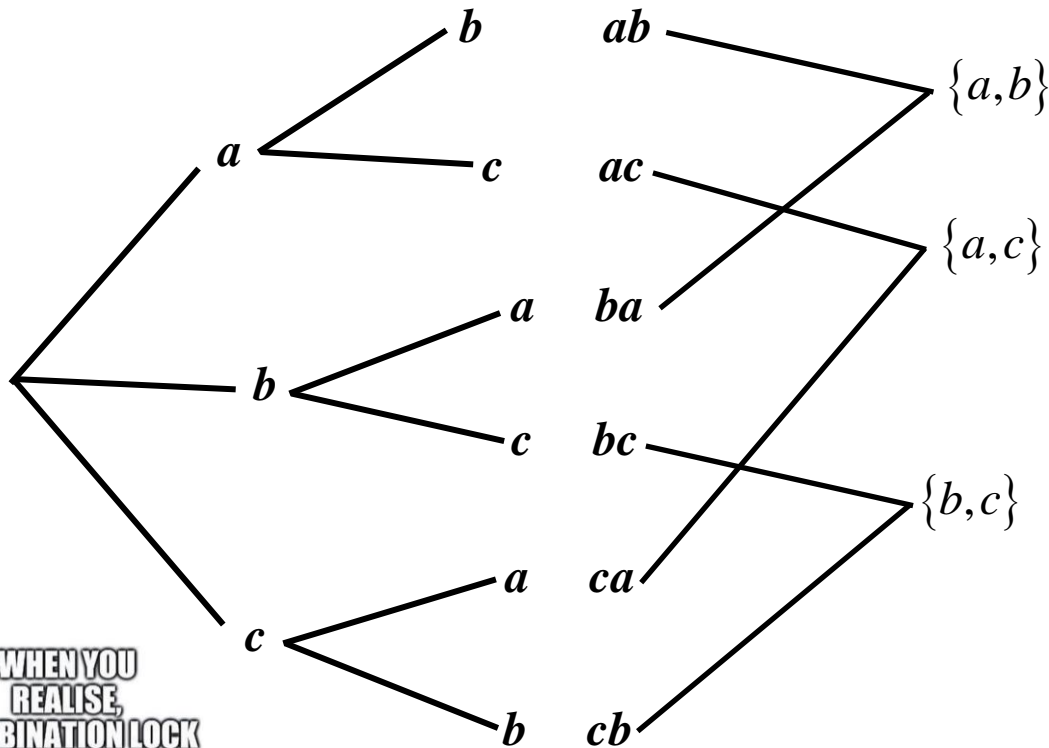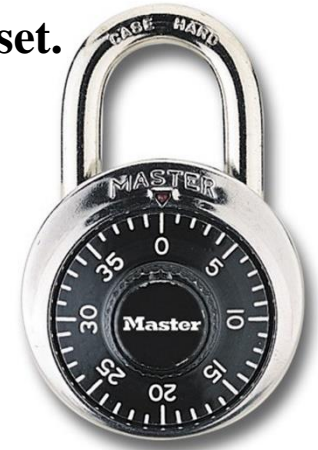## Combinations:

A combination is a selection of objects without regard to order, i.e. a subset.

Example:

Find all the combinations of the objects $\{a,b,c\}$ of size 2.

There are 6 permutations of size 2 from the 3 objects, but only 3 combinations of size 2 from the 3 objects.
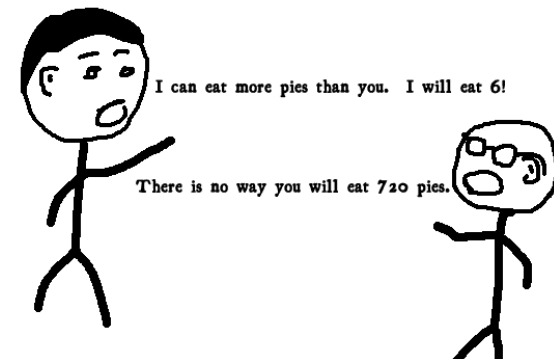
PIE-EATING CONTEST

I can eat more pies than you. I will eat 6!

There is no way you will eat 720 pies.

In general, the number of combinations of size r from n objects is abbreviated as $_nC_r$. So far, we know that $_3C_2 = 3$, and $_3C_2 = 3 = \dfrac{6}{2} = \dfrac{_3P_2}{2!}$. This is true in general, and leads to a nice formula for $_nC_r$. $_nC_r = \dfrac{n!}{r! \cdot (n-r)!}$

Examples:

1. A three-person committee is to be selected from a group of 10 people. How many different committees are possible?

2. In a certain lottery, you must select 6 numbers from the numbers 1-50. How many different lottery selections are possible?

3. A group consists of 7 men and 8 women. A committee of 4 people will be selected.

a) How many different 4-person committees are possible?

b) How many different 4-person committees consisting of 4 women are possible?

c) How many different 4-person committees consisting of 3 women and 1 man are possible?

Which 3 women?	Which 1 man?

d) How many different 4-person committees consisting of 2 women and 2 men are possible?

Which 2 women?	Which 2 men?

e) How many different 4-person committees have at least 1 man?

Probability:

Experiment:

Any process that produces random results.

Example:

Flip a coin twice and record the results.

Heads/Heads, Heads/Tails, Tails/Heads, Tails/Tails

Sample Space:

The set of all possible outcomes of an experiment. It's abbreviated with the letter *S*, and it's like the universal set for an experiment.

Example:

From the previous experiment

$$S = \{HH, HT, TH, TT\}$$

Event:

Any subset of the sample space

Example:

From the previous experiment

$$S = \{HH, HT, TH, TT\}$$

E is the event that heads occurs. $E = \{$ $\}$

F is the event that tails occurs. $F = \{$ $\}$

G is the event of getting the same result on both flips. $G = \{$ $\}$

J is the event of getting different results on the two flips. $J = \{$ $\}$

A probability is a number between 0 and 1(inclusive) that indicates the likelihood that an event will occur.

A probability of 1 means the event must occur.

A probability of 0 means the event won't occur.

A probability of ½ means the event is just as likely to occur as not to occur.

The closer the probability value is to 1, the more likely the event will occur, and the closer the probability value is to 0, the less likely the event will occur.

Theoretical Probability and the Equally Likely assumption:

In many experiments, the outcomes in the sample space all have the same probability of occurring. Certain conditions in the experiments will allow you to make this equally likely assumption.

When you can assume equally likely outcomes, the probability of an event is determined using counting.

$$P(E) = \frac{n(E)}{n(S)}$$

Examples:

1. A _fair_ die is rolled. $S = \{1,2,3,4,5,6\}$

a) $P(\text{rolling a 1})$

b) $P(\text{rolling a 2 or a 3})$

c) $P(\text{rolling an odd number})$

d) $P(\text{rolling a 7})$

2. A card is _randomly selected_ from a standard 52-card deck.

a) P(selecting an ace)   **b)** P(selecting a red card)   **c)** P(selecting a club)

d) P(selecting a face card)   **e)** P(selecting an ace or a diamond)

J Q K ♦   A♥   A♣   A♠   A♦
♠   2♦
♥   3♦
♣   4♦
                                                          5♦
                                                          6♦
                                                          7♦
                                                          8♦
                                                          9♦
                                                         10♦
                                                          J♦
                                                          Q♦
                                                          K♦

PROBABILITY
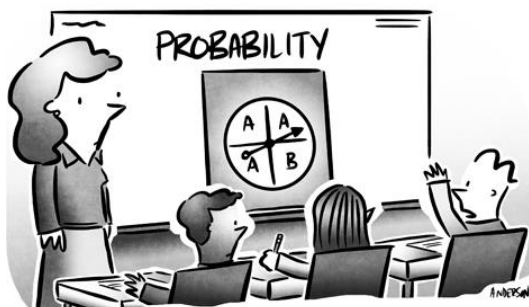
"I know mathematically that A is more likely,
but I gotta say, I feel like B wants it more."

3. A _fair coin_ is flipped twice. $S = \{HH, HT, TH, TT\}$

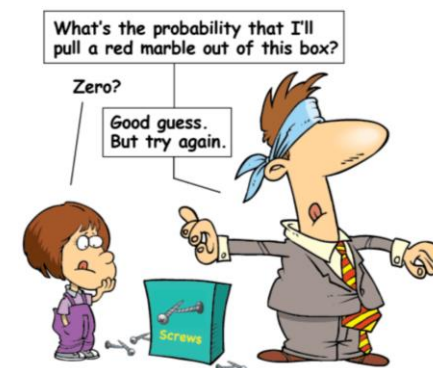**a)** $P(\text{heads occurs})$

b) $P(\text{tails occurs})$

c) $P(\text{same result on both flips})$

d) $P(\text{different result on the two flips})$

e) $P(\text{two heads occur})$

f) $P(\text{three heads occur})$

What's the probability that I'll pull a red marble out of this box?

Zero?

Good guess. But try again.

Screws

4. A survey of North Harris students had the following results.

	Pepperoni	Sausage	Mushroom	Total
Freshman	25	15	5	45
Sophomore	30	20	5	55
Total	55	35	10	100

A student from the survey is _selected at random_.

a) $P(\text{freshman})$

b) $P(\text{sausage})$

c) $P(\text{freshman and pepperoni})$

d) $P(\text{sausage or mushroom})$

e) $P(\text{freshman or pepperoni})$

**Empirical Probability:**

The experiment is performed a bunch of times, _n_, and the results are recorded.

$$P(E) = \frac{\# \text{ of times } E \text{ occurs}}{n}$$

Example:

A coin is flipped 1,000 times with the following results: 450 heads and 550 tails. Find the empirical probability of flipping a tail.