Amortization Tables:

Sometimes in the course of paying back a loan, not all the payments are the same. Since the exact amount of the payment must be rounded to the nearest penny, the small difference is accounted for in the final payment, which might be slightly different than all the other payments. Also, for tax purposes, it's nice to have a complete break-down of the portions of each payment that go toward reducing the principal(amount borrowed) and paying off the earned interest. All of these goals can be achieved with an amortization table.

Examples: 1. Create an amortization table for a loan of $1,000 at 3% to be repaid with 6 monthly payments.

The first thing we'll do is calculate the monthly payment: $1{,}000\left[\dfrac{\frac{.03}{12}}{1-\left(1+\frac{.03}{12}\right)^{-6}}\right]=$

Next, we'll create a blank amortization table that we'll fill-in as we progress through the payments.

Payment #	Payment	Principal portion	Interest portion	Remaining balance
0				
1				
2				
3				
4				
5				
6				

The row corresponding to the 0^{th} payment is all blank except for the remaining balance, which is the amount of the loan. Also, all the payments will be the same except possibly the 6^{th} payment, and the remaining balance after the 6^{th} payment will be $0.

Payment #	Payment	Principal portion	Interest portion	Remaining balance
0	-	-	-	$1,000
1	$168.13			
2	$168.13			
3	$168.13			
4	$168.13			
5	$168.13			
6				$0

The next thing we'll do is calculate the interest portion of the first payment by multiplying the remaining balance of $1,000 by the interest rate per month, .03/12. The principal portion is the payment amount minus the interest portion. The new remaining balance is the original balance of $1,000 minus the principal portion of the payment.

Payment #	Payment	Principal portion	Interest portion	Remaining balance
0	-	-	-	$1,000
1	$168.13	$165.63	$2.50	$834.37
2	$168.13			
3	$168.13			
4	$168.13			
5	$168.13			
6				$0

This process is continued through the next-to-last payment, the 5th payment.

Payment #	Payment	Principal portion	Interest portion	Remaining balance
0	-	-	-	$1,000
1	$168.13	$165.63	$2.50	$834.37
2	$168.13	$166.04	$2.09	$668.33
3	$168.13	$166.46	$1.67	$501.87
4	$168.13	$166.88	$1.25	$334.99
5	$168.13	$167.29	$.84	$167.70
6				$0

Now for the 6th payment, we know that the principal portion must equal the remaining balance of $167.70, so to get the 6th payment we'll add the interest portion to the $167.70.

Payment #	Payment	Principal portion	Interest portion	Remaining balance
0	-	-	-	$1,000
1	$168.13	$165.63	$2.50	$834.37
2	$168.13	$166.04	$2.09	$668.33
3	$168.13	$166.46	$1.67	$501.87
4	$168.13	$166.88	$1.25	$334.99
5	$168.13	$167.29	$.84	$167.70
6	$168.12	$167.70	$.42	$0

So you can see that the final payment is slightly different than the others. Also, if we calculated the total interest paid by multiplying the payment of $168.13 by 6 and subtracting the loan amount of $1,000, we'd be off by a penny from the actual interest amount of $8.77.

2. Create an amortization table for a loan of $5,000 at 4% to be repaid with 5 monthly payments.

The first thing we'll do is calculate the monthly payment: $5,000\left[\dfrac{\frac{.04}{12}}{1-\left(1+\frac{.04}{12}\right)^{-5}}\right]=$

Payment #	Payment	Principal portion	Interest portion	Remaining balance
0	-	-	-	$5,000
1				
2				
3				
4				
5				$0

In general, the principal portions of the payments will increase, while the interest portions will decrease. Sometimes people want to know specific information about a particular payment or remaining balance without having to create the whole table or a large portion of it. A computer can do this very quickly.

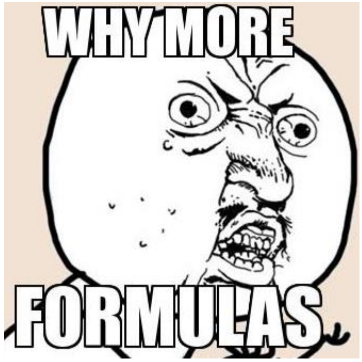
As an example, suppose that you wanted to know the remaining balance after the 2nd payment in the \$1,000 loan from the first example. It would be the same as a loan amount with the same payment of $1{,}000\left[\dfrac{\frac{.03}{12}}{1-\left(1+\frac{.03}{12}\right)^{-6}}\right]$ **but with 4 remaining payments. So the remaining balance can be solved from**

$$1{,}000\left[\dfrac{\frac{.03}{12}}{1-\left(1+\frac{.03}{12}\right)^{-6}}\right]=\text{Remaining balance}\left[\dfrac{\frac{.03}{12}}{1-\left(1+\frac{.03}{12}\right)^{-4}}\right]$$

, which yields

$$\text{Remaining balance}=1{,}000\left[\dfrac{1-\left(1+\frac{.03}{12}\right)^{-4}}{1-\left(1+\frac{.03}{12}\right)^{-6}}\right]=\$668.33$$

, which is exactly what we got in the first example. Sometimes the result will be exact, like this example, while other times it will be very close. There are general formulas for approximating all of the entries in an amortization table, and they can all be found in the Financial Formulas link on the course page.



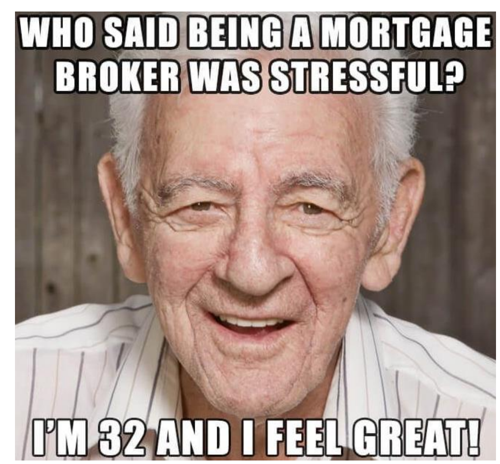
As another example, let's look at the formula for the principal portion of the k^{th} payment: $PPmt = i \cdot PV \cdot \left[\dfrac{(1+i)^{k-1-n}}{1-(1+i)^{-n}}\right]$**. Let's use it to approximate the principal portion of the 3rd payment in the second example of a loan of $5,000.**

$$PPmt = \tfrac{.04}{12} \cdot 5,000 \cdot \left[\frac{\left(1+\tfrac{.04}{12}\right)^{3-1-5}}{1-\left(1+\tfrac{.04}{12}\right)^{-5}}\right]$$

$$= \tfrac{.04}{12} \cdot 5,000 \cdot \left[\frac{\left(1+\tfrac{.04}{12}\right)^{-3}}{1-\left(1+\tfrac{.04}{12}\right)^{-5}}\right]$$

$$= \$999.99$$

This is the same amount that we got in the second example.



Excel or a graphing calculator can be used to find exact values for the remaining balance, interest portion, and principal portion. If you type in the amount of the loan in cell A1, and in cell A2 you type in "=A1 – *Pmt* + ROUND(*iA1,2)", with the values substituted for *Pmt* and *i*, you can click, hold, and drag to get all the remaining balances up to the next-to-the-last one. Of course, the last remaining balance is $0. Here it is for the second example:**

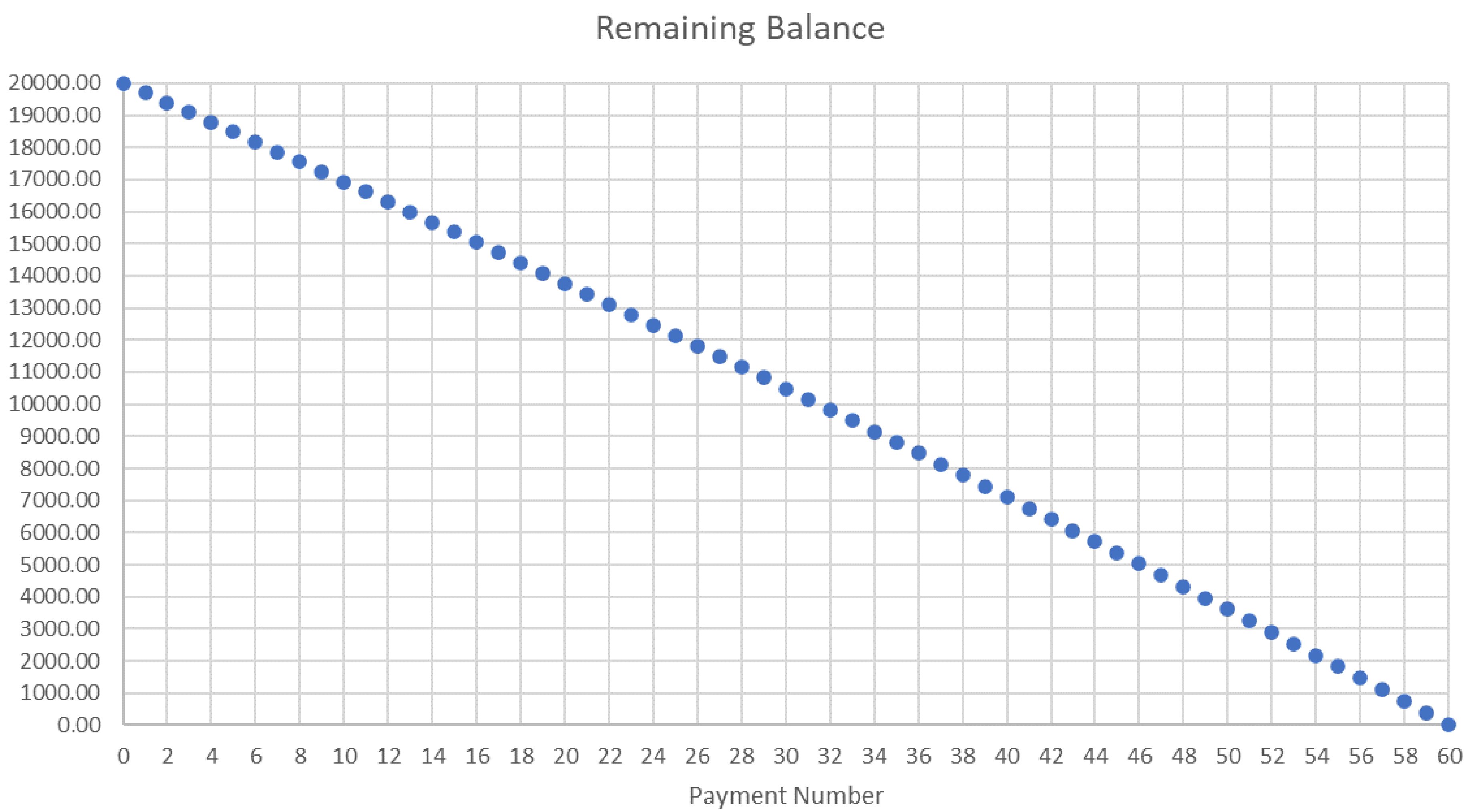
	A
1	$5,000.00
2	$4,006.65
3	$3,009.99
4	$2,010.00
5	$1,006.68

Once the remaining balances are known, it's easy to get the interest portions and principal portions of the payments.

Using this idea on the car loan of $20,000 at 3.9% compounded monthly for 5 years from the previous lecture, we get

Payment Number	Remaining Balance	Payment Number	Remaining Balance	Payment Number	Remaining Balance
0	$20,000.00	25	$12,136.89	50	$3,609.37
1	$19,697.57	26	$11,808.90	51	$3,253.67
2	$19,394.16	27	$11,479.85	52	$2,896.81
3	$19,089.76	28	$11,149.73	53	$2,538.79
4	$18,784.37	29	$10,818.54	54	$2,179.61
5	$18,477.99	30	$10,486.27	55	$1,819.26
6	$18,170.61	31	$10,152.92	56	$1,457.74
7	$17,862.23	32	$9,818.49	57	$1,095.05
8	$17,552.85	33	$9,482.97	58	$731.18
9	$17,242.47	34	$9,146.36	59	$366.13
10	$16,931.08	35	$8,808.66	60	$0.00
11	$16,618.68	36	$8,469.86		
12	$16,305.26	37	$8,129.96		
13	$15,990.82	38	$7,788.95		
14	$15,675.36	39	$7,446.83		
15	$15,358.87	40	$7,103.60		
16	$15,041.36	41	$6,759.26		
17	$14,722.81	42	$6,413.80		
18	$14,403.23	43	$6,067.21		
19	$14082.61	44	$5,719.50		
20	$13,760.95	45	$5,370.66		
21	$13,438.24	46	$5,020.68		
22	$13,114.48	47	$4,669.57		
23	$12,789.67	48	$4,317.32		
24	$12,463.81	49	$3,963.92		

Here's the graph of the Remaining Balance over the 60 payments.



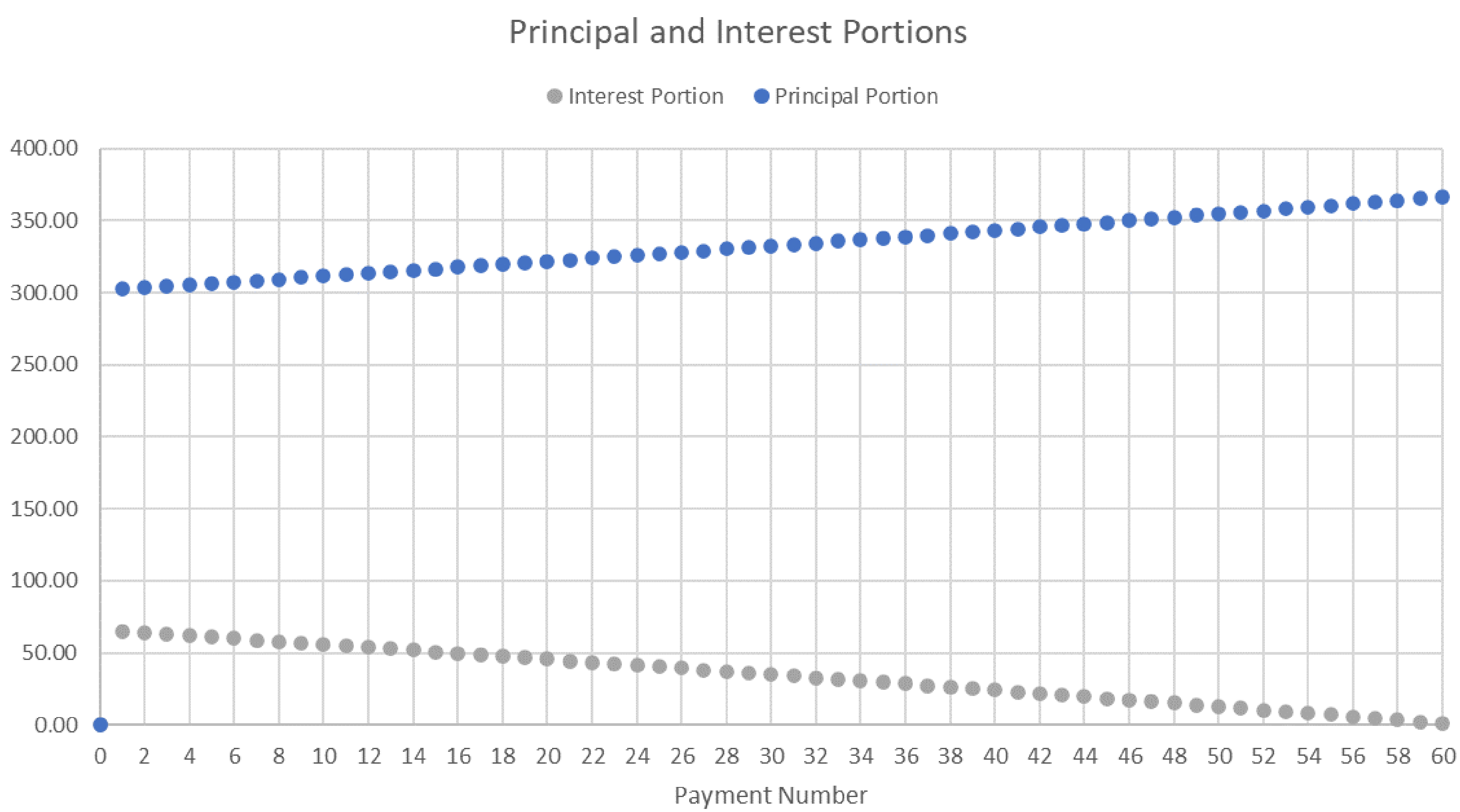
Here are the interest and principal portions of the payments.

Payment Number	Interest Portion	Principal Portion
0	$0.00	$0.00
1	$65.00	$302.43
2	$64.02	$303.41
3	$63.03	$304.40
4	$62.04	$305.39
5	$61.05	$306.38
6	$60.05	$307.38
7	$59.05	$308.38
8	$58.05	$309.38
9	$57.05	$310.38
10	$56.04	$311.39
11	$55.03	$312.40
12	$54.01	$313.42
13	$52.99	$314.44
14	$51.97	$315.46
15	$50.94	$316.49
16	$49.92	$317.51
17	$48.88	$318.55
18	$47.85	$319.58
19	$46.81	$320.62
20	$45.77	$321.66
21	$44.72	$322.71
22	$43.67	$323.76
23	$42.62	$324.81
24	$41.57	$325.86
25	$40.51	$326.92
26	$39.44	$327.99

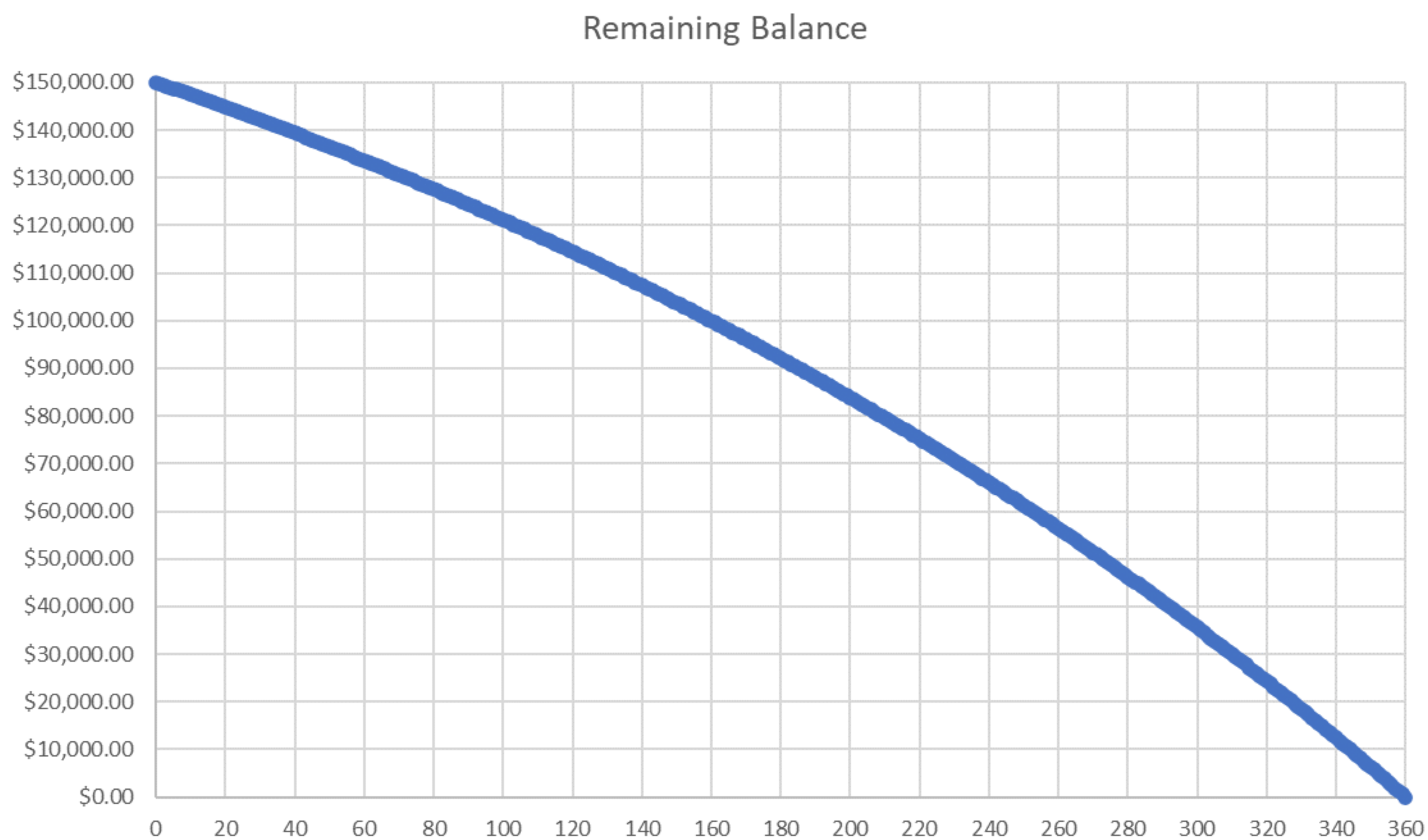
27	**$38.38**	**$329.05**
28	**$37.31**	**$330.12**
29	**$36.24**	**$331.19**
30	**$35.16**	**$332.27**
31	**$34.08**	**$333.35**
32	**$33.00**	**$334.43**
33	**$31.91**	**$335.52**
34	**$30.82**	**$336.61**
35	**$29.73**	**$337.70**
36	**$28.63**	**$338.80**
37	**$27.53**	**$339.90**
38	**$26.42**	**$341.01**
39	**$25.31**	**$342.12**
40	**$24.20**	**$343.23**
41	**$23.09**	**$344.34**
42	**$21.97**	**$345.46**
43	**$20.84**	**$346.59**
44	**$19.72**	**$347.71**
45	**$18.59**	**$348.84**
46	**$17.45**	**$349.98**
47	**$16.32**	**$351.11**
48	**$15.18**	**$352.25**
49	**$14.03**	**$353.40**
50	**$12.88**	**$354.55**
51	**$11.73**	**$355.70**
52	**$10.57**	**$356.86**
53	**$9.41**	**$358.02**
54	**$8.25**	**$359.18**
55	**$7.08**	**$360.35**
56	**$5.91**	**$361.52**

57	$4.74	$362.69
58	$3.56	$363.87
59	$2.38	$365.05
60	$1.19	$366.24

Here's the graph of the interest and principal portions over the 60 payments.



For the home loan of $150,000 at 3.1% compounded monthly for 30 years from the previous lecture, we get the following graph of the remaining balance.



Notice that the remaining balance doesn't drop to half the original loan amount until the 220th payment out of 360 payments.

Here's the graph of the interest and principal portions over the 360 payments.