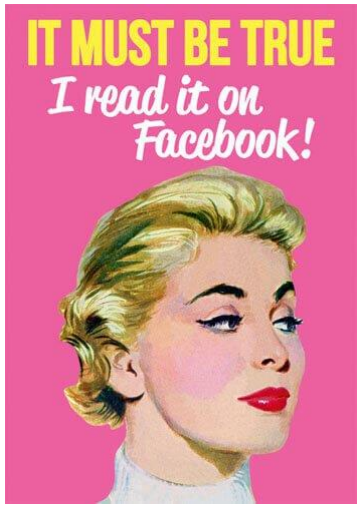
Inductive Reasoning:

Drawing a conclusion by looking at specific examples, identifying a pattern(intuition), and assuming that it's correct and will continue.



The conclusion might be wrong!

Deductive Reasoning:

Drawing a conclusion by starting with true information, and using the rules of logic and mathematics to arrive at the conclusion.

The conclusion must be correct!

Examples:

1. Choose a number. Add three. Multiply by two. Subtract four. Divide by two.

Starting Number	Ending number
1	**2**
2	**3**
3	**4**
4	**5**

a) Use inductive reasoning to state a conjecture(guess) that relates the ending number to the starting number.

The ending number is one more than the starting number.

b) Use deductive reasoning to verify the conjecture from part a).

For n, the starting number, the ending number is $\frac{2(n+3)-4}{2}$, but

$$\frac{2(n+3)-4}{2} = \frac{2n+6-4}{2} = \frac{2n+2}{2} = \frac{2n}{2} + \frac{2}{2} = n+1$$

One more than the starting number.

2. Use inductive reasoning to identify a pattern in the following lists of numbers, and use the pattern to find the next two numbers in the list.

a) 2,4,6,8,…

Starting with 2, keep adding 2 to get the next number.

OR ***(adding or multiplying?)***

Multiply the position number in the list by 2 to get the number in the list.

10,12

b) 1,4,9,16,…

Starting with 1 add consecutive odd numbers starting with 3 to get the next numbers in the list.

OR ***(adding or squaring?)***

Square the position number in the list to get the number in that position.

25,36

c) 1,1,2,3,5,8,13,21,…

Starting with 1 and 1, keep adding the two previous numbers to get the next number in the list.

13 + 21 = 34, 21 + 34 = 55

d) 1,2,3,…

Starting with 1 keep adding 1 to get the next number in the list.

OR

The position number is the value in the list.

4,5

Be careful: The formula $n+(n-1)(n-2)(n-3)$ ***was used to produce the numbers in the list.***

So the next two numbers in the list should be 10 and 29.

3. Identify a pattern in the following formulas for the sums of counting numbers.

$$1=\frac{1\cdot 2}{2}$$

$$1+2=\frac{2\cdot 3}{2}$$

$$1+2+3=\frac{3\cdot 4}{2}$$

$$1+2+3+4=\frac{4\cdot 5}{2}$$

a) Use the pattern(inductive reasoning) to find the value of the following.

$$1+2+3+4+5+6+7+8+9+10=\frac{\boxed{10}\cdot\boxed{11}}{2}$$

$$1+2+3+4+5+6+7+8+9+10+11+\cdots+20=\frac{\boxed{20}\cdot\boxed{21}}{2}$$

$$1+2+3+4+5+6+7+8+9+10+11+\cdots+n=\frac{\boxed{n}\cdot\boxed{(n+1)}}{2}$$

b) Use deductive reasoning to show that the pattern is correct.

$$S = 1 + 2 + 3 + 4 + \cdots + n$$

$$\begin{aligned} S &= 1 + 2 + 3 + 4 + \cdots + n \\ +S &= n + (n-1) + (n-2) + (n-3) + \cdots + 1 \\ \hline 2S &= \underbrace{(n+1) + (n+1) + (n+1) + (n+1) + \cdots + (n+1)}_{n \text{ times}} \end{aligned}$$

$$S = \frac{n(n+1)}{2}$$

4. Examine the following table, and use inductive reasoning to fill-in the question mark.

# of 1's	Squared	Expanded	Sum of the digits
1	1^2	1	$\boxed{1} = 1^2$
2	11^2	121	$\boxed{4} = 2^2$
3	111^2	12321	$\boxed{9} = 3^2$
4	1111^2	1234321	$\boxed{16} = 4^2$
5	11111^2	123454321	$\boxed{25} = 5^2$
6	111111^2	12345654321	$\boxed{36} = 6^2$
7	1111111^2	1234567654321	$\boxed{49} = 7^2$
8	11111111^2	123456787654321	$\boxed{64} = 8^2$
9	111111111^2	12345678987654321	$\boxed{81} = 9^2$
10	1111111111^2	1234567900987654321	$\boxed{100} = 10^2$

Are you correct? No, the actual sum of the digits is 82. Again, inductive reasoning can sometimes lead to incorrect conclusions.

Problem Solving

Polya's Four Step Process:

1. ***Understand the problem.*** Read it. Figure out what you're given. Figure out what you need to determine.

2. ***Devise a plan.*** Guess and check and refine, look for a pattern, make a systematic list, use a drawing or diagram, eliminate possibilities, use a formula,…

3. ***Carry out the plan, and solve the problem.*** Proceed with step 2, and get a result.

4. ***Look back, and check the answer.*** Make sure the result of step 3 actually satisfies the conditions of the problem. Make sure your solution is reasonable.

Examples:

1. A television sells for \$750. Instead of paying the total amount at the time of purchase, you can put \$100 down and make \$50 a month payments for 14 months. How much extra will you pay for the TV using the payment plan?

Total amount you pay with the plan: $\$100 + \$50 \cdot 14$

Cost of the TV: \$750

$$\text{Extra} = \$100 + \$50 \cdot 14 - \$750 = \$800 - \$750 = \boxed{\$50}$$

2. At the beginning of a year, the odometer of a car read 25,124 miles. At the end of the year, it read 37,364 miles. If the car averaged 24 miles per gallon, how many gallons of gasoline did it use during the year?

$$24 \text{ mpg} = \frac{\text{total miles driven}}{\text{total gallons of gasoline used}}$$

$$24 \text{ mpg} = \frac{37{,}364 - 25{,}124}{\text{total gallons of gasoline used}}$$

$$\text{total gallons of gasoline used} = \frac{37{,}364 - 25{,}124}{24} = \frac{12{,}240}{24} = \boxed{510 \textit{ gallons}}$$

3. How many different ways can you make change for a quarter using only nickels and dimes?

Systematic listing:

Way	Number of Dimes	Number of Nickels
1	0	5
2	1	3
3	2	1

4. Complete the following Magic Square.

5	**22**	**18**
28	**15**	**2**
12	**8**	**25**

The sum of each row, column, and diagonal must be $5+15+25=45$.

$25+18=43$, so we need a 2. $15+18=33$, so we need a 12. $5+18=23$, so we need a 22. $22+15=37$, so we need an 8. $12+5=17$, so we need a 28.

5. You throw three darts at a dart board. Each dart hits the board and scores a 2, 4, or 6. How many different total scores can you make?

Systematic listing:

2's	4's	6's	Total score	Different Total scores
3	0	0	6	1
2	1	0	8	2
2	0	1	10	3
1	2	0	10	
1	0	2	14	4
1	1	1	12	5
0	3	0	12	
0	0	3	18	6
0	1	2	16	7
0	2	1	14	

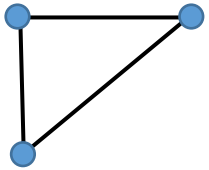
So, there are 7 different scores possible.

6. There are 5 people in a room. Each person shakes the hand of every other person exactly once. How many handshakes occur? First we'll look at a diagram.

2 people

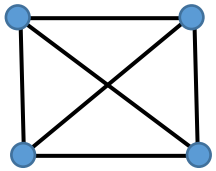
1 handshake

3 people



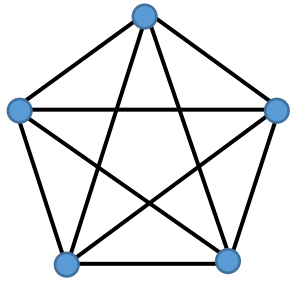
3 handshakes

4 people



6 handshakes

5 people



? handshakes

Notice that each time a new person is added, they have to shake hands with the people who are already there. This increases the number of handshakes by the number of people who are already there.

Number of People	2	3	4	5
Number of Handshakes	1	$1+2=\boxed{3}$	$1+2+3=\boxed{6}$	$1+2+3+4=\boxed{10}$

How many handshakes with 6 people? 7 people? n people?

15 21 $\dfrac{n(n-1)}{2}$