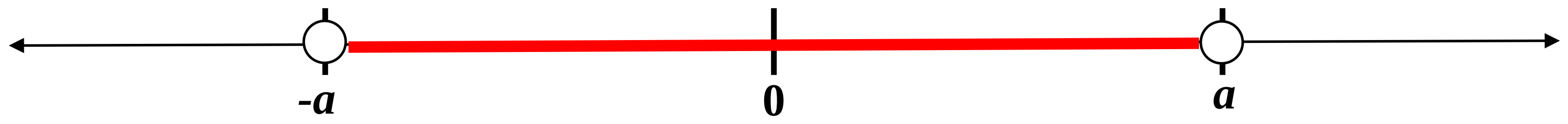
Absolute Value Inequalities:

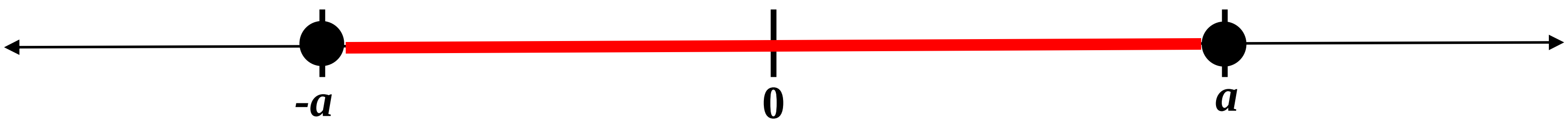
Remember that absolute value represents distance on the number line from zero.

If $a > 0$**, then what numbers would satisfy** $|x| < a$**?**



So $|x| < a$ **is equivalent to** $-a < x < a$**.**

If $a > 0$, then what numbers would satisfy $|x| \le a$?



So $|x| \le a$ is equivalent to $-a \le x \le a$.

If $a \le 0$, then what numbers would satisfy $|x| < a$? None

Can a distance be less than zero? No

If $a < 0$, then what numbers would satisfy $|x| \le a$? None

Can a distance be less than zero? No

What about $|x| \le 0$?

This can only be true if $|x| = 0$, and so $x = 0$.

Examples:

1. $|x| < 5$

$\boxed{-5 < x < 5} \Rightarrow \boxed{(-5,5)}$

2. $|x| \leq 3$

$\boxed{-3 \leq x \leq 3} \Rightarrow \boxed{(-3,3)}$

3. $|x| < -1$

Distance can't be negative, so $\boxed{\text{no solution}}$.

4. $|x+4| \le 10$

$$\underbrace{-10 \le x+4 \le 10}_{\text{subtract 4}} \Rightarrow \boxed{-14 \le x \le 6} \Rightarrow \boxed{[-14,6]}$$

5. $|5x+2| < 3$

$$\underbrace{-3 < 5x+2 < 3}_{\text{subtract 2}} \Rightarrow \underbrace{-5 < 5x < 1}_{\text{divide by 5}} \Rightarrow \boxed{-1 < x < \frac{1}{5}} \Rightarrow \boxed{\left(-1, \frac{1}{5}\right)}$$

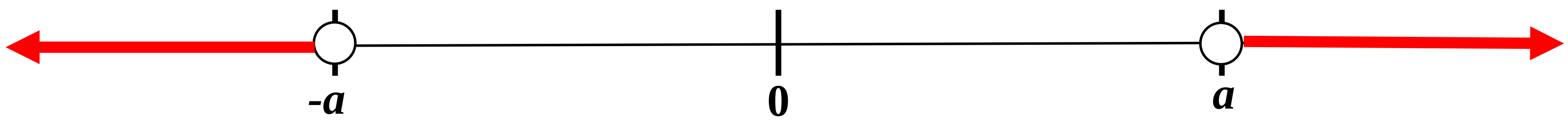
6. $|7-2x| \le 0$

Distance can't be negative, so $|7-2x| = 0 \Rightarrow 7-2x = 0 \Rightarrow 2x = 7 \Rightarrow x = \boxed{\frac{7}{2}}$

7. $|3x-7|+6 \le 5$

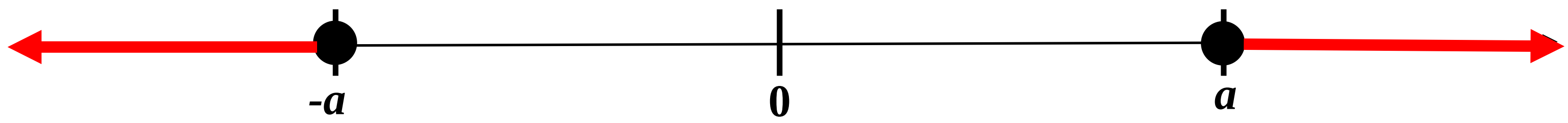
Subtract 6 to isolate the absolute value. $|3x-7| \le -1$, but distance can't be negative, so $\boxed{\text{no solution}}$.

If $a > 0$, then what numbers would satisfy $|x| > a$?



So $|x| > a$ is equivalent to $x < -a \quad or \quad x > a$.

If $a > 0$**, then what numbers would satisfy** $|x| \geq a$**?**



So $|x| \leq a$ **is equivalent to** $x \leq -a \quad or \quad x \geq a$**.**

If $a < 0$**, then what numbers would satisfy** $|x| > a$**?** All real numbers

Wouldn't all numbers have their distance to zero be greater than a negative number? Yes

If $a \leq 0$**, then what numbers would satisfy** $|x| \geq a$**?** All real numbers

Wouldn't all numbers have their distance to zero be greater than or equal to zero or a negative number? Yes

What about $|x| > 0$?

This is true for all real numbers except zero.

Examples:

1. $|x| > 10$

$\boxed{x < -10 \text{ or } x > 10} \Rightarrow \boxed{(-\infty,-10) \cup (10,\infty)}$

2. $|x| \geq 6$

$\boxed{x \leq -6 \text{ or } x \geq 6} \Rightarrow \boxed{(-\infty,-6] \cup [6,\infty)}$

3. $|x| > -2$

Distance will always be larger than a negative number, so $\boxed{\text{All real numbers}} \Rightarrow \boxed{(-\infty,\infty)}$.

4. $|x-1|>0$

Distance from zero for a number will always be positive except for the number zero.

$x-1=0 \Rightarrow x=1 \Rightarrow$ all real numbers except for $1 \Rightarrow \boxed{(-\infty,1)\cup(1,\infty)}$

5. $|3x-4|>8$

$$\underbrace{3x-4<-8}_{\text{add 4}} \text{ or } \underbrace{3x-4>8}_{\text{add 4}} \Rightarrow \underbrace{3x<-4}_{\text{divide by 3}} \text{ or } \underbrace{3x>12}_{\text{divide by 3}} \Rightarrow \boxed{x<-\frac{4}{3} \text{ or } x>4} \Rightarrow \boxed{\left(-\infty,-\frac{4}{3}\right)\cup(4,\infty)}$$

6. $|2-9x|\geq 17$

$$\underbrace{2-9x\leq -17}_{\text{subtract 2}} \text{ or } \underbrace{2-9x\geq 17}_{\text{subtract 2}} \Rightarrow \underbrace{-9x\leq -19}_{\text{divide by -9 and reverse}} \text{ or } \underbrace{-9x\geq 15}_{\text{divide by -9 and reverse}}$$

$$\Rightarrow \boxed{x\geq \frac{19}{9} \text{ or } x\leq -\frac{5}{3}} \Rightarrow \boxed{\left(-\infty,-\frac{5}{3}\right]\cup\left[\frac{19}{9},\infty\right)}$$

7. $|x-7|+3\geq 4$

Subtract 3 to isolate the absolute value.

$$|x-7|\geq 1 \Rightarrow \underbrace{x-7\leq -1}_{\text{add 7}} \text{ or } \underbrace{x-7\geq 1}_{\text{add 7}} \Rightarrow \boxed{x\leq 6 \text{ or } x\geq 8} \Rightarrow \boxed{(-\infty,6]\cup[8,\infty)}$$