

Simple Interest:

$$I = Prt$$



***I* is the interest earned in dollars.**

***P* is the amount of money loaned or borrowed, called the principal or present value**

***r* is the annual interest rate as a decimal.**

***t* is the amount of time in years.**

Examples:

- 1. You borrow \$580 at 4.3% for 2 years. How much interest will you owe?**
- 2. You deposit \$1250 into an account paying 2.3% for 6 months. How much interest will you earn?**

If you add the amount of money loaned or borrowed to the amount of interest earned, you get the total amount or future value, A.

$$\begin{aligned} A &= P + I = P + Prt \\ &= P(1 + rt) \end{aligned}$$



Examples:

- 1. You borrow \$650 at 4.3% for 90 days. How much will you owe?**
- 2. You deposit \$930 into an account paying 2.3% for 48 weeks. How much money will be in the account?**



More examples:

- 1. What simple annual interest rate, rounded to the nearest hundredth of a percent will turn \$500 into \$600 in 2 years?**

$$A = P(1 + rt) \Rightarrow \frac{A}{P} = 1 + rt \Rightarrow r = \frac{\frac{A}{P} - 1}{t}$$

- 2. How many years, rounded to the nearest tenth of a year, will it take for \$200 to turn into \$250 at an annual rate of 5%?**

$$A = P(1 + rt) \Rightarrow \frac{A}{P} = 1 + rt \Rightarrow t = \frac{\frac{A}{P} - 1}{r}$$



Present Value with Simple Interest:

If you solve the total amount/future value equation for P , you get the present value formula for simple interest.

$$A = P(1 + rt) \Rightarrow P = \frac{A}{1 + rt}$$

Example:

You would like to have \$500 2 years from now by depositing money into an account paying 3.1%. How much should you deposit now?

Compound Interest:

In compound interest, interest earns interest. There are time periods called compounding periods, and an interest rate per compounding period called i .

P	First compounding period	$P + iP = P(1 + i)$	Second compounding period	$P(1 + i)^2$	Third compounding period	$P(1 + i)^3$	...
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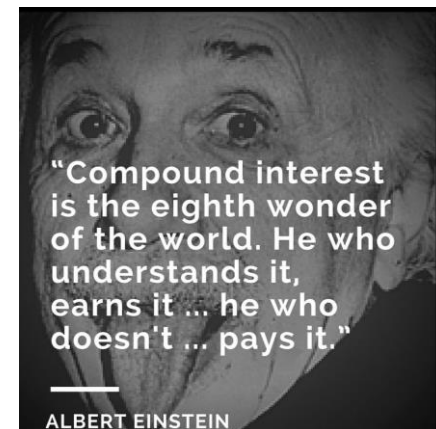
So if the process is allowed to continue for n compounding periods, then the total amount or future value will be

$$A = P(1 + i)^n.$$

In most problems, you will be given the value of r , called the annual nominal rate, and the number of compounding periods per year, m . To get the value of i , simply divide

r by m : $i = \frac{r}{m}$.

$$A = P \left(1 + \frac{r}{m} \right)^n$$



"Compound interest is the eighth wonder of the world. He who understands it, earns it ... he who doesn't ... pays it."

ALBERT EINSTEIN

Examples:

- 1. You borrow \$650 at 2.5% compounded monthly for 2 years. How much will you owe? How much of what you owe is interest?**
- 2. You deposit \$900 into an account paying 3.1% compounded semi-annually for 4 years. How much money will be in the account? How much of the money in the account is interest?**
- 3. You deposit \$500 into an account paying 2.1% compounded quarterly for 6 years. How much money will be in the account? How much of the money in the account is interest?**



More examples:

1. What nominal rate compounded annually will turn \$350 into \$425 in 4 years?

$$A = P(1+i)^n \Rightarrow \frac{A}{P} = (1+i)^n \Rightarrow \sqrt[n]{\frac{A}{P}} = 1+i \Rightarrow \boxed{i = \sqrt[n]{\frac{A}{P}} - 1}$$

2. Between which two compounding periods will \$500 turn into \$600 at 6% compounded semi-annually?

$$A = P(1+i)^n \Rightarrow \frac{A}{P} = (1+i)^n \Rightarrow \log\left(\frac{A}{P}\right) = n \log(1+i) \Rightarrow \boxed{n = \frac{\log\left(\frac{A}{P}\right)}{\log(1+i)}}$$



Present Value with Compound Interest:

If you solve the total amount/future value equation for P , you get the present value formula for compound interest.

$$A = P \left(1 + \frac{r}{m} \right)^n \Rightarrow P = \frac{A}{\left(1 + \frac{r}{m} \right)^n}$$

Example:

You would like to have \$500 2 years from now by depositing money into an account paying 3.1% compounded weekly. How much should you deposit now?

Comparing Compound Interest Investments:

To compare different compound interest investment schemes, you can find the simple interest rate that generates the same amount of money as the compound scheme in 1 year. This rate is called the Effective Rate.

$$P\left(1 + \frac{r}{m}\right)^m = P(1 + r_e)$$

$$\left(1 + \frac{r}{m}\right)^m = (1 + r_e)$$

$$r_e = \left(1 + \frac{r}{m}\right)^m - 1$$

The larger the effective rate, the more money that will be produced by the compound investment.

Example:

Find the effective rate of 3.1% compounded quarterly.