Math 1332 Review 1(answers)

1. Identify a pattern in the given list of numbers, and use inductive reasoning to find the next two numbers in the list:

a) 4,9,14,19,…

A pattern is to start with 4 and continue to add 5 to the previous number.
24,29

b) $\frac{3}{4},\frac{3}{5},\frac{1}{2},\frac{3}{7},\ldots$

A pattern is to have a numerator of 3, a denominator which starts with 4 and continues to increase by 1, and you reduce the fraction, if possible.
$\frac{3}{8},\frac{1}{3}$

2. Consider the following procedure: Select a number. Double the number. Add four to the product. Divide the sum by two. Subtract two from the quotient.

a) Perform these operations on the given numbers in the table, and write in your results:

The original number	The result of the operations
0	0
1	1
–1	–1
3	3

b) Using inductive reasoning, write a conjecture that relates the result of the operations to the original number.

The result of the operations is the original number.

c) Use deductive reasoning to prove your conjecture in part b).

If the original number is n, then the result of the operations is $\frac{2n+4}{2}-2$. And

$$\frac{2n+4}{2}-2=\frac{2(n+2)}{2}-2=n+2-2=n.$$

3. Notice that $\frac{1}{1\cdot 2}=\frac{1}{2}$, $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}=\frac{2}{3}$, $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}=\frac{3}{4}$.

a) Use inductive reasoning to find a formula for $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)}$

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

b) Use your formula in part a) to find the value of $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{1000\cdot 1001}$.

$$\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{1000\cdot 1001}=\frac{1000}{1001}$$

4. A car rents for \$175 per week plus \$.30 per mile. Find the rental cost for a 3 week trip of 1,200 miles.

3 weeks costs $3 \cdot \$175 = \$525$ and 1,200 miles costs $1{,}200 \cdot \$.30 = \$360$, so the rental cost is $\$525 + \$360 = \boxed{\$885}$

5. Fill in the blank with either $\in$ or $\notin$ to make each statement true.

a) $93 \boxed{\in} \{1,2,3,4,\ldots,100\}$ **b)** $\{d\} \boxed{\notin} \{a,b,c,e,f\}$

6. Fill in the blank with either $\subseteq$ or $\not\subseteq$ to make each statement true.

a) $\{penny, nickel, dime\} \boxed{\subseteq} \{half\text{-}dollar, quarter, dime, nickel, penny\}$

b) $\varnothing \boxed{\subseteq} \{x \mid x \text{ is an odd natural number}\}$ **c)** $\{-1,0,1\} \boxed{\not\subseteq} \{-3,-2,-1,1,2,3\}$

7. Determine whether $\subseteq$ or $\subset$, both, or neither can be placed in each blank to make the statement true.

a) $\{1,2\} \boxed{\subseteq} \{1,1,2,2\}$

b) $\{x \mid x \text{ is a student in this class}\} \boxed{both} \{x \mid x \text{ is a student at North Harris}\}$

8. Determine if the pair of sets are equal, equivalent, both, or neither.

a) $A = \{x \mid x \text{ is a lowercase letter that comes before } f \text{ in the English alphabet}\}$

$B = \{a,b,c,d,e\}$

$\boxed{\text{both}}$

b) $A = \{x \mid x \in \mathbb{N} \text{ and } x \text{ is between 3 and 7, exclusive}\}$

$B = \{a,b,c\}$

$\boxed{\text{equivalent}}$

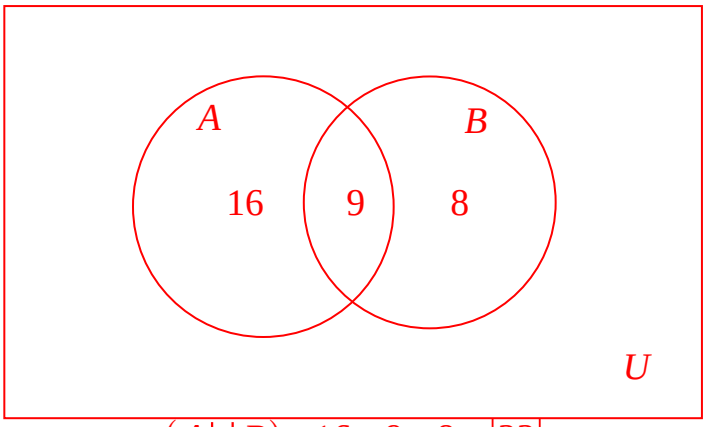
9. Find the number of subsets and the number of proper subsets of the set $\{1,2,3,4\}$.

$\boxed{\begin{array}{l}\#\text{ of subsets} = 2^4 = 16 \\ \#\text{ of proper subsets} = 2^4 - 1 = 16 - 1 = 15\end{array}}$

10. Set A contains 25 elements, set B contains 17 elements, and 9 elements are common to both sets A and B. How many elements are in $A \cup B$?

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$
$$= 25 + 17 - 9 = \boxed{33}$$

or



$$n(A \cup B) = 16 + 9 + 8 = \boxed{33}$$

11. Let $U = \{1,2,3,4,5,6,7,8\}$, $A = \{1,2,3,4\}$, $B = \{1,2,4,5\}$, and $C = \{1,5\}$. List the elements of the following sets:

a) $A \cap B$

$$\{1,2,3,4\} \cap \{1,2,4,5\} = \boxed{\{1,2,4\}}$$

b) $A \cup B'$

$$\{1,2,3,4\} \cup \{1,2,4,5\}' = \{1,2,3,4\} \cup \{3,6,7,8\}$$
$$= \boxed{\{1,2,3,4,6,7,8\}}$$

c) $A' \cap B$

$$\{1,2,3,4\}' \cap \{1,2,4,5\} = \{5,6,7,8\} \cap \{1,2,4,5\}$$
$$= \boxed{\{5\}}$$

d) $(A \cup B)'$

$$(\{1,2,3,4\} \cup \{1,2,4,5\})' = \{1,2,3,4,5\}'$$
$$= \boxed{\{6,7,8\}}$$

e) $A' \cap B'$

$$\{1,2,3,4\}' \cap \{1,2,4,5\}' = \{5,6,7,8\} \cap \{3,6,7,8\}$$
$$= \boxed{\{6,7,8\}}$$

f) $A \cup (B \cap C)$

$$\{1,2,3,4\} \cup (\{1,2,4,5\} \cap \{1,5\})$$
$$= \{1,2,3,4\} \cup \{1,5\} = \boxed{\{1,2,3,4,5\}}$$

g) $(A \cap C)' \cup B$

$$(\{1,2,3,4\} \cap \{1,5\})' \cup \{1,2,4,5\}$$
$$= \{1\}' \cup \{1,2,4,5\}$$
$$= \{2,3,4,5,6,7,8\} \cup \{1,2,4,5\}$$
$$= \boxed{\{1,2,3,4,5,6,7,8\}}$$

h) $A' \cap B' \cap C'$

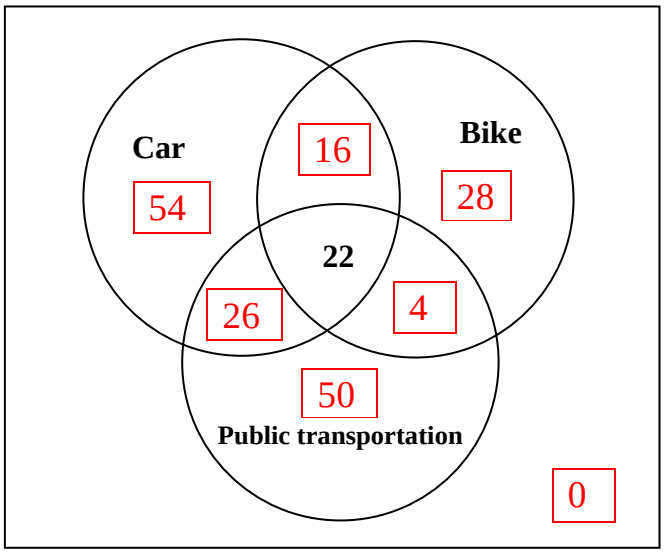
$$\{1,2,3,4\}' \cap \{1,2,4,5\}' \cap \{1,5\}'$$
$$= \{5,6,7,8\} \cap \{3,6,7,8\} \cap \{2,3,4,6,7,8\}$$
$$= \boxed{\{6,7,8\}}$$

12. At the fast-food restaurant Whyaburger, you can choose some, all, or none of the following ingredients on your Whyaburger: {pickles, onions, jalapenos, cheese, lettuce, tomatoes, bacon}. How many different Whyaburgers are possible?

$$2^7 = \boxed{128}$$

13. A survey of 200 students at a nonresidential college revealed the following results:

Of those surveyed 118 used cars, 102 used public transportation, and 70 used bikes. Forty-eight students used cars and public transportation, 38 used cars and bikes, and 26 used public transportation and bikes. Twenty-two students used all three modes of transportation.



a) Complete the Venn Diagram:

b) How many used only public transportation?

$$\boxed{50}$$

c) How many used cars and public transportation, but not bikes?

$$\boxed{26}$$

d) How many used cars or public transportation, but not bikes?

$$50 + 26 + 54 = \boxed{130}$$

e) How many used exactly two of these modes of transportation?

$$26 + 16 + 4 = \boxed{46}$$

f) How many did not use any of these modes of transportation?

$$\boxed{0}$$

14. A popular brand of pen comes in red, blue, or black ink. The writing tip can be chosen from extra bold, bold, regular, fine, or micro. How many different choices of pens do you have with this brand?

$$3 \cdot 5 = \boxed{15}$$

15. Six acts are scheduled to perform in a variety show. How many different ways are there to schedule their appearances?

$$_6P_6 = 6! = \boxed{720}$$

Or

$$\underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = \boxed{720}$$

16. A club with 15 members is to choose four officers – president, vice-president, secretary, and treasurer. In how many different ways can these offices be filled, if no member can hold more than one office?

$$_{15}P_4 = \frac{15!}{11!} = \boxed{32{,}760}$$

Or

$$\underline{15} \cdot \underline{14} \cdot \underline{13} \cdot \underline{12} = \boxed{32{,}760}$$

17. How many different six-digit numbers can be formed by rearranging the digits in 335,557?

$$\frac{6!}{2! \cdot 3!} = \frac{5!}{2!} = \frac{120}{2} = \boxed{60}$$

18. A political discussion group consists of 4 Republicans and 6 Democrats. If a committee of four people is to be selected, how many different committees consisting of the following are possible?

a) all four are Democrats.

$$_6C_4 = \frac{6!}{4! \cdot 2!} = \frac{6 \cdot 5}{2} = \boxed{15}$$

b) two are Democrats and two are Republicans.

$$_6C_2 \cdot {}_4C_2 = \frac{6!}{2! \cdot 4!} \cdot \frac{4!}{2! \cdot 2!} = \frac{6!}{8} = \frac{720}{8} = \boxed{90}$$