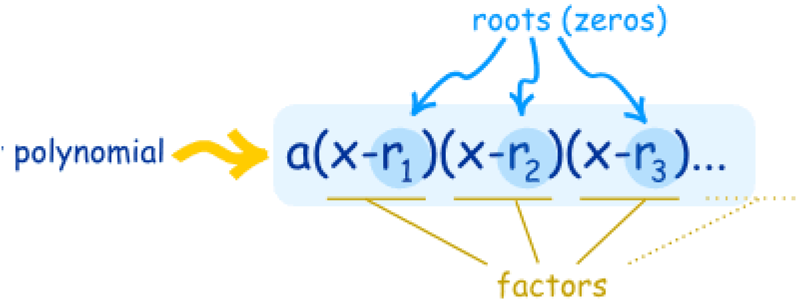
Linear Factorization Theorem:



Every polynomial function $f(x)=a_n x^n + a_{n-1}x^{n-1}+\cdots+a_1 x + a_0$ **of degree** $n \geq 1$ **can be factored into** $f(x)=a_n(x-c_1)(x-c_2)\cdots(x-c_n)$ **, where** $c_1, c_2, \ldots, c_n$ **are the *n* zeros of** $f(x)$**.**

Examples:

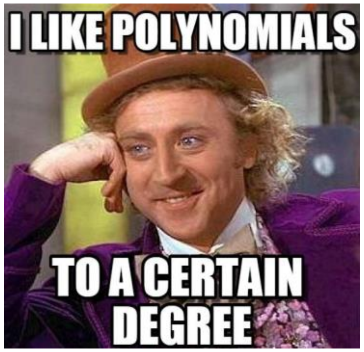
1. Write the polynomial function $f(x)=x^3+3x^2-x-3$ **in linear factored form.**

2. Write the polynomial function $f(x) = 2x^3 + 2x$ in linear factored form.

3. Find a polynomial function, $f(x)$, of degree 3 with zeros of -2, 3, and 5 and with $f(0) = 60$.

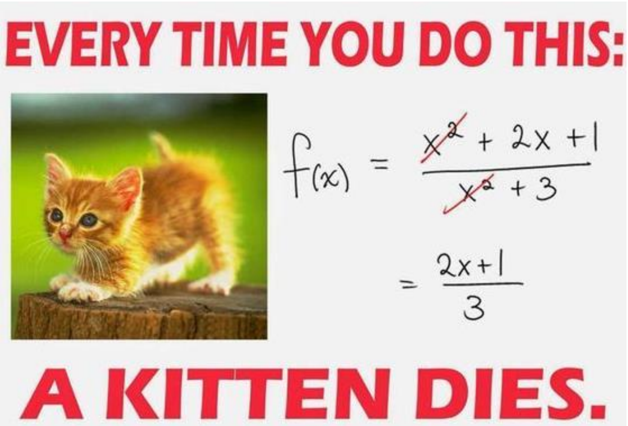
4. Find a polynomial function with real coefficients, $f(x)$, of degree 3 with zeros of 2 and *i* and with $f(0)=6$.

5. Find a polynomial function with real coefficients, $f(x)$, of degree 3 with zeros of 1 and $1+i$ and with $f(0)=-2$.



Rational Functions:

$$f(x)=\frac{p(x)}{q(x)}$$, **where $p(x)$ and $q(x)$ are polynomial functions with no common real zeros.**

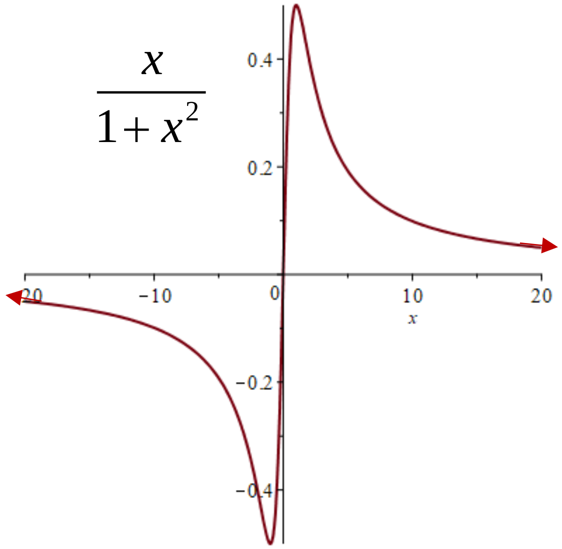


Asymptotes: **(Polynomials don't have them.)**

Lines that the graph of a rational function approach.

Horizontal Asymptote: **(End Behavior)**

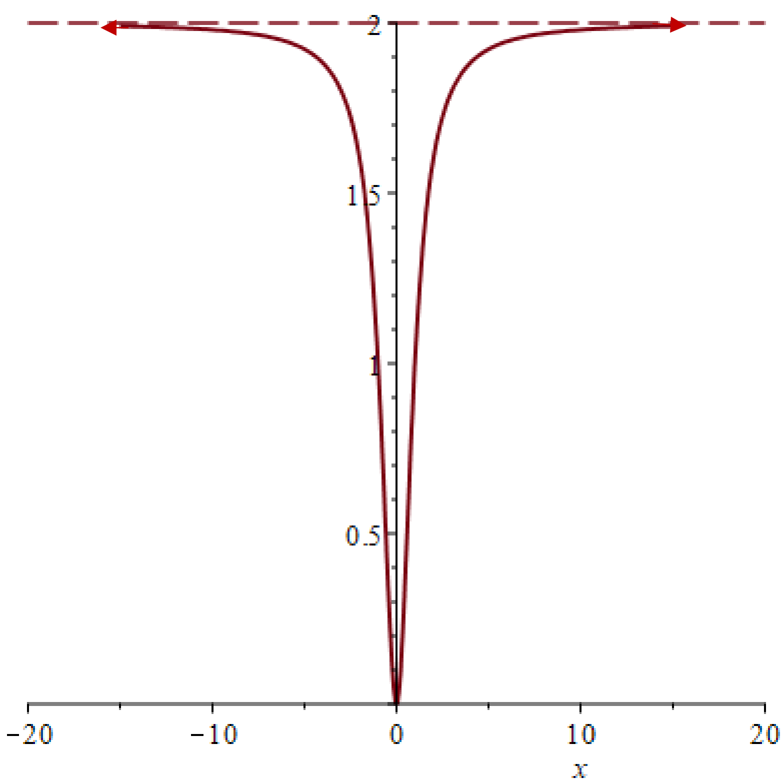
If the degree of $p(x)$ is less than the degree of $q(x)$, then $y=0$ (*x*-axis) is the horizontal asymptote.



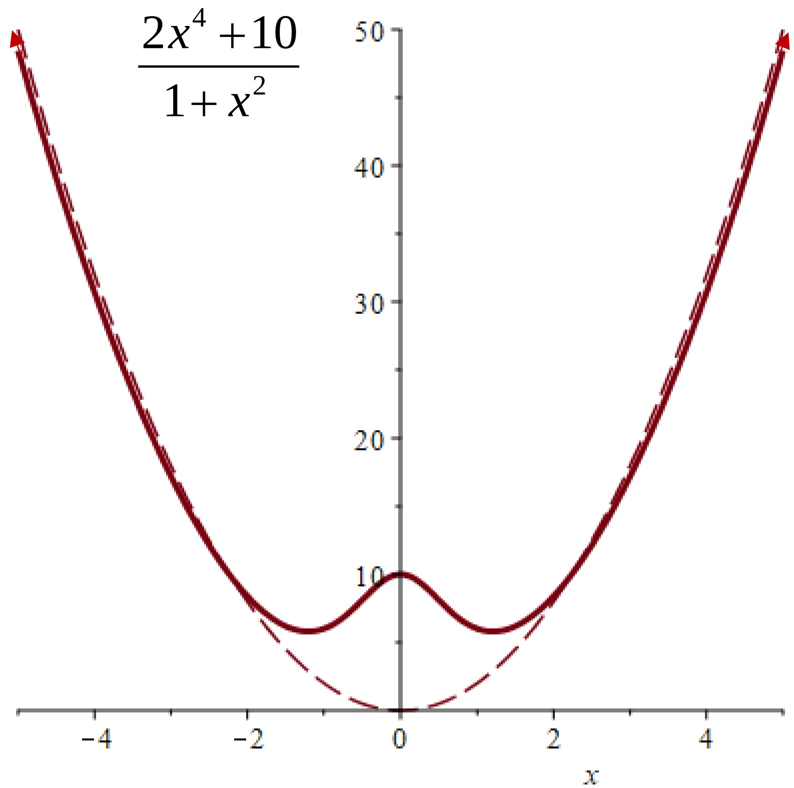
If the degree of $p(x)$ is equal to the degree of $q(x)$, then

$$y = \frac{\text{leading coefficient of } p(x)}{\text{leading coefficient of } q(x)}$$ **is the horizontal asymptote.**

$$\frac{2x^2}{1+x^2}$$

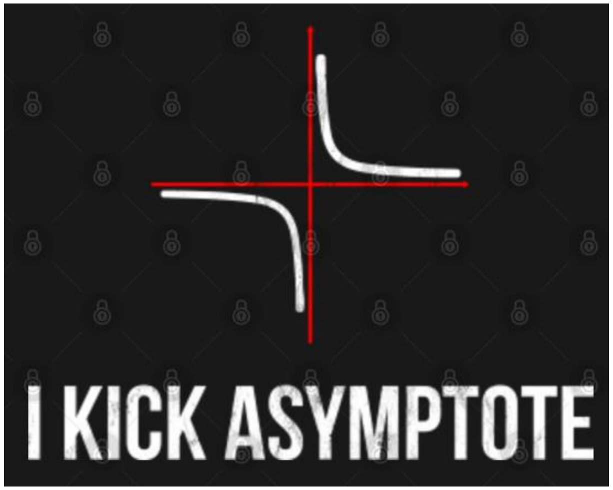


If the degree of $p(x)$ is greater than the degree of $q(x)$, then there is no horizontal asymptote. The end behavior is like the polynomial function formed from the ratio of the leading terms of $p(x)$ and $q(x)$.



The end behavior is like $\frac{2x^4}{x^2} = 2x^2$.

Examples: Determine the horizontal asymptote/end behavior of the following rational functions.



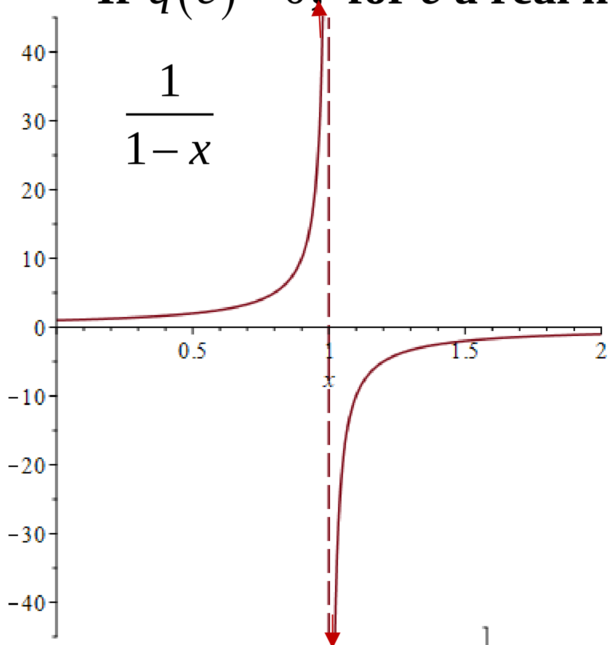
1. $f\left(x\right)=\dfrac{2x^3+x-1}{3x^3+x^2}$

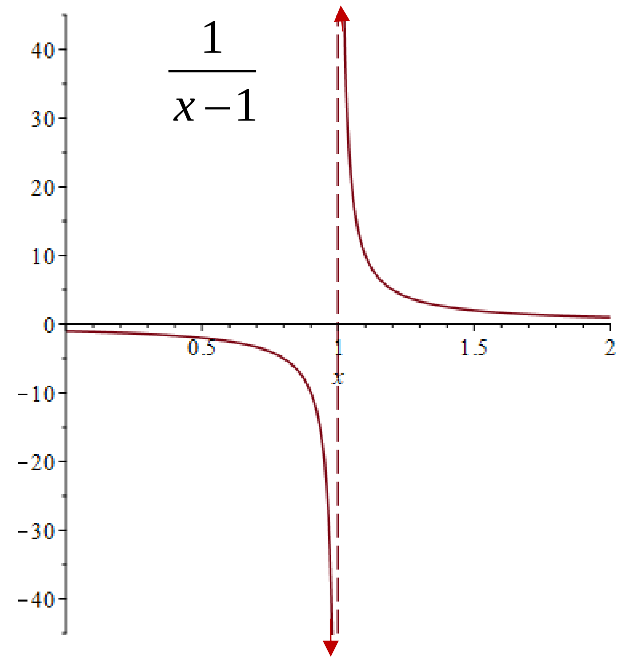
2. $f\left(x\right)=\dfrac{2x^2+x-1}{3x^3+x^2}$

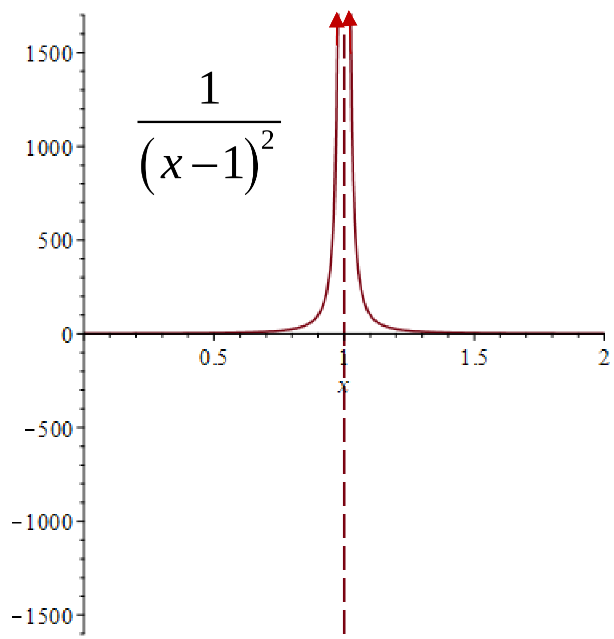
3. $f\left(x\right)=\dfrac{2x^5+x-1}{3x^3+x^2}$

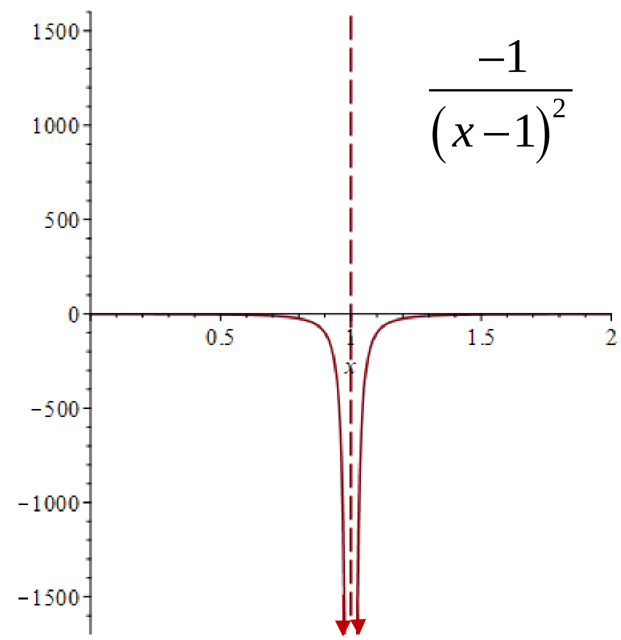
<u>Vertical Asymptote:</u> (Upward/downward explosions)

If $q(c)=0$, for *c* a real number, then $x=c$ is a vertical asymptote.





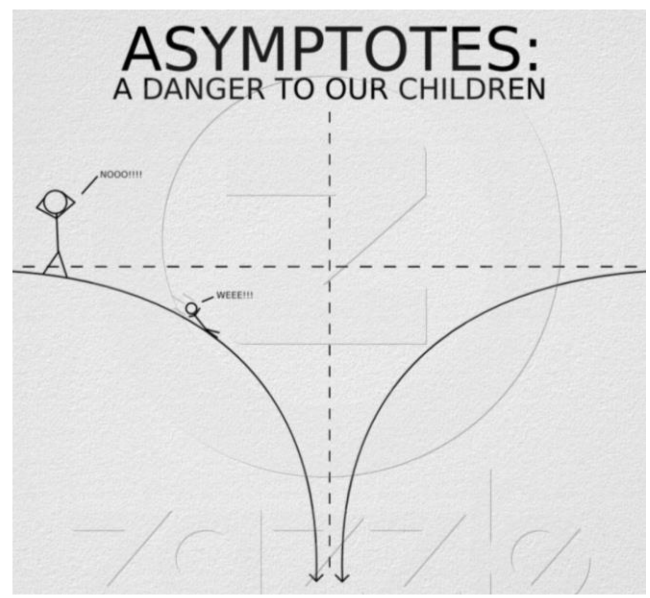




Examples: Determine the vertical asymptotes/explosive behavior of the following rational functions.

1. $f\left(x\right)=\dfrac{3x+5}{x-6}$

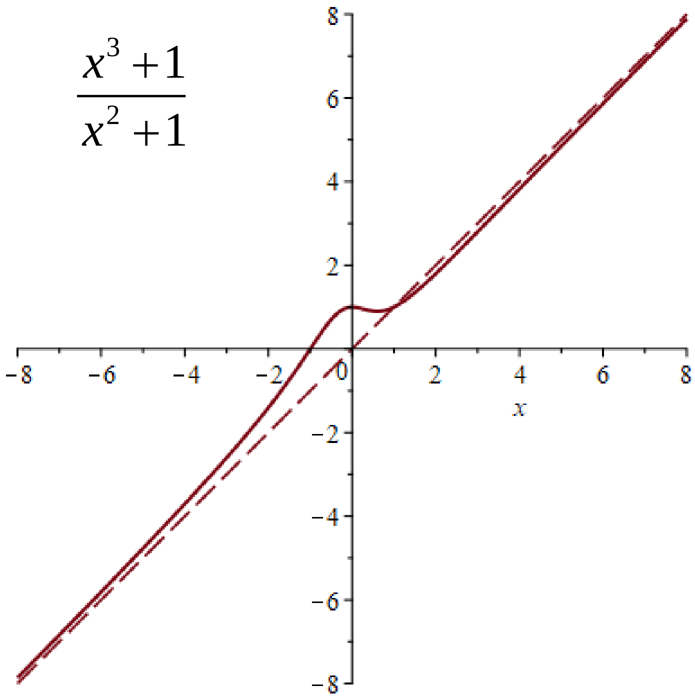
2. $f\left(x\right)=\dfrac{x^3+1}{x^2-5x-14}$

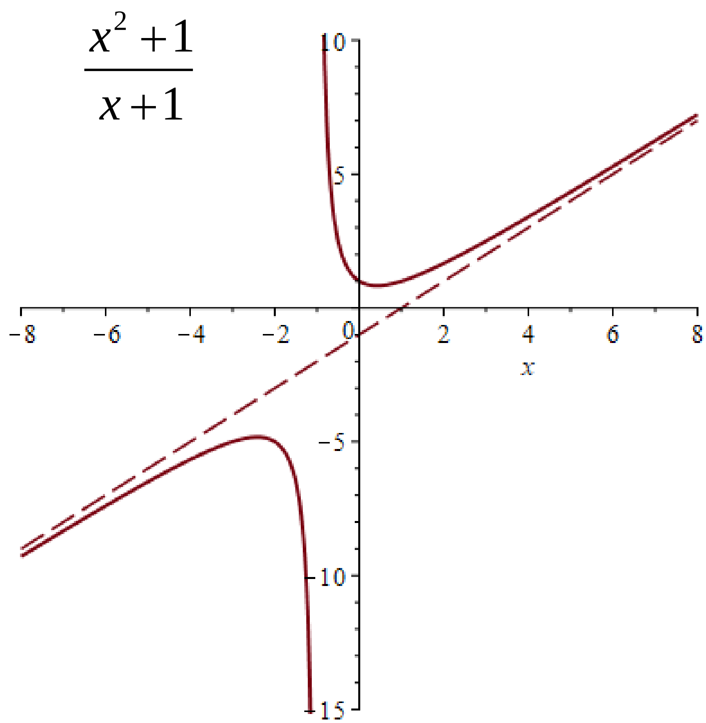


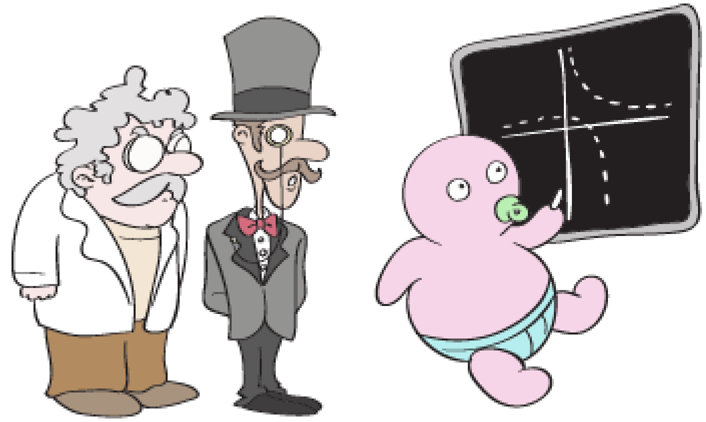
3. $f\left(x\right)=\dfrac{x^3}{x^4-1}$

Slant/Oblique Asymptote:

If the degree of $p(x)$ is 1 more than the degree of $q(x)$, then the rational function will have a slant/oblique asymptote. The equation of the slant asymptote can be determined using polynomial long division.







Examples:

1. $f\left(x\right)=\dfrac{2x^2+x-1}{x+2}$

2. $f\left(x\right)=\dfrac{x^3+8}{x^2-5x+6}$