Methods of Factoring Trinomials

$$6x^2+19x+15$$

1. Trial and Error Method:

This is the usual first method taught in most textbooks.

The factor pairs of the leading coefficient and the constant term are determined.

Factors of 6	Factors of 15
1,6	1,15
2,3	3,5

Then the possible binomial products are intelligently constructed and multiplied out.

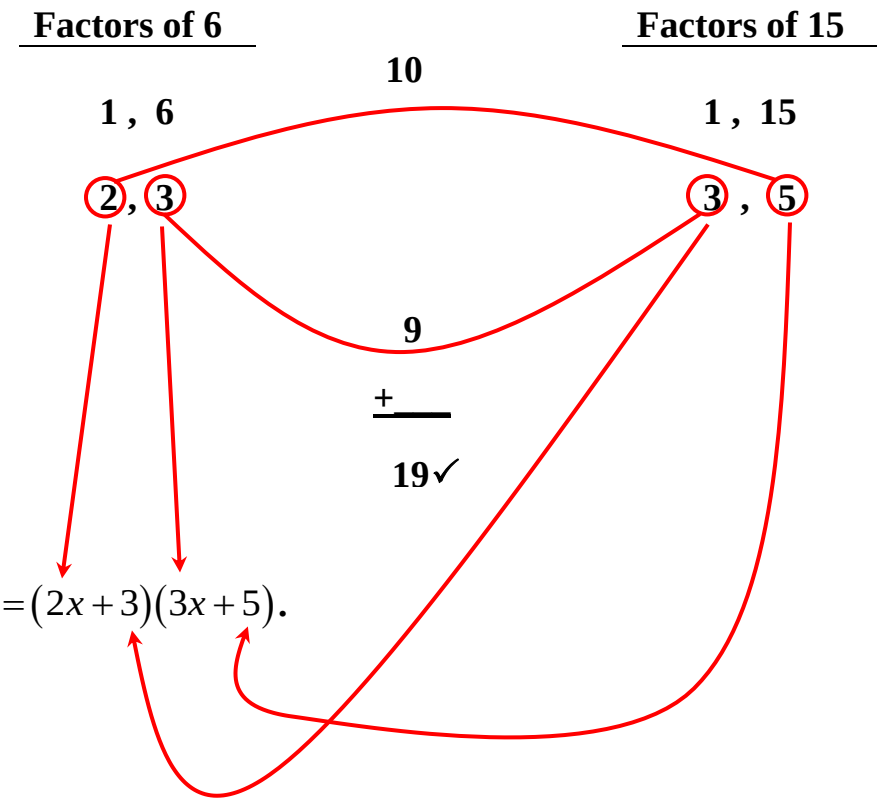
$$(x+1)(6x+15)$$
$$(x+15)(6x+1)=6x^2+91x+15$$
$$(x+3)(6x+5)=6x^2+23x+15$$
$$(x+5)(6x+3)$$
$$(2x+1)(3x+15)$$
$$(2x+15)(3x+1)=6x^2+47x+15$$
$$(2x+3)(3x+5)=6x^2+19x+15\checkmark$$
$$(2x+5)(3x+3)$$

So $6x^2+19x+15=(2x+3)(3x+5)$**.**

2. Listing and Pairing Method:

This is a streamlined version of Trial and Error that I came up with.

The factor pairs of the leading coefficient and constant term are listed. Then they are paired up, multiplied, and added to produce the middle coefficient.



So $6x^2+19x+15=(2x+3)(3x+5)$**.**

3. Grouping Method:

This method uses Factoring by Grouping which is usually taught as a method for factoring 4-term polynomials. The trinomial is converted into an equivalent 4-term polynomial, and then Factoring by Grouping is used.

$$6x^2+19x+15$$

90

9, 10

$$6x^2+9x+10x+15$$

$$\left(6x^2+9x\right)+\left(10x+15\right)$$

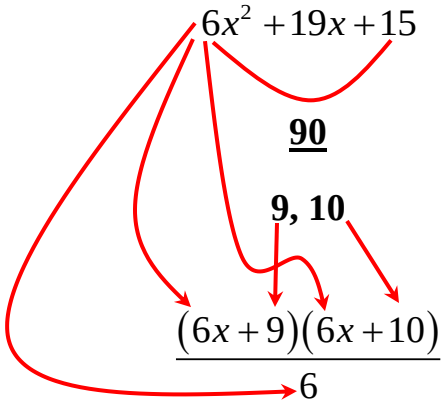
$$3x\left(2x+3\right)+5\left(2x+3\right)$$

$$\left(2x+3\right)\left(3x+5\right)$$

So $6x^2+19x+15=\left(2x+3\right)\left(3x+5\right)$**.**

4. First Magic Method:

This method begins like the Grouping Method, but it's somewhat more efficient.



All common factors are removed from each binomial.

$$\frac{2\cdot 3(2x+3)(3x+5)}{6}$$

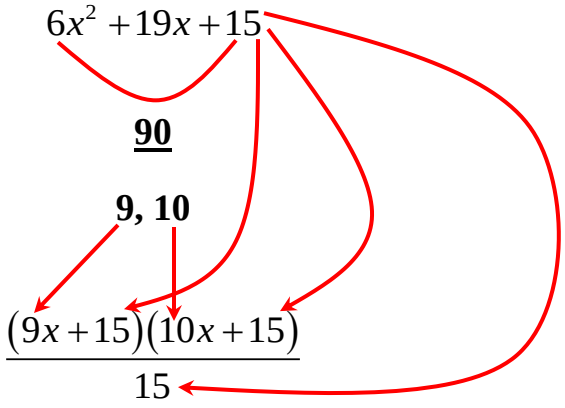
The fraction is reduced.

$$(2x+3)(3x+5)$$

So $6x^2+19x+15=(2x+3)(3x+5)$**.**

5. Second Magic Method:

This method begins like the Grouping Method, but it's somewhat more efficient.



All common factors are removed from each binomial.

$$\frac{5\cdot 3(3x+5)(2x+3)}{15}$$

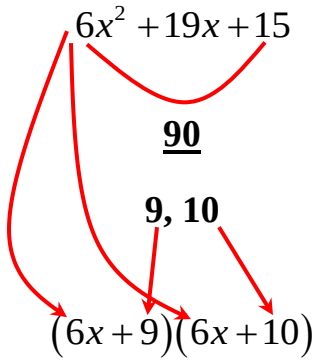
The fraction is reduced.

$$(3x+5)(2x+3)$$

So $6x^2+19x+15=(2x+3)(3x+5)$**.**

6. First Black Magic Method:(Caution: You must remove all common factors first!)

This method is a more efficient form of the Magic Method.



All common factors are removed from each binomial, and then discarded.

$$2\cdot 3\,(2x+3)(3x+5)$$
discard

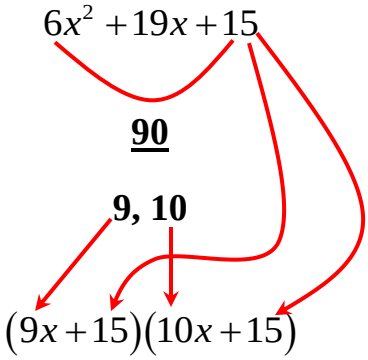
This leads to

$$(2x+3)(3x+5)$$

So $6x^2+19x+15=(2x+3)(3x+5)$**.**

7. Second Black Magic Method:(Caution: You must remove all common factors first!)

This method is a more efficient form of the Magic Method.



All common factors are removed from each binomial, and then discarded.

$$5\cdot 3\,(3x+5)(2x+3)$$
discard

This leads to

$$(3x+5)(2x+3)$$

So $6x^2+19x+15=(2x+3)(3x+5)$**.**

8. Bottoms-up Method: (Caution: You must remove all common factors first!)

This method reduces the factoring of a general trinomial into the factoring of a trinomial with a leading coefficient of 1.

$$6x^2+19x+15$$

90

Rewrite as $x^2+19x+90$

And now factor $(x+9)(x+10)$

Now divide the two constant terms by the original leading coefficient.

$$\left(x+\frac{9}{6}\right)\left(x+\frac{10}{6}\right)$$

Reduce the two constant terms.

$$\left(x+\frac{3}{2}\right)\left(x+\frac{5}{3}\right)$$

Now bring the denominators of the constant terms up as the leading coefficients of each binomial factor.

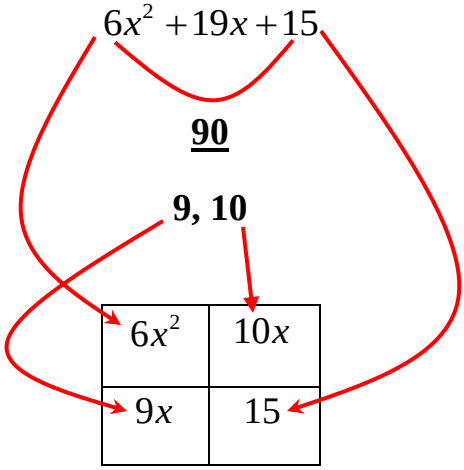
$$\left(x+\frac{3}{2}\right)\left(x+\frac{5}{3}\right)$$

$$(2x+3)(3x+5)$$

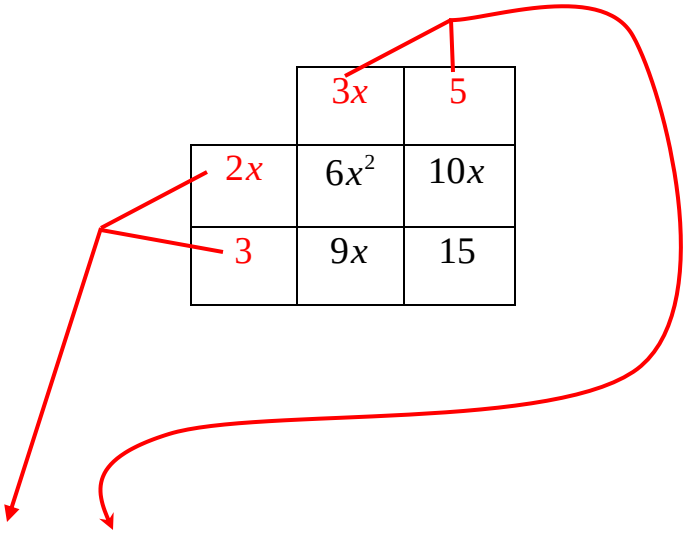
So $6x^2+19x+15=(2x+3)(3x+5)$**.**

9. The Box Method: (Caution: You must remove all common factors first!)

This method is like the Grouping Method, but without actually grouping.



Now find all common factors for each row and each column.



So $6x^2+19x+15=(2x+3)(3x+5)$**.**