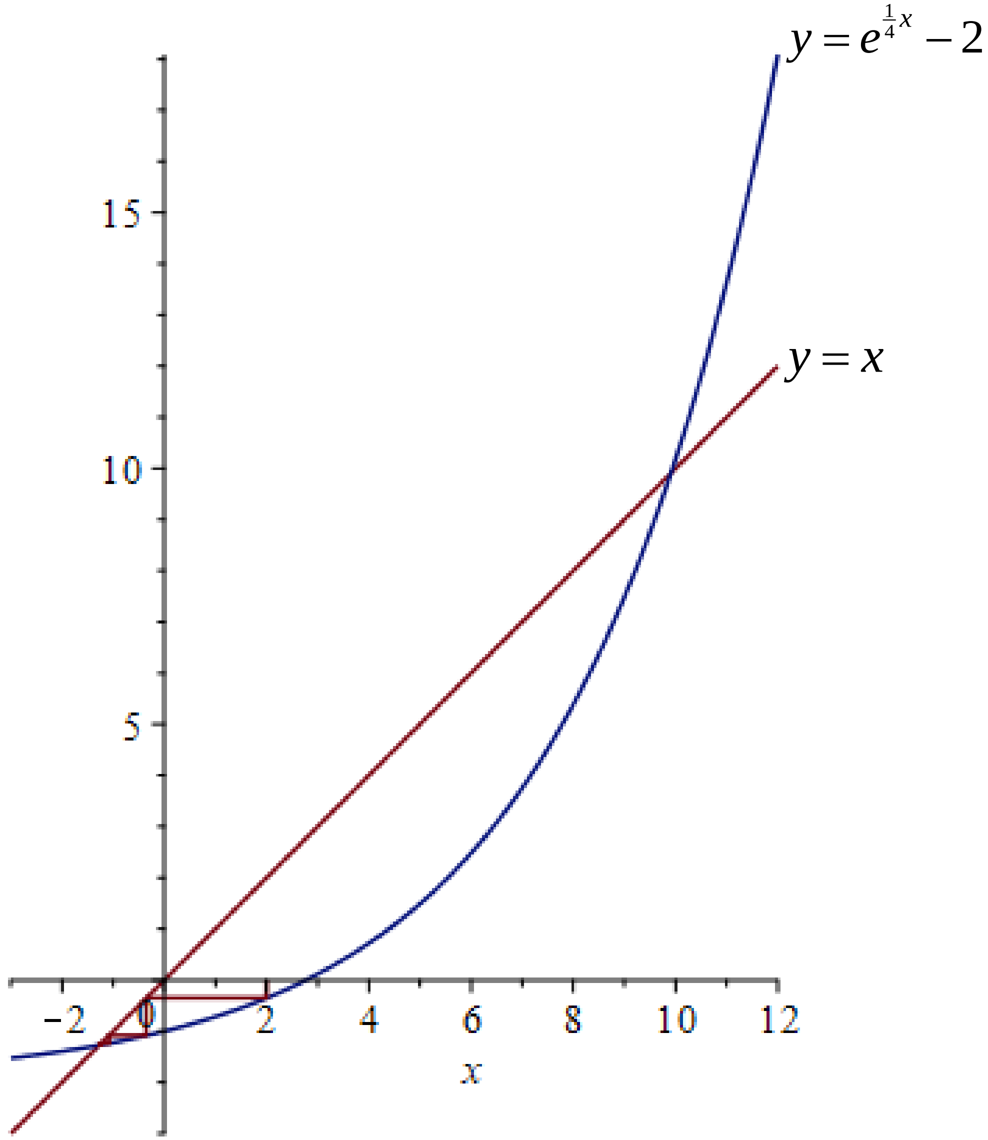
x_1	**-3**
x_2	**-3.5**
x_3	**-5.125**
x_4	**-12.1328125**
x_5	**-72.60256958**
x_6	**-2634.566555**
x_7	**-3470469.466**
x_8	**-6.02208×10^{12}**
x_9	**-1.81327×10^{25}**
x_{10}	**-1.64398×10^{50}**
x_{11}	**-1.3513×10^{100}**
x_{12}	**-9.1305×10^{199}**



The equation $e^{\frac{1}{4}x} - 2 = x$ has two solutions. Classify them as attracting or repelling. Demonstrate with the starting values of $x_1 = 2, 6, 8, 10, -2$.

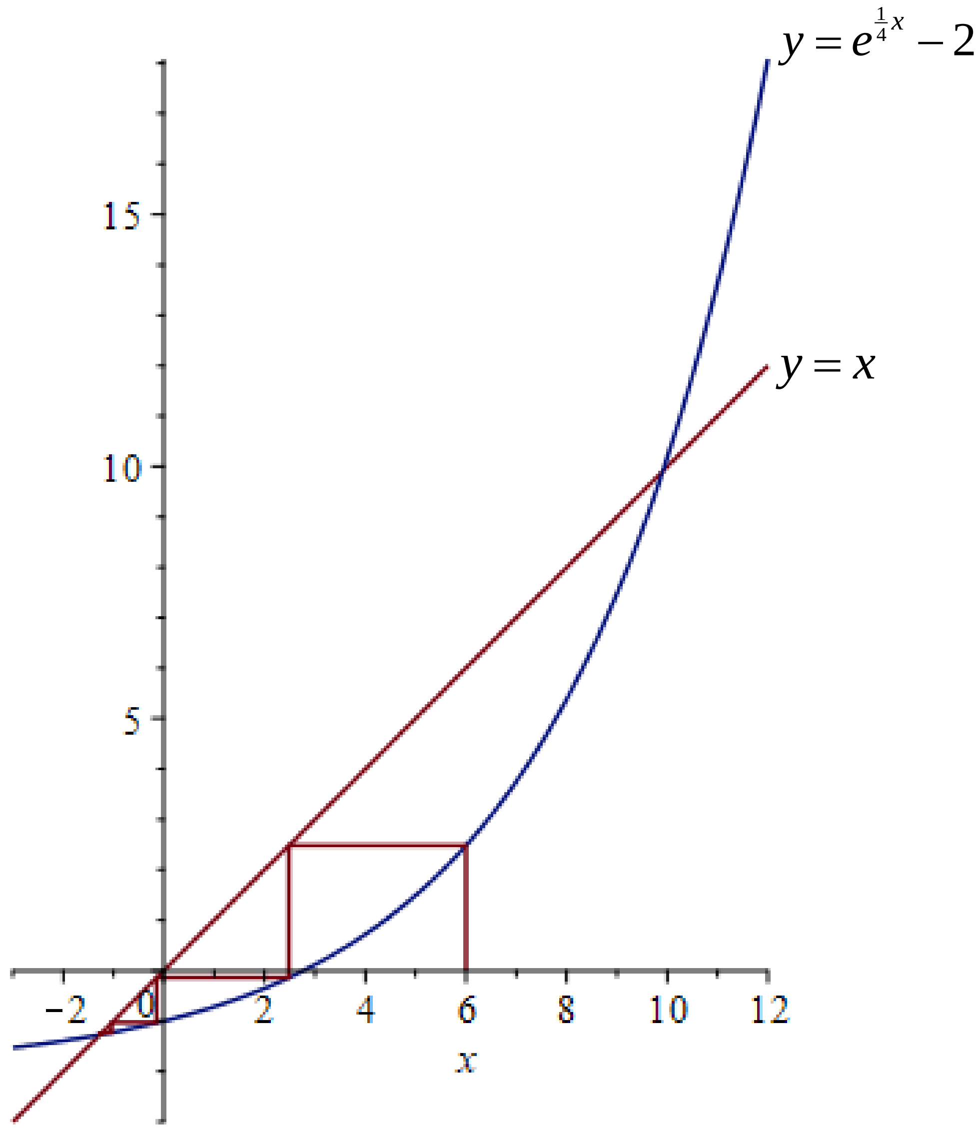
$x_1 = 2$

x_1	**2**
x_2	**-0.351278729**
x_3	**-1.084073981**
x_4	**-1.237397608**
x_5	**-1.266075709**
x_6	**-1.271318781**
x_7	**-1.272273287**
x_8	**-1.272446921**
x_9	**-1.272478502**
x_{10}	**-1.272484246**
x_{11}	**-1.272485291**
x_{12}	**-1.272485481**
x_{13}	**-1.272485516**
x_{14}	**-1.272485522**
x_{15}	**-1.272485523**
x_{16}	**-1.272485523**



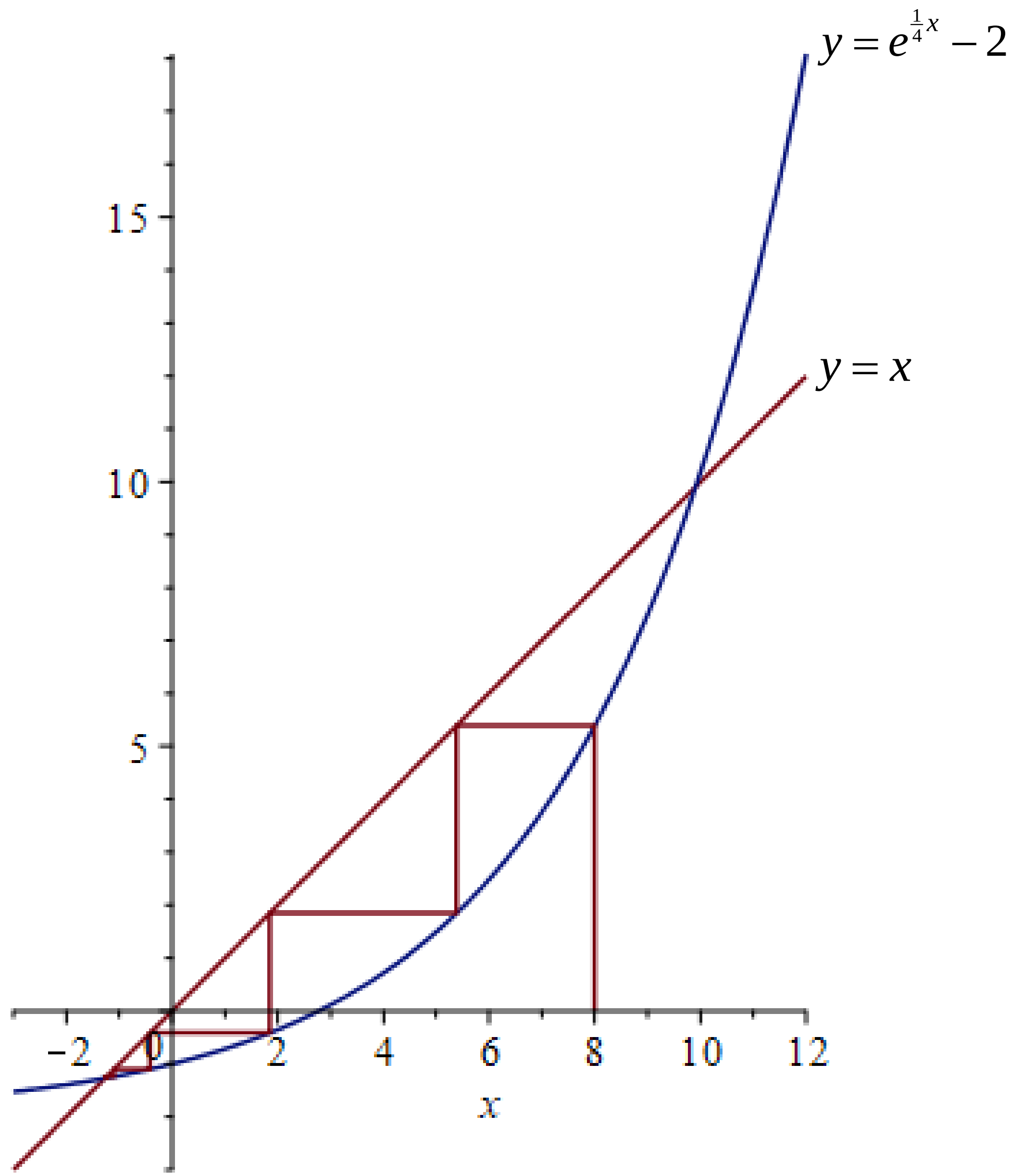
$x_1 = 6$

x_1	**6**
x_2	**2.48168907**
x_3	**-0.140286827**
x_4	**-1.034463822**
x_5	**-1.227880506**
x_6	**-1.264327422**
x_7	**-1.271000225**
x_8	**-1.272215329**
x_9	**-1.272436379**
x_{10}	**-1.272476585**
x_{11}	**-1.272483898**
x_{12}	**-1.272485228**
x_{13}	**-1.272485469**
x_{14}	**-1.272485513**
x_{15}	**-1.272485521**
x_{16}	**-1.272485523**
x_{17}	**-1.272485523**



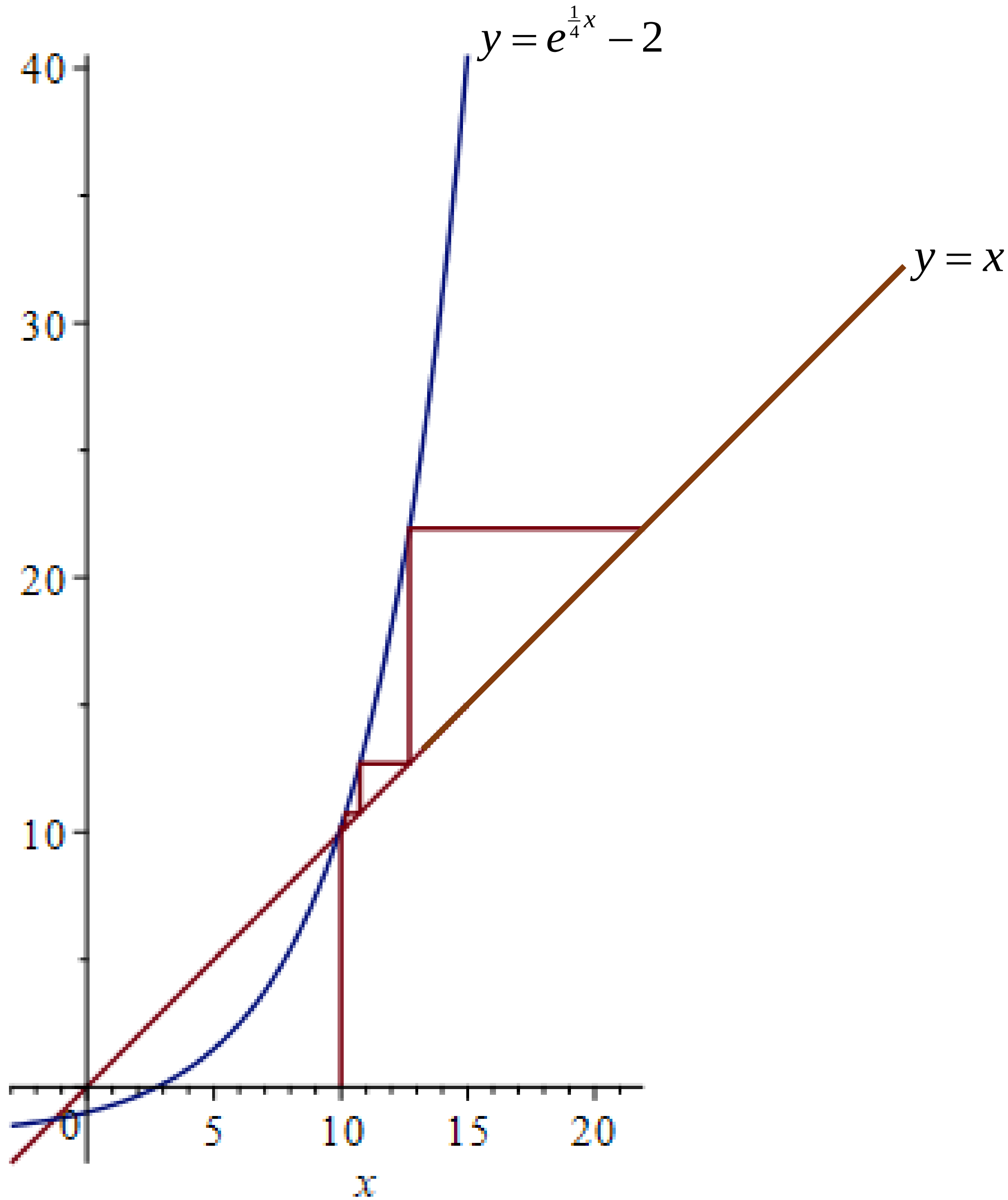
$x_1 = 8$

x_1	**8**
x_2	**5.389056099**
x_3	**1.846886134**
x_4	**-0.41319663**
x_5	**-1.098142864**
x_6	**-1.240075138**
x_7	**-1.266566821**
x_8	**-1.271408241**
x_9	**-1.272289562**
x_{10}	**-1.272449881**
x_{11}	**-1.272479041**
x_{12}	**-1.272484344**
x_{13}	**-1.272485309**
x_{14}	**-1.272485484**
x_{15}	**-1.272485516**
x_{16}	**-1.272485522**
x_{17}	**-1.272485523**
x_{18}	**-1.272485523**



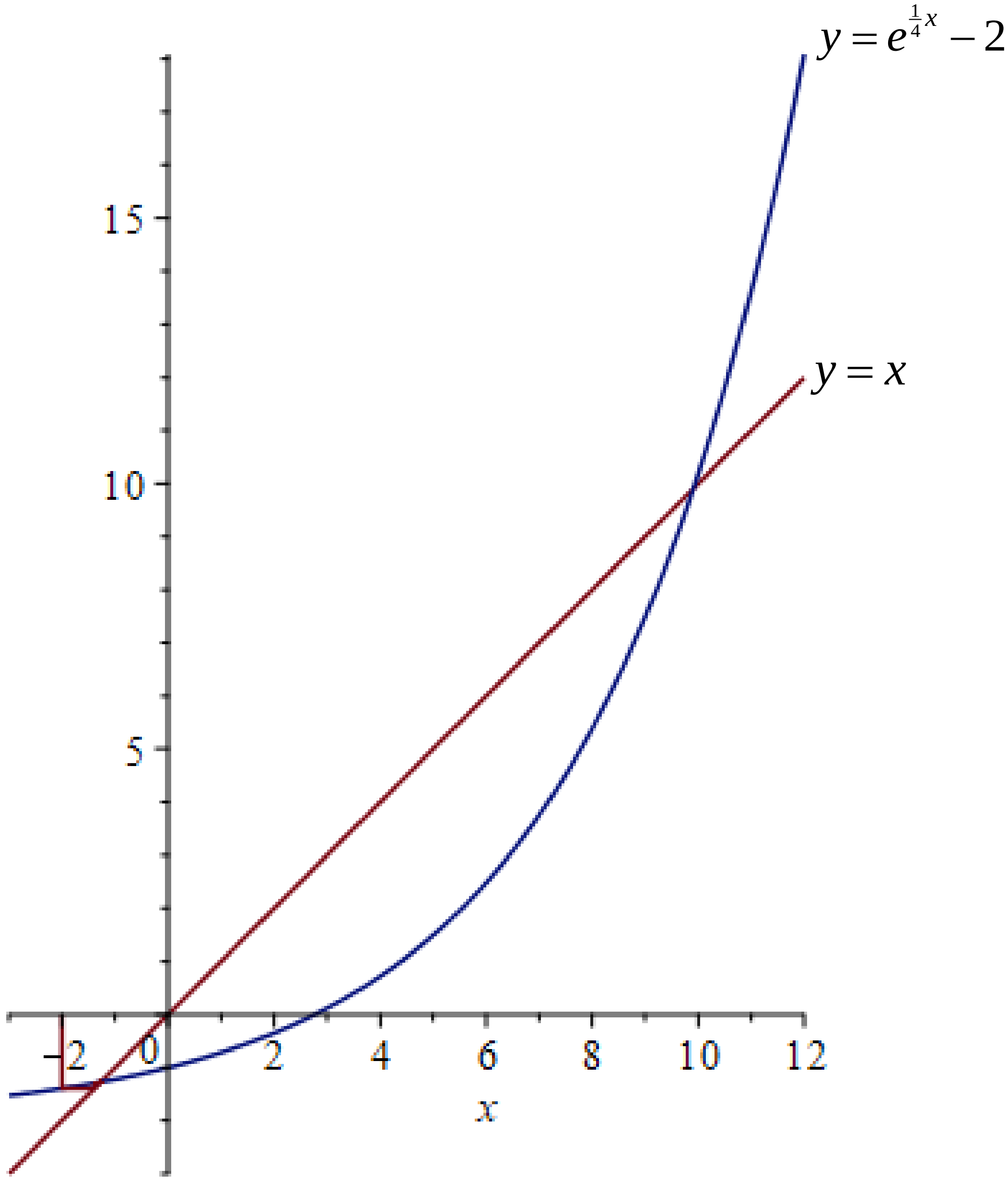
$x_1 = 10$

x_1	**10**
x_2	**10.18249396**
x_3	**10.75117584**
x_4	**12.69921308**
x_5	**21.9221132**
x_6	**237.9734517**
x_7	**6.8808 x 10^{25}**

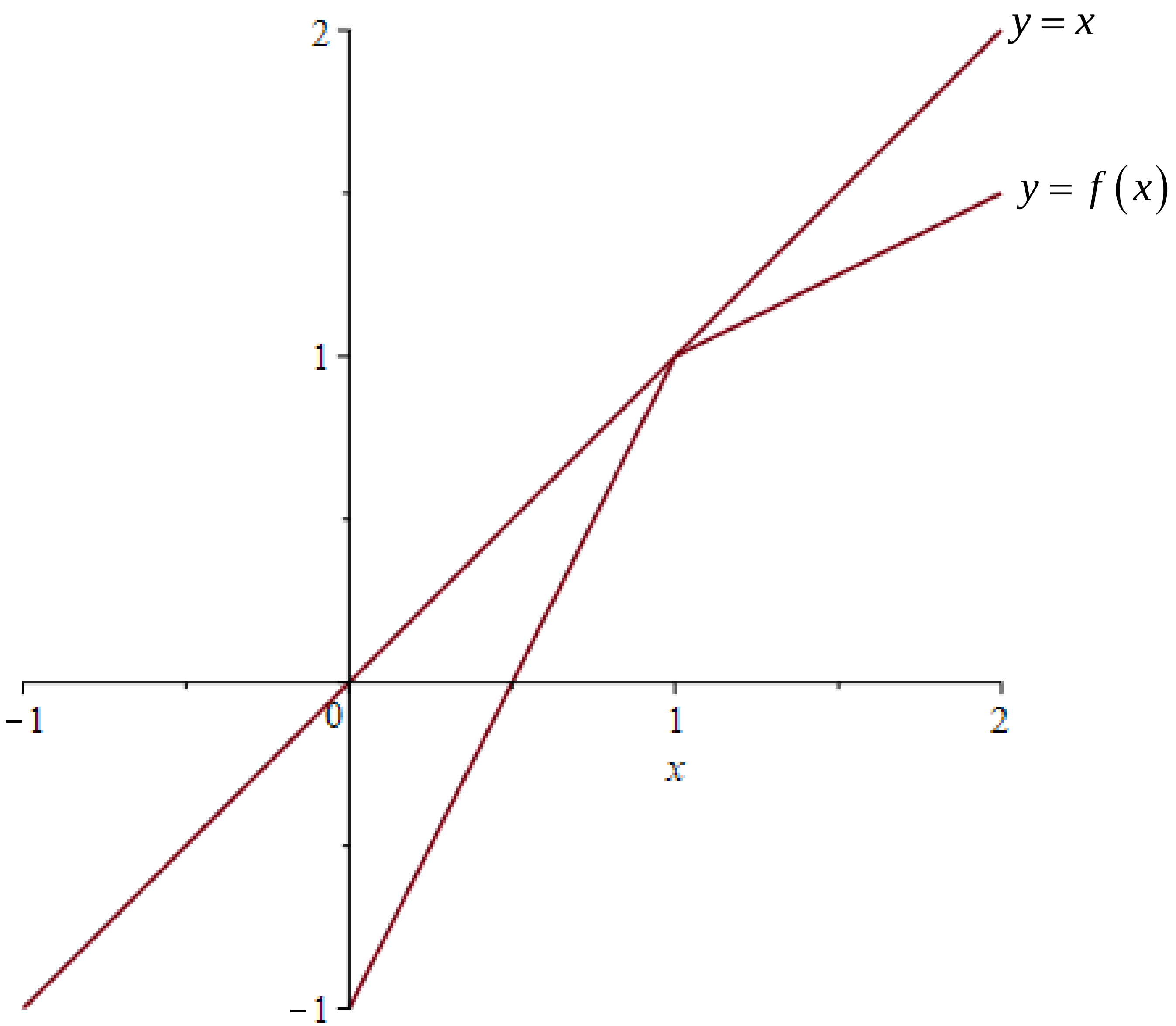


$x_1 = -2$

x_1	**-2**
x_2	**-1.39346934**
x_3	**-1.29416045**
x_4	**-1.27641707**
x_5	**-1.27320024**
x_6	**-1.2726155**
x_7	**-1.27250916**
x_8	**-1.27248982**
x_9	**-1.27248631**
x_{10}	**-1.27248567**
x_{11}	**-1.27248555**
x_{12}	**-1.27248553**
x_{13}	**-1.27248552**
x_{14}	**-1.27248552**

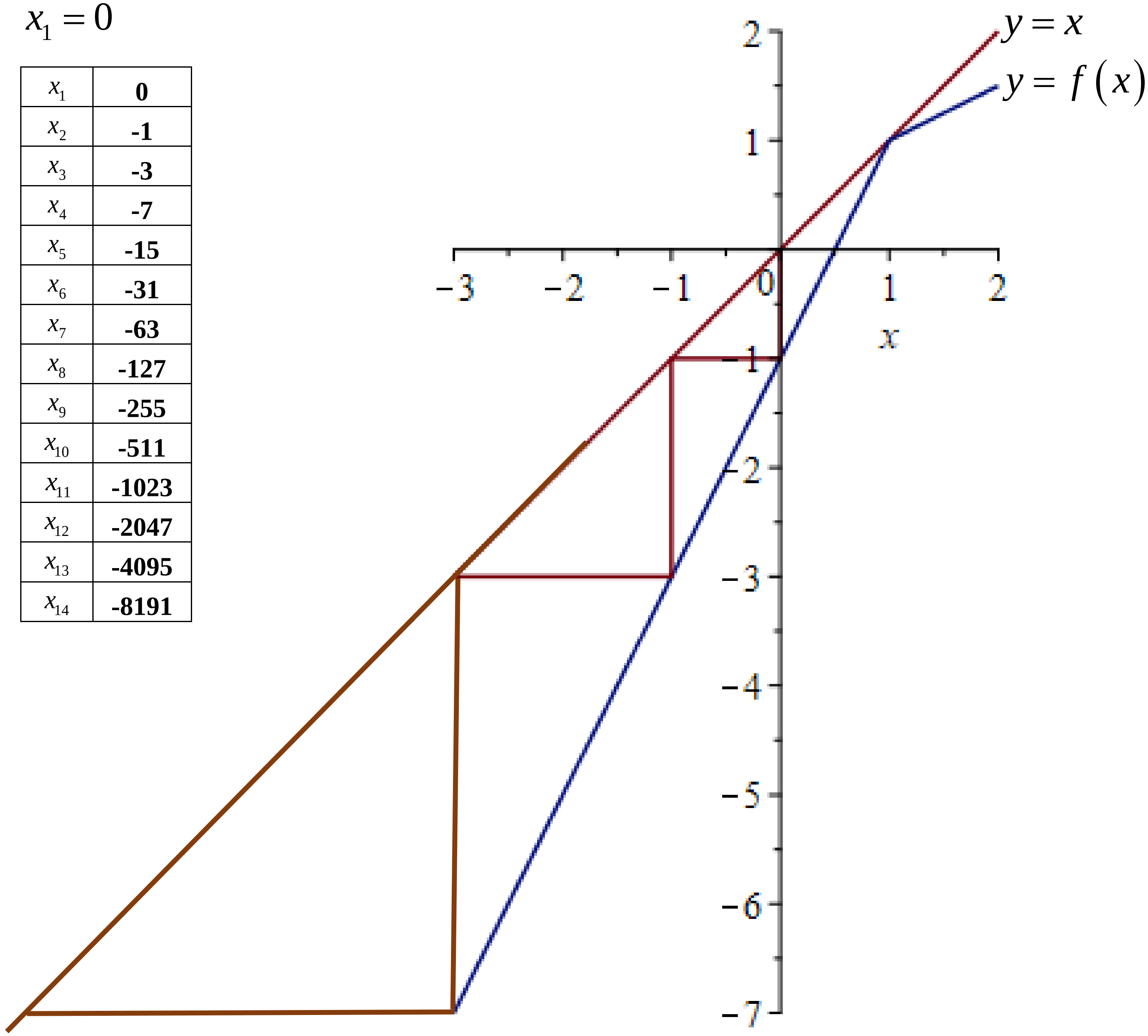


Other things can happen. $x = f(x) = \begin{cases} 2x-1; x \le 1 \\ \frac{1}{2}x + \frac{1}{2}; x \ge 1 \end{cases}$ **What's the exact solution?**



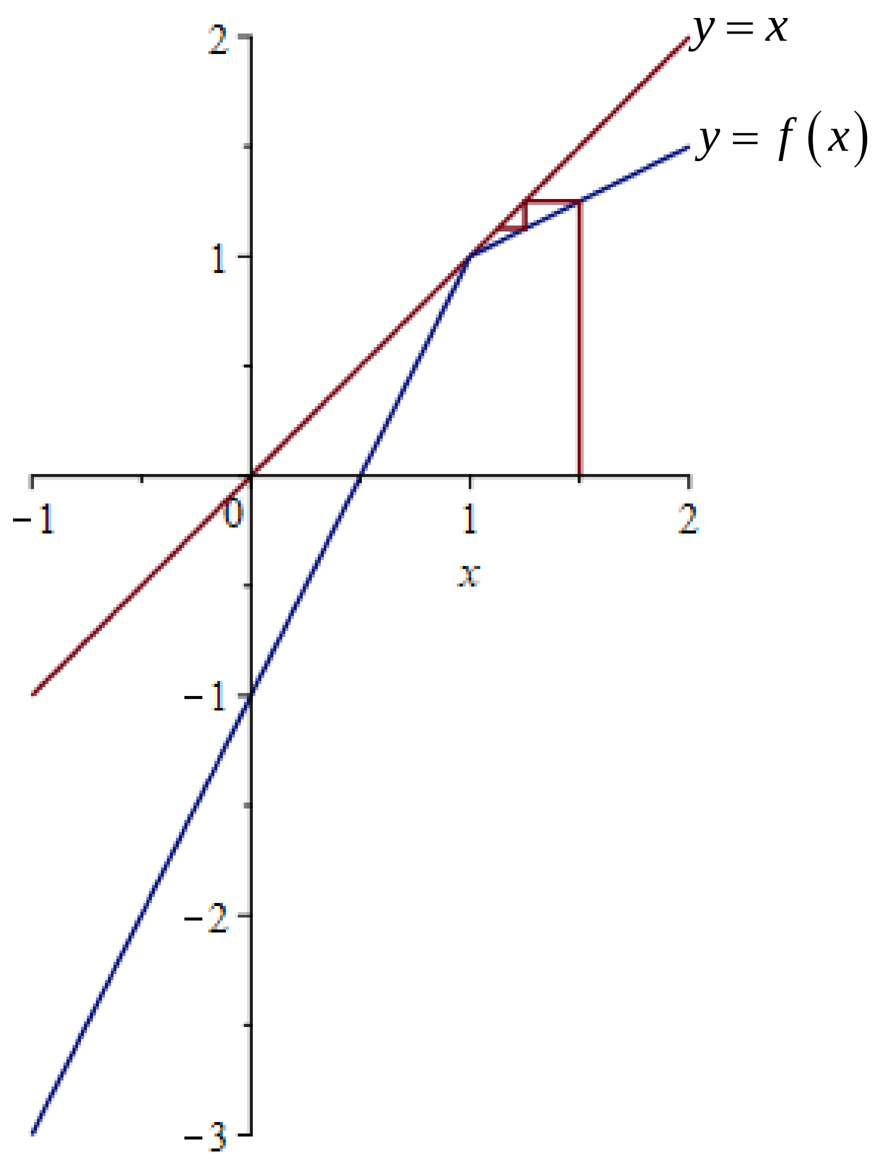
$x_1 = 0$

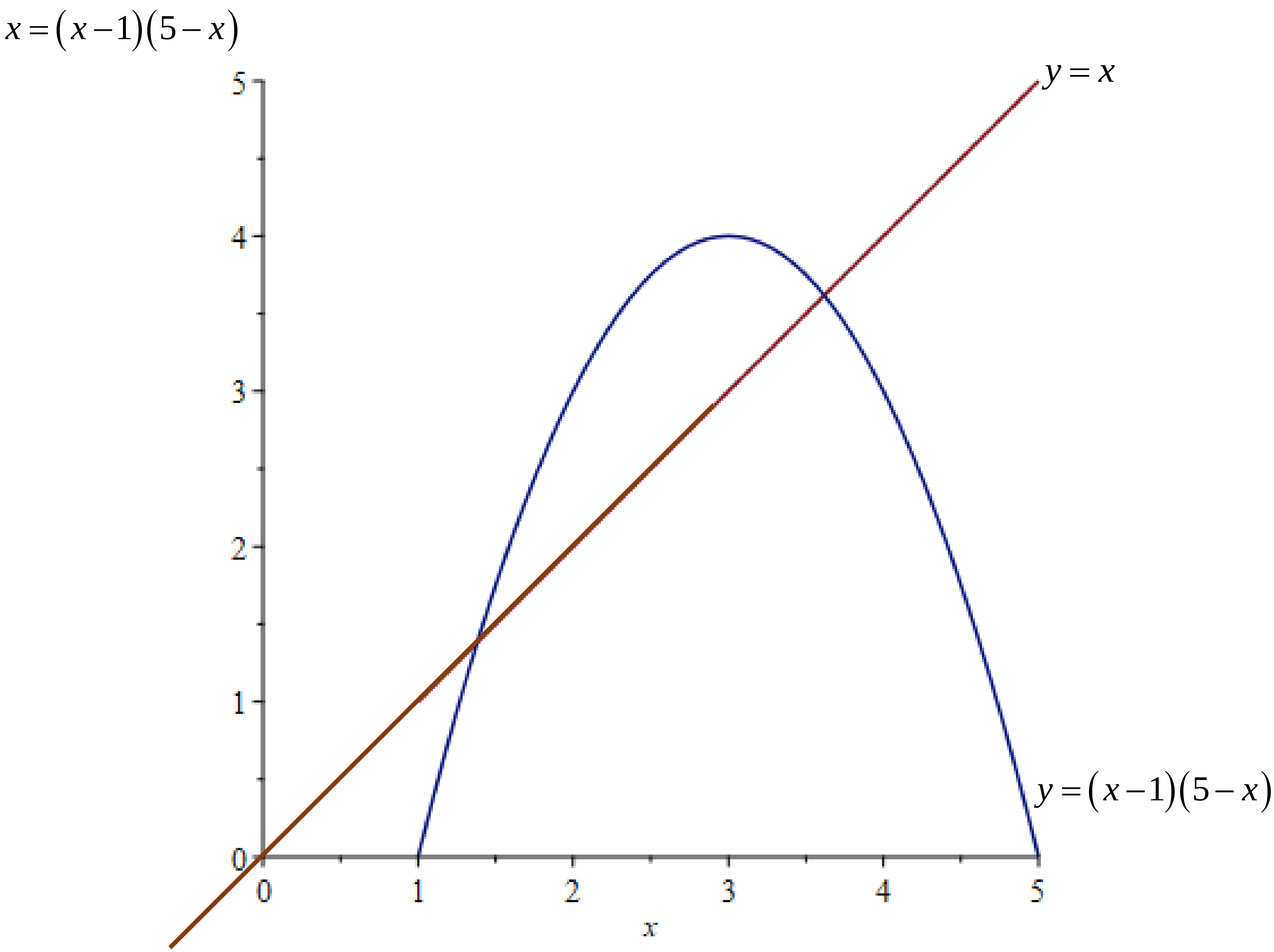
x_1	**0**
x_2	**-1**
x_3	**-3**
x_4	**-7**
x_5	**-15**
x_6	**-31**
x_7	**-63**
x_8	**-127**
x_9	**-255**
x_{10}	**-511**
x_{11}	**-1023**
x_{12}	**-2047**
x_{13}	**-4095**
x_{14}	**-8191**



$x_1 = 1.5$

x_1	**1.5**
x_2	**1.25**
x_3	**1.125**
x_4	**1.0625**
x_5	**1.03125**
x_6	**1.015625**
x_7	**1.0078125**
x_8	**1.00390625**
x_9	**1.001953125**

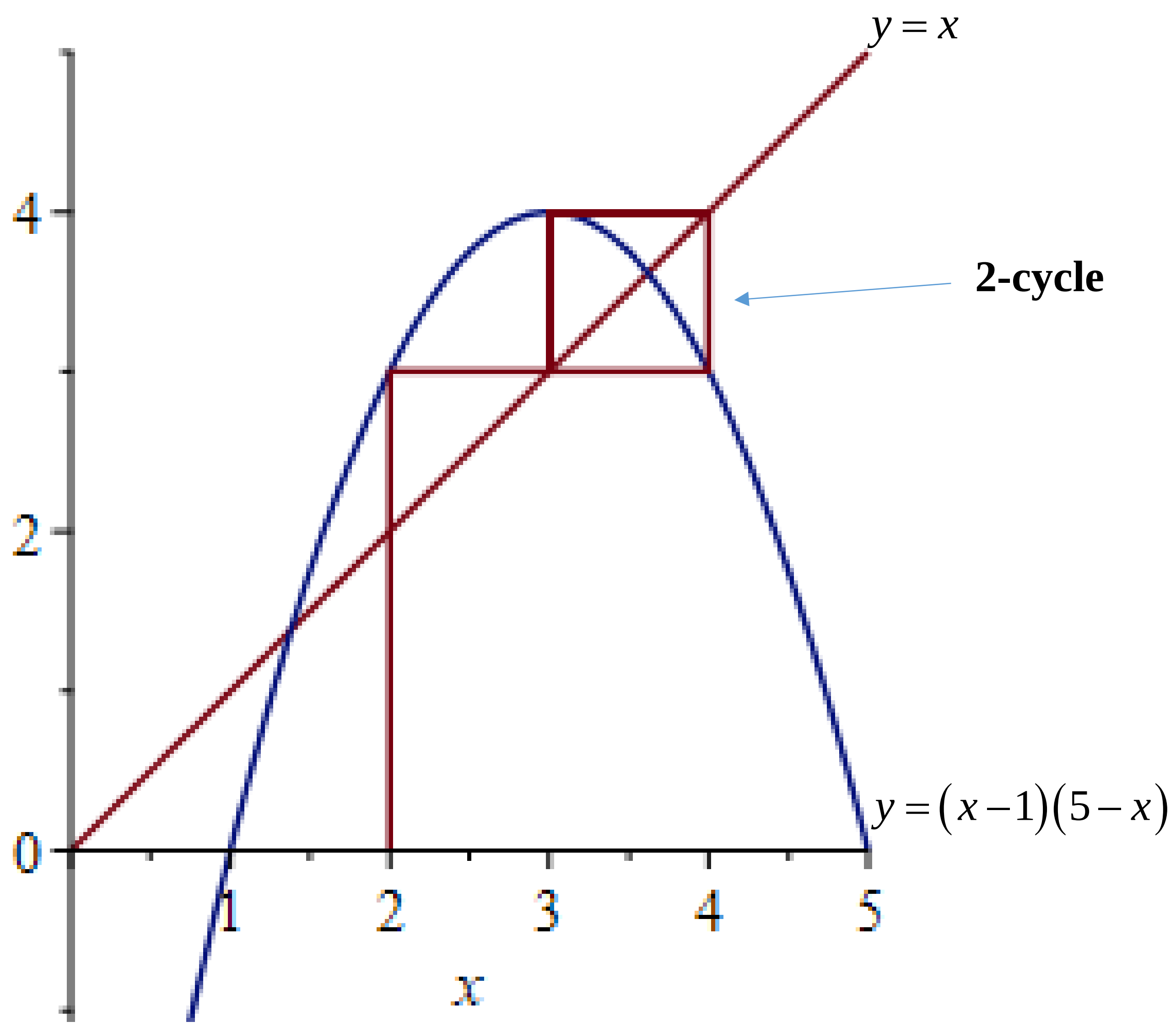




$x=(x-1)(5-x)$
$y=x$
$y=(x-1)(5-x)$
0
1
2
3
4
5
x

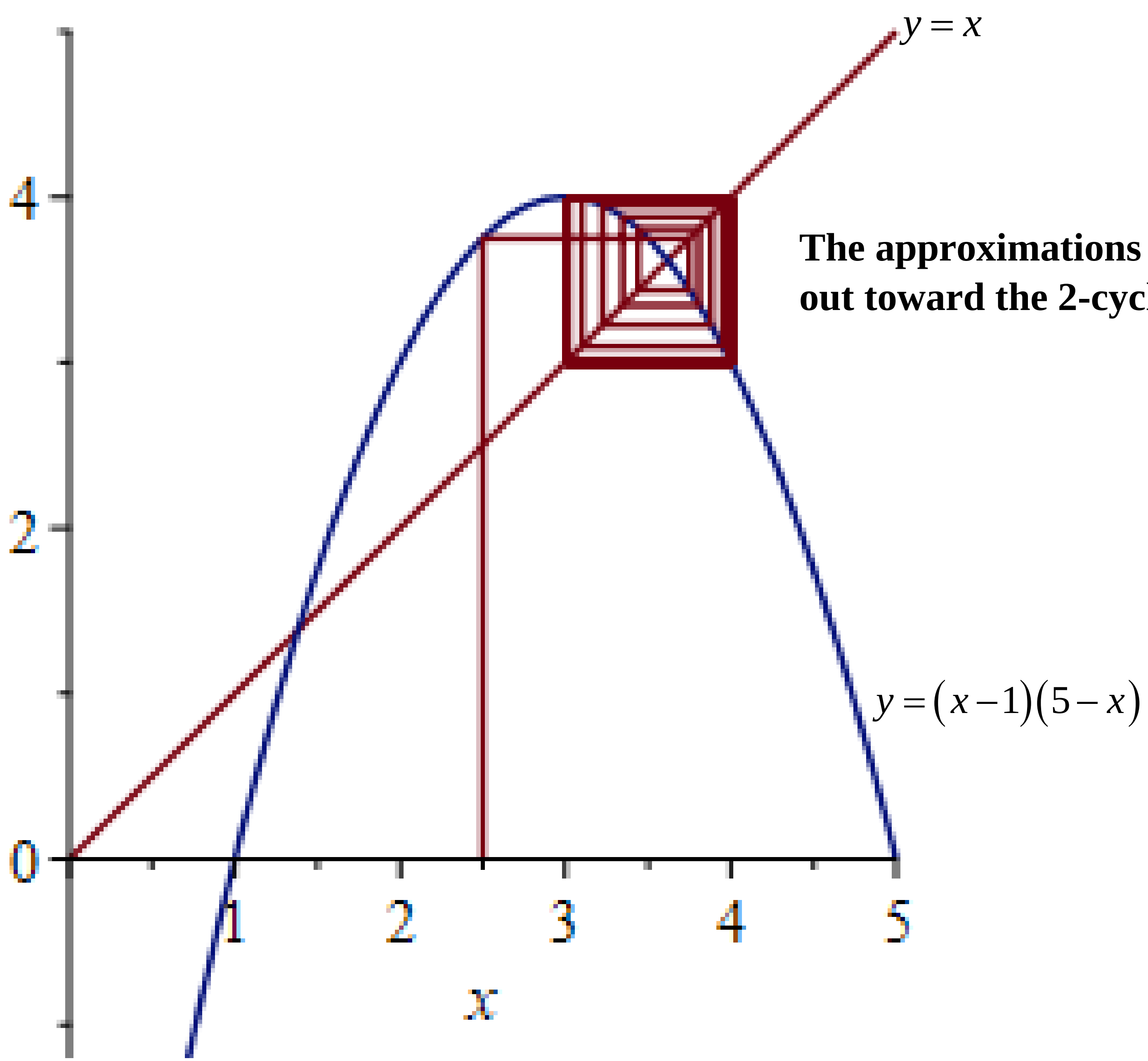
$x_1 = 2$

x_1	**2**
x_2	**3**
x_3	**4**
x_4	**3**
x_5	**4**
x_6	**3**
x_7	**4**
x_8	**3**
x_9	**4**



$x_1 = 2.5$

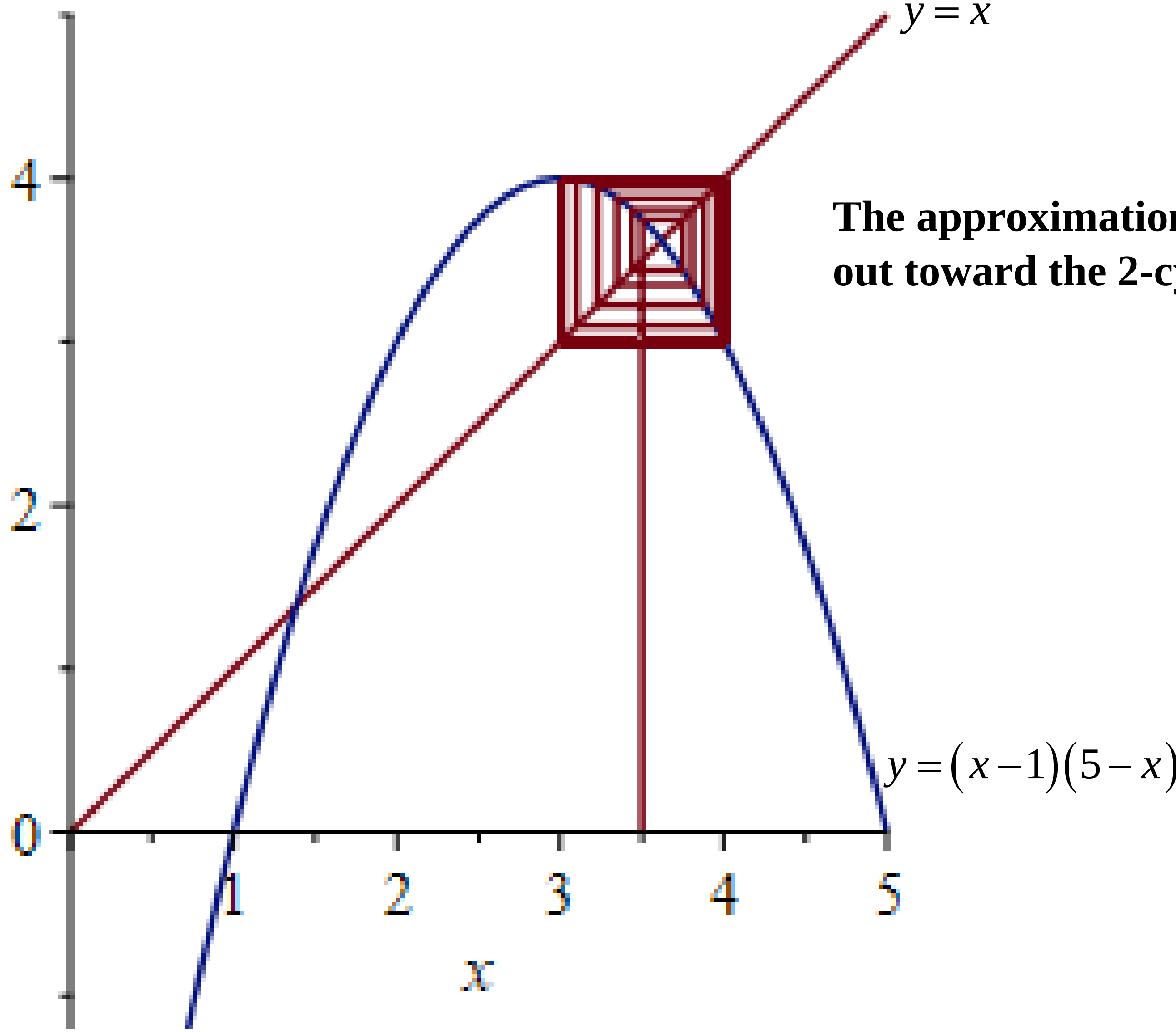
x_1	**2.5**
x_2	**3.75**
x_3	**3.4375**
x_4	**3.80859375**
x_5	**3.34617614746093**
x_6	**3.88016207492910**
x_7	**3.22531472185650**
x_8	**3.94923327611473**
x_9	**3.09895618751650**
x_{10}	**3.99020767295220**
x_{11}	**3.01948876442659**
x_{12}	**3.99962018806113**
x_{13}	**3.00075947962064**
x_{14}	**3.99999942319071**
x_{15}	**3.00000115361826**
x_{16}	**3.99999999999867**
x_{17}	**3.00000000000266**
x_{18}	**4.00000000000000**
x_{19}	**3.00000000000000**
x_{20}	**4.00000000000000**
x_{21}	**3.00000000000000**



The approximations spiral out toward the 2-cycle.

$x_1 = 3.5$

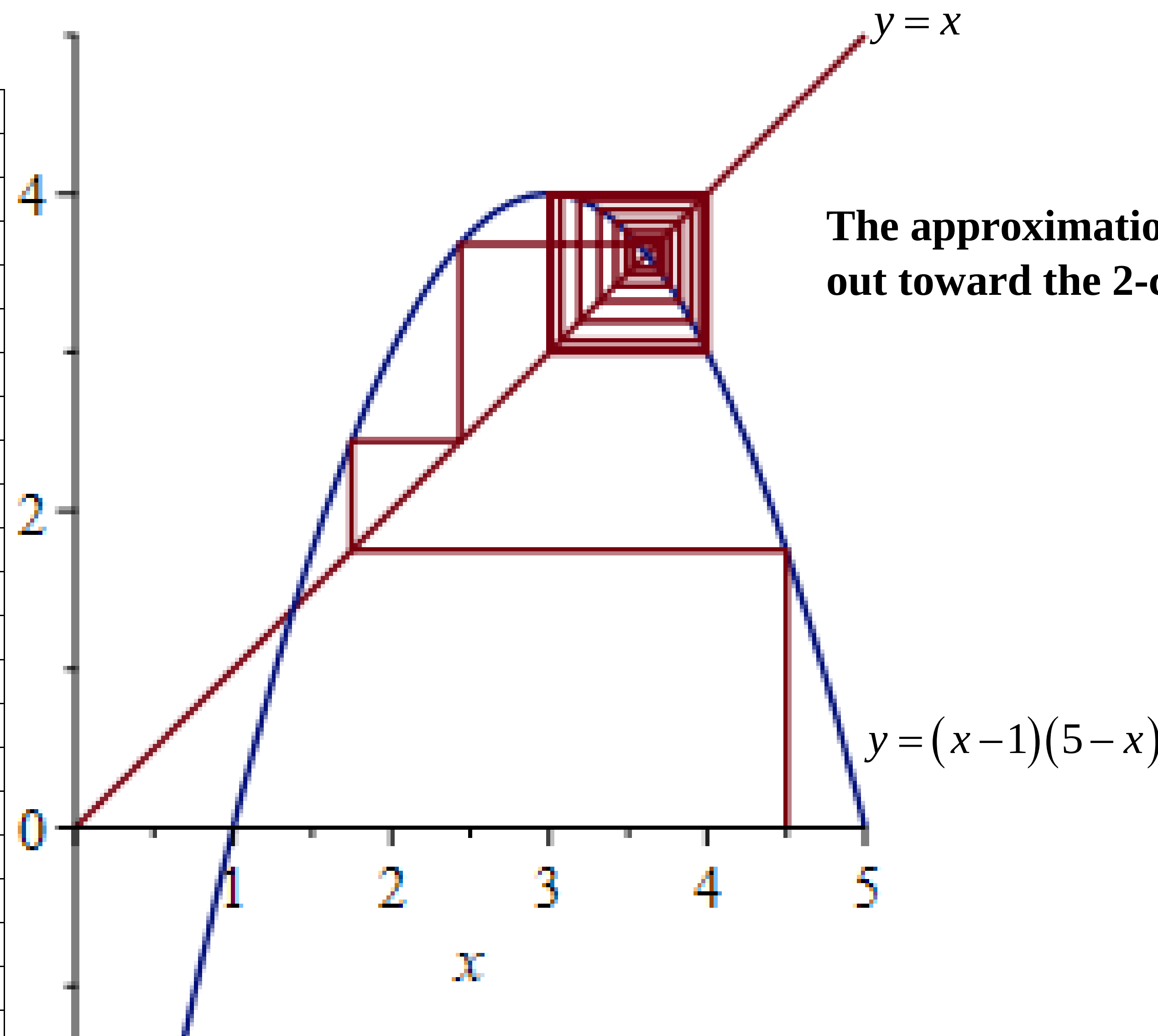
x_1	**3.5**
x_2	**3.75**
x_3	**3.4375**
x_4	**3.80859375**
x_5	**3.34617614746093**
x_6	**3.88016207492910**
x_7	**3.22531472185650**
x_8	**3.94923327611473**
x_9	**3.09895618751650**
x_{10}	**3.99020767295220**
x_{11}	**3.01948876442659**
x_{12}	**3.99962018806113**
x_{13}	**3.00075947962064**
x_{14}	**3.99999942319071**
x_{15}	**3.00000115361826**
x_{16}	**3.99999999999867**
x_{17}	**3.00000000000266**
x_{18}	**4.00000000000000**
x_{19}	**3.00000000000000**
x_{20}	**4.00000000000000**
x_{21}	**3.00000000000000**



The approximations spiral out toward the 2-cycle.

$x_1 = 4.5$

x_1	**4.5**
x_2	**1.75**
x_3	**2.4375**
x_4	**3.68359375**
x_5	**3.53269958496093**
x_6	**3.71623115218244**
x_7	**3.48701293664341**
x_8	**3.76281839954196**
x_9	**3.41810808932024**
x_{10}	**3.82518562564498**
x_{11}	**3.31906868322890**
x_{12}	**3.89819517538258**
x_{13}	**3.19324542691946**
x_{14}	**3.96265620497471**
x_{15}	**3.07329303102368**
x_{16}	**3.99462813160336**
x_{17}	**3.01071487982321**
x_{18}	**3.99988519135037**
x_{19}	**3.00022960411823**
x_{20}	**3.99999994728195**
x_{21}	**3.00000010543610**
x_{22}	**3.99999999999999**
x_{23}	**3.00000000000002**
x_{24}	**4.00000000000000**



The approximations spiral out toward the 2-cycle.

Starting guesses near the 2-cycle generate approximations that get closer to the 2-cycle, so it's called an attracting 2-cycle.