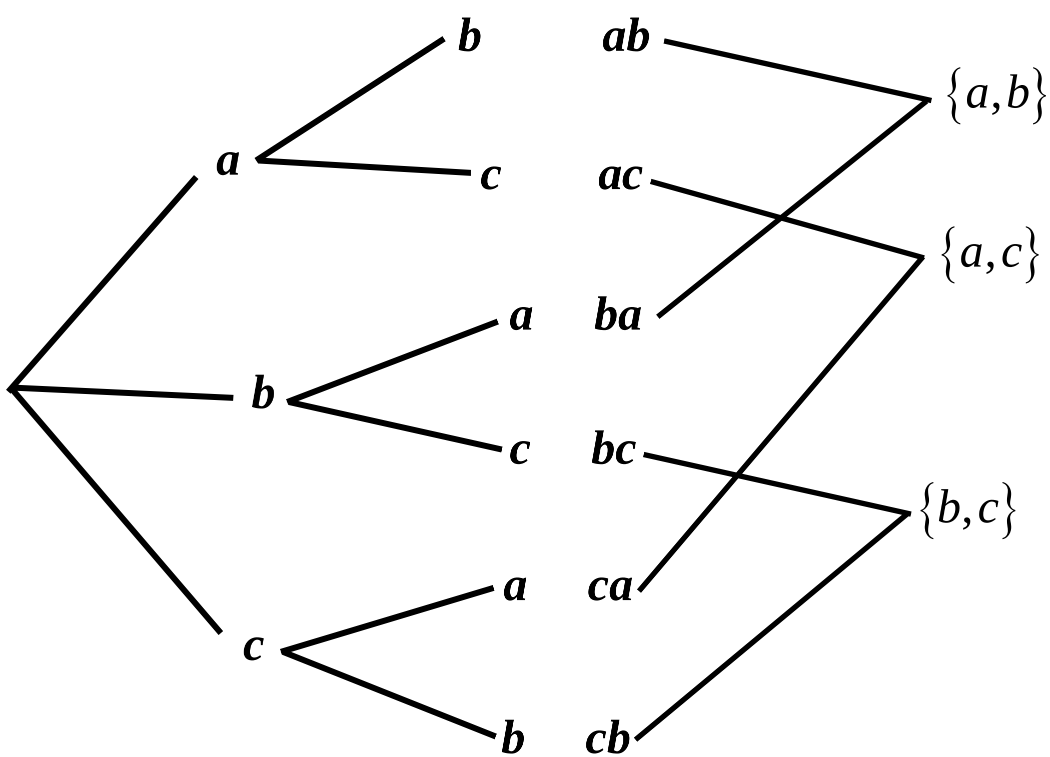
Combinations:

A combination is a selection of objects without regard to order, i.e. a subset.

Example:

Find all the combinations of the objects $\{a,b,c\}$ of size 2.



There are 6 permutations of size 2 from the 3 objects, but only 3 combinations of size 2 from the 3 objects.

In general, the number of combinations of size *r* from *n* objects is abbreviated as $_{n}C_{r}$. So far, we know that $_{3}C_{2} = 3$, and $_{3}C_{2} = 3 = \frac{6}{2} = \frac{_{3}P_{2}}{2!}$. This is true in general, and leads to a nice formula for $_{n}C_{r}$. $_{n}C_{r} = \frac{n!}{r! \cdot (n-r)!}$

Most scientific calculators have a key for determining the number of combinations.

<u>Examples:</u>

1. A three-person committee is to be selected from a group of 10 people. How many different committees are possible?

$$_{10}C_{3} = \frac{10!}{3! \cdot 7!} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = \boxed{120}$$

2. In a certain lottery, you must select 6 numbers from the numbers 1-50. How many different lottery selections are possible?

$$_{50}C_{6} = \frac{50!}{6! \cdot 44!} = \frac{50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \boxed{15{,}890{,}700}$$

3. A group consists of 7 men and 8 women. A committee of 4 people will be selected.

a) How many different 4-person committees are possible?

$$_{15}C_4 = \frac{15!}{4! \cdot 11!} = \frac{15 \cdot 14 \cdot 13 \cdot 12}{4 \cdot 3 \cdot 2 \cdot 1} = \boxed{1{,}365}$$

b) How many different 4-person committees consisting of 4 women are possible?

$$_{8}C_4 = \frac{8!}{4! \cdot 4!} = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2 \cdot 1} = \boxed{70}$$

c) How many different 4-person committees consisting of 3 women and 1 man are possible?

$_{8}C_3 = 56$	$_{7}C_1 = 7$
Which 3 women?	**Which 1 man?**

$$56 \cdot 7 = \boxed{392}$$

d) How many different 4-person committees consisting of 2 women and 2 men are possible?

$_8C_2 = 28$	$_7C_2 = 21$
Which 2 women?	**Which 2 men?**

$28 \cdot 21 = \boxed{588}$

e) How many different 4-person committees have at least 1 man?

1 man	2 men	3 men	4 men	Total
$_8C_3 \cdot {_7C_1} = 392$	$_8C_2 \cdot {_7C_2} = 588$	$_8C_1 \cdot {_7C_3} = 280$	$_7C_4 = 35$	1,295

OR

= the number of committees with at least one man

= the total number of committees – the number of committees with no men

$= 1{,}365 - 70 = \boxed{1{,}295}$

Sometimes it's easier to calculate quantities indirectly!

<u>Probability:</u>

<u>Experiment:</u>

Any process that produces random results.

Example:

Flip a coin twice and record the results.

Heads/Heads, Heads/Tails, Tails/Heads, Tails/Tails

<u>Sample Space:</u>

The set of all possible outcomes of an experiment. It's abbreviated with the letter *S*, and it's like the universal set for an experiment.

Example:

From the previous experiment

$$S = \{HH, HT, TH, TT\}$$

<u>Event:</u>

Any subset of the sample space

Example:

From the previous experiment

$$S = \{HH, HT, TH, TT\}$$

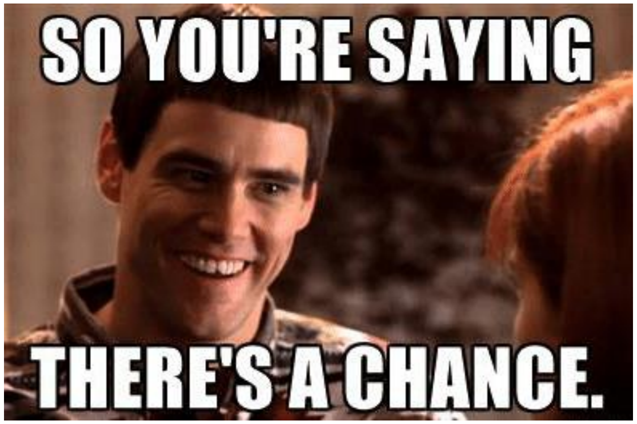
E **is the event that heads occurs.** $E = \{HH, HT, TH\}$

F **is the event that tails occurs.** $F = \{HT, TH, TT\}$

G **is the event of getting the same result on both flips.** $G = \{HH, TT\}$

J **is the event of getting different results on the two flips.** $J = \{HT, TH\}$

A probability is a number between 0 and 1(inclusive) that indicates the likelihood that an event will occur.



A probability of 1 means the event must occur.

A probability of 0 means the event won't occur.

A probability of ½ means the event is just as likely to occur as not to occur.

The closer the probability value is to 1, the more likely the event will occur, and the closer the probability value is to 0, the less likely the event will occur.

Theoretical Probability and the Equally Likely assumption:

In many experiments, the outcomes in the sample space all have the same probability of occurring. Certain conditions in the experiments will allow you to make this equally likely assumption.

When you can assume equally likely outcomes, the probability of an event is determined using counting.

$$P(E) = \frac{n(E)}{n(S)}$$

Examples:

1. A *fair* die is rolled. $S = \{1, 2, 3, 4, 5, 6\}$

a) $P(\text{rolling a 1})$

$$\frac{1}{6}$$

b) $P(\text{rolling a 2 or a 3})$

$$\frac{2}{6} = \frac{1}{3}$$

c) $P(\text{rolling an odd number})$

$$\frac{3}{6} = \frac{1}{2}$$

d) $P(\text{rolling a 7})$

$$\frac{0}{6} = 0$$

2. A card is *randomly selected* from a standard 52-card deck.

a) $P(\text{selecting an ace})$

$$\frac{4}{52} = \frac{1}{13}$$

b) $P(\text{selecting a red card})$

$$\frac{26}{52} = \frac{1}{2}$$

c) $P(\text{selecting a club})$

$$\frac{13}{52} = \frac{1}{4}$$

d) $P(\text{selecting a face card})$

J Q K ♦ ♠ ♥ ♣

$$\frac{12}{52} = \frac{3}{13}$$

e) $P(\text{selecting an ace or a diamond})$

A♥ A♣ A♠ A♦

2♦ 3♦ 4♦ 5♦ 6♦ 7♦ 8♦ 9♦ 10♦ J♦ Q♦ K♦

$$\frac{16}{52} = \frac{4}{13}$$

3. A *fair coin* is flipped twice. $S = \{HH, HT, TH, TT\}$

a) $P(\text{heads occurs})$

$$\frac{3}{4}$$

b) $P(\text{tails occurs})$

$$\frac{3}{4}$$

c) $P(\text{same result on both flips})$

$$\frac{2}{4} = \frac{1}{2}$$

d) $P(\text{different result on the two flips})$

$$\frac{2}{4} = \frac{1}{2}$$

e) $P(\text{two heads occur})$

$$\frac{1}{4}$$

f) $P(\text{three heads occur})$

$$\frac{0}{4} = 0$$

4. A survey of North Harris students had the following results.

	Pepperoni	Sausage	Mushroom	Total
Freshman	**25**	**15**	**5**	**45**
Sophomore	**30**	**20**	**5**	**55**
Total	**55**	**35**	**10**	**100**

A student from the survey is *selected at random*.

a) $P(\text{freshman})$

$$\frac{45}{100} = \frac{9}{20}$$

b) $P(\text{sausage})$

$$\frac{35}{100} = \frac{7}{20}$$

c) $P(\text{freshman and pepperoni})$

$$\frac{25}{100} = \frac{1}{4}$$

d) $P(\text{sausage or mushroom})$

$$\frac{45}{100} = \frac{9}{20}$$

e) $P(\text{freshman or pepperoni})$

$$\frac{75}{100} = \frac{3}{4}$$

Empirical Probability:

The experiment is performed a bunch of times, *n*, and the results are recorded.

$$P(E) = \frac{\#\text{ of times } E \text{ occurs}}{n}$$

Example:

A coin is flipped 1,000 times with the following results: 450 heads and 550 tails. Find the empirical probability of flipping a tail.

$$\frac{550}{1000} = \frac{11}{20}$$

Calculator Advice:

Most calculators have keys for factorial, number of permutations, and number of combinations. The factorial key will usually look like $x!$, $n!$, or just !. The number of permutations key will look like ${}_nP_r$, and the number of combinations key will look like ${}_nC_r$. To use the factorial key, just enter the number and then press the factorial key. To use the others, enter the n value, press the appropriate key, then enter the r value, and press the = or enter key.

Some calculators have these operations hidden. Press the MATH key and look under the probability heading or press the PRB key and select the operation that you want using the arrow key.