

Methods of Factoring Trinomials

$$6x^2 + 19x + 15$$

1. Trial and Error Method:

This is the usual first method taught in most textbooks.

The factor pairs of the leading coefficient and the constant term are determined.

Factors of 6	Factors of 15
1,6	1,15
2,3	3,5

Then the possible binomial products are intelligently constructed and multiplied out.

$$(x+1)(6x+15)$$

$$(x+15)(6x+1) = 6x^2 + 91x + 15$$

$$(x+3)(6x+5) = 6x^2 + 23x + 15$$

$$(x+5)(6x+3)$$

$$(2x+1)(3x+15)$$

$$(2x+15)(3x+1) = 6x^2 + 47x + 15$$

$$(2x+3)(3x+5) = 6x^2 + 19x + 15 \checkmark$$

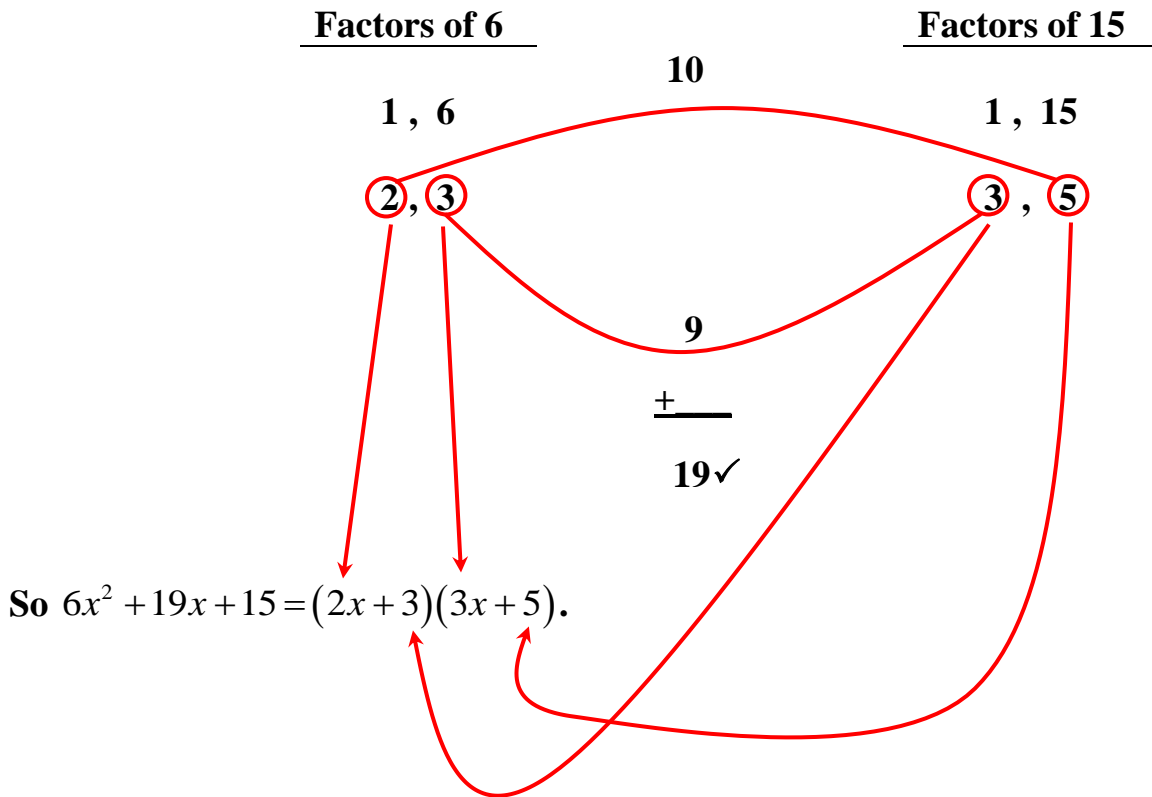
$$(2x+5)(3x+3)$$

So $6x^2 + 19x + 15 = (2x+3)(3x+5)$.

2. Listing and Pairing Method:

This is a streamlined version of Trial and Error that I came up with.

The factor pairs of the leading coefficient and constant term are listed. Then they are paired up, multiplied, and added to produce the middle coefficient.



3. Grouping Method:

This method uses Factoring by Grouping which is usually taught as a method for factoring 4-term polynomials. The trinomial is converted into an equivalent 4-term polynomial, and then Factoring by Grouping is used.

$$\begin{array}{c} 6x^2 + 19x + 15 \\ \text{ } \\ \underline{90} \\ \text{ } \\ \mathbf{9, 10} \\ \text{ } \\ \begin{array}{c} \swarrow \quad \searrow \\ 6x^2 + 9x + 10x + 15 \\ (6x^2 + 9x) + (10x + 15) \\ 3x(2x + 3) + 5(2x + 3) \\ (2x + 3)(3x + 5) \end{array} \end{array}$$

So $6x^2 + 19x + 15 = (2x + 3)(3x + 5)$.

4. First Magic Method:

This method begins like the Grouping Method, but it's somewhat more efficient.

$$6x^2 + 19x + 15$$

90

9, 10

$$\frac{(6x + 9)(6x + 10)}{6}$$

All common factors are removed from each binomial.

$$\frac{2 \cdot 3(2x + 3)(3x + 5)}{6}$$

The fraction is reduced.

$$(2x + 3)(3x + 5)$$

So $6x^2 + 19x + 15 = (2x + 3)(3x + 5).$

5. Second Magic Method:

This method begins like the Grouping Method, but it's somewhat more efficient.

$$\begin{array}{c} 6x^2 + 19x + 15 \\ \underline{90} \\ 9, 10 \\ \hline (9x + 15)(10x + 15) \\ 15 \end{array}$$

All common factors are removed from each binomial.

$$\frac{5 \cdot 3(3x + 5)(2x + 3)}{15}$$

The fraction is reduced.

$$(3x + 5)(2x + 3)$$

So $6x^2 + 19x + 15 = (2x + 3)(3x + 5)$.

6. First Black Magic Method:(Caution: You must remove all common factors first!)

This method is a more efficient form of the Magic Method.

$$6x^2 + 19x + 15$$

90

9, 10

$$(6x + 9)(6x + 10)$$

All common factors are removed from each binomial, and then discarded.

$$2 \cdot 3 (2x + 3)(3x + 5)$$

discard

This leads to

$$(2x + 3)(3x + 5)$$

So $6x^2 + 19x + 15 = (2x + 3)(3x + 5).$

7. Second Black Magic Method:(Caution: You must remove all common factors first!)

This method is a more efficient form of the Magic Method.

$6x^2 + 19x + 15$

90

9, 10

$(9x + 15)(10x + 15)$

The diagram illustrates the factoring process. At the top is the quadratic expression $6x^2 + 19x + 15$. Below it is the number 90, which is underlined. Below 90 are the numbers 9 and 10. At the bottom are the binomial factors $(9x + 15)(10x + 15)$. Red arrows show the following connections: one arrow from the coefficient 6 to the coefficient 9 in the first binomial; one arrow from the coefficient 15 in the quadratic to the constant 15 in the first binomial; one arrow from the coefficient 15 in the quadratic to the coefficient 10 in the second binomial; and one arrow from the coefficient 6 to the constant 15 in the second binomial.

All common factors are removed from each binomial, and then discarded.

$$\underset{\text{discard}}{5 \cdot 3} (3x + 5)(2x + 3)$$

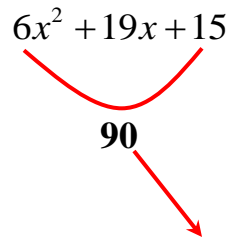
This leads to

$$(3x + 5)(2x + 3)$$

So $6x^2 + 19x + 15 = (2x + 3)(3x + 5).$

8. Bottoms-up Method: (Caution: You must remove all common factors first!)

This method reduces the factoring of a general trinomial into the factoring of a trinomial with a leading coefficient of 1.

$$6x^2 + 19x + 15$$


90

Rewrite as

$$x^2 + 19x + 90$$

And now factor

$$(x + 9)(x + 10)$$

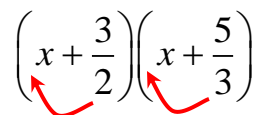
Now divide the two constant terms by the original leading coefficient.

$$\left(x + \frac{9}{6}\right)\left(x + \frac{10}{6}\right)$$

Reduce the two constant terms.

$$\left(x + \frac{3}{2}\right)\left(x + \frac{5}{3}\right)$$

Now bring the denominators of the constant terms up as the leading coefficients of each binomial factor.

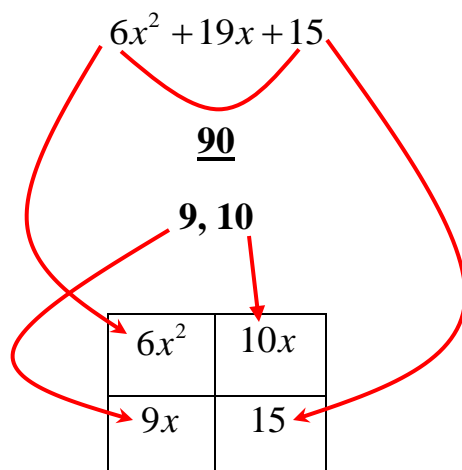
$$\left(x + \frac{3}{2}\right)\left(x + \frac{5}{3}\right)$$


$$(2x + 3)(3x + 5)$$

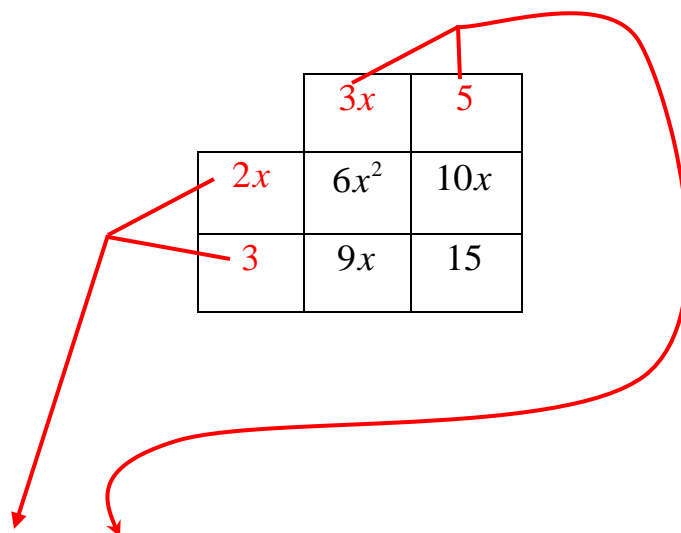
So $6x^2 + 19x + 15 = (2x + 3)(3x + 5).$

9. The Box Method: (Caution: You must remove all common factors first!)

This method is like the Grouping Method, but without actually grouping.



Now find all common factors for each row and each column.



So $6x^2 + 19x + 15 = (2x + 3)(3x + 5).$