Sign Charts for Rational Functions:

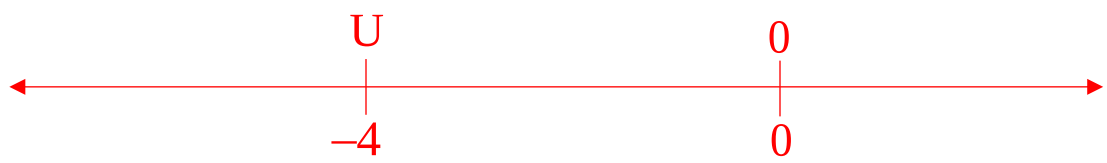
The behavior of a rational function in the vicinity of its vertical asymptotes can be determined by the sign of the function values. To make a sign chart for a rational function, draw a number line and locate the real zeros of the numerator and label them with a *0*, since the function value is zero there. Locate the real zeros of the denominator, and label them with a *u*, since the function is undefined at these values. Use what you know about the graphs of polynomial functions to determine the sign of the rational function on the intervals in between and on the edges.

Examples:

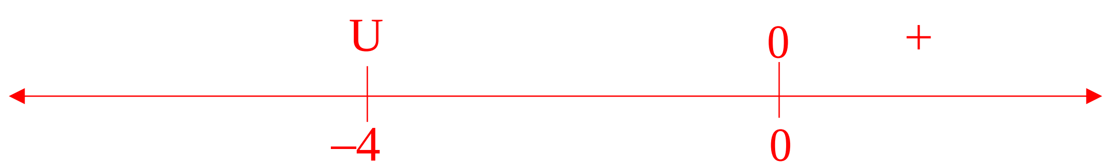
The arrows indicate the end behavior of the numerator and denominator, so we can quickly know the sign of the rational function left and right. In the middle we can use the fact that an odd exponent signals a sign change, while an even exponent signals no sign change.

1. $f(x) = \dfrac{3x}{x+4}$

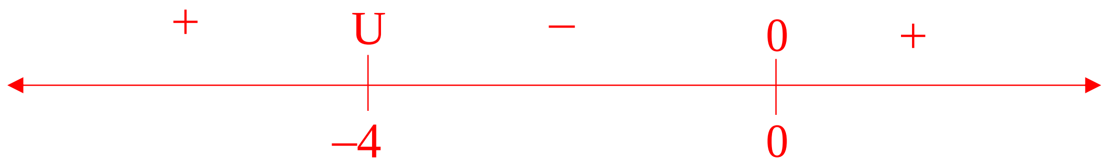
The first thing to do is label the zeros of the numerator and denominator on a number line.



We can either choose a value larger than zero, a value between -4 and zero, and a value smaller than -4 to figure out the sign pattern of the rational function, or we can use what we know about polynomials. $3x$ and $x+4$ are eventually positive to the right(end behavior), so their ratio must be positive to the right.

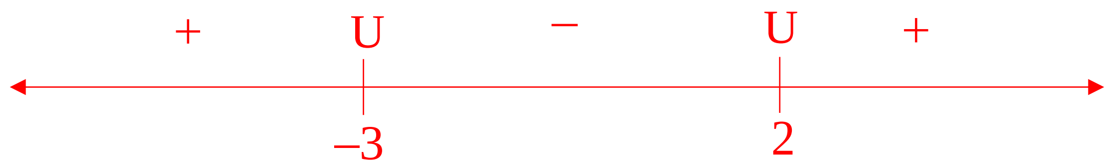


Polynomial functions change sign(cross the *x*-axis) for odd exponents and don't change sign for even exponents. The exponent on *x* is odd, and the exponent on *x* + 4 is odd. This means that as we cross zero, the sign must change, and as we cross -4, the sign must change.

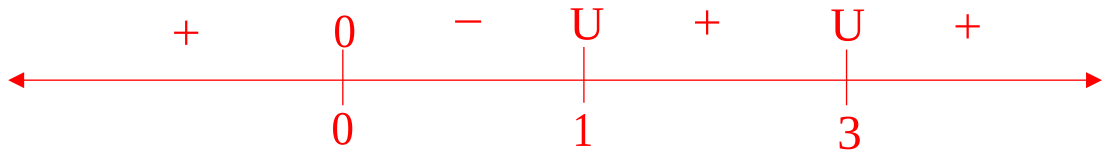


2. $f\left(x\right)=\dfrac{6}{x^2+x-6}$

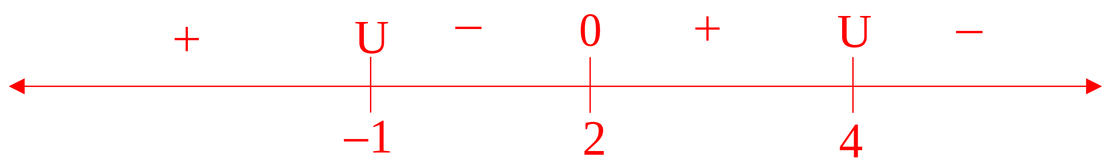
$$\frac{6}{x^2+x-6}=\frac{6}{\left(x+3\right)\left(x-2\right)}$$



3. $f(x)=\dfrac{3x}{(x-1)(x-3)^2}$



4. $f(x)=\dfrac{2-x}{(x+1)(x-4)}$



Sketching graphs of rational functions:

1. Find and draw the horizontal, vertical, and slant asymptotes as dashed lines.

2. Label the *x*-intercepts.

3. Label the *y*-intercept.

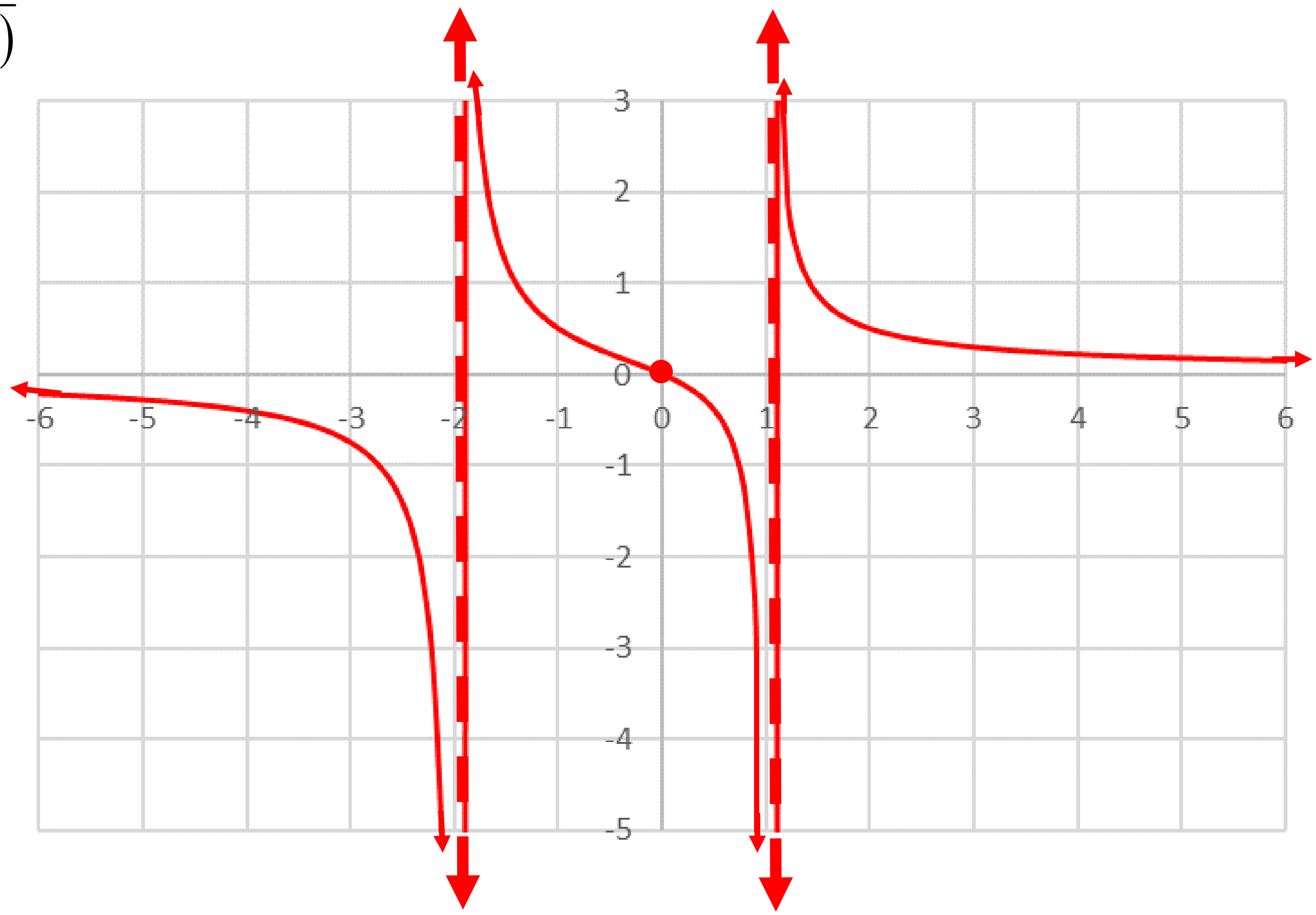
4. Create the sign chart.

5. Connect all the dots in a reasonable manner using the sign chart as a guide.

The goal in sketching the graph of a rational function is to plot as few points as possible(the *x* and *y* intercepts), and use the *x*-intercept behavior and the behavior near the asymptotes to capture the qualitative behavior of the graph. Don't worry about the vertical scaling, just produce a reasonably complete graph.

Examples:

1. $f(x) = \dfrac{x}{(x-1)(x+2)}$



Asymptotes:

H.A.: $y = 0$

V.A.: $x = 1, x = -2$

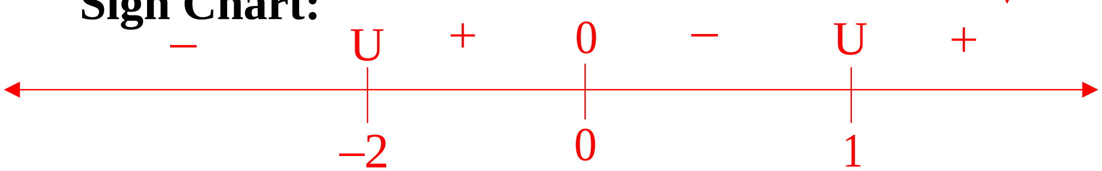
S.A.: none

Intercepts:

***x*-int(s):** 0

***y*-int:** 0

Sign Chart:



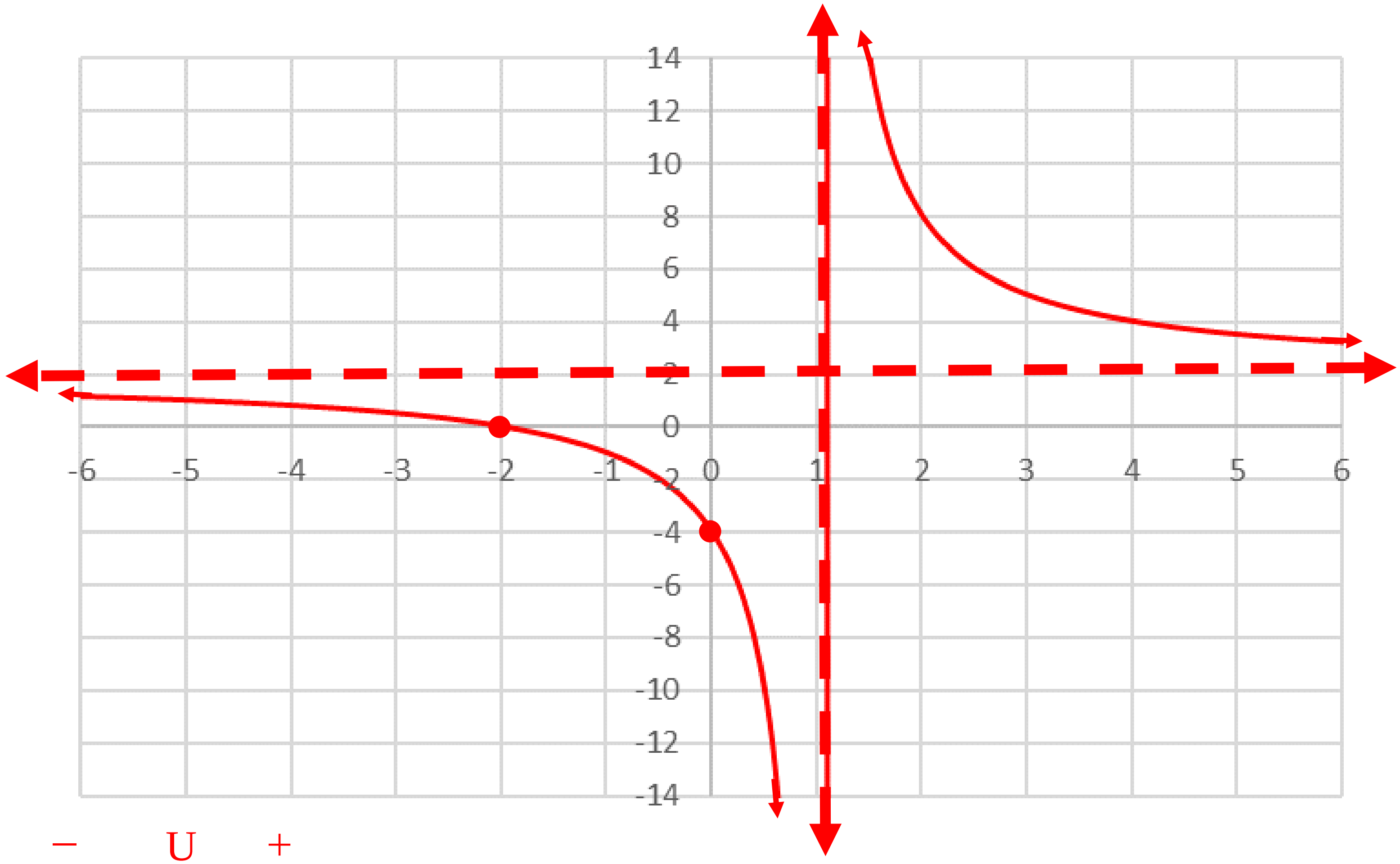
2. $f(x)=\dfrac{2x+4}{x-1}=\dfrac{2(x+2)}{x-1}$

Asymptotes:

H.A.: $y=2$

V.A.: $x=1$

S.A.: none

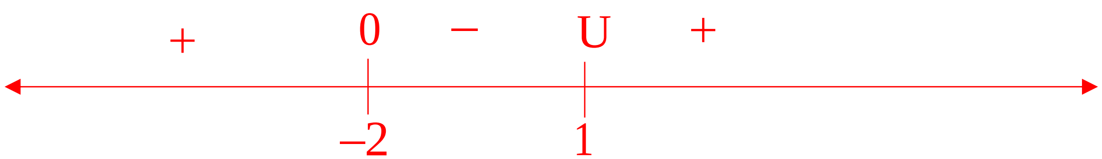


Intercepts:

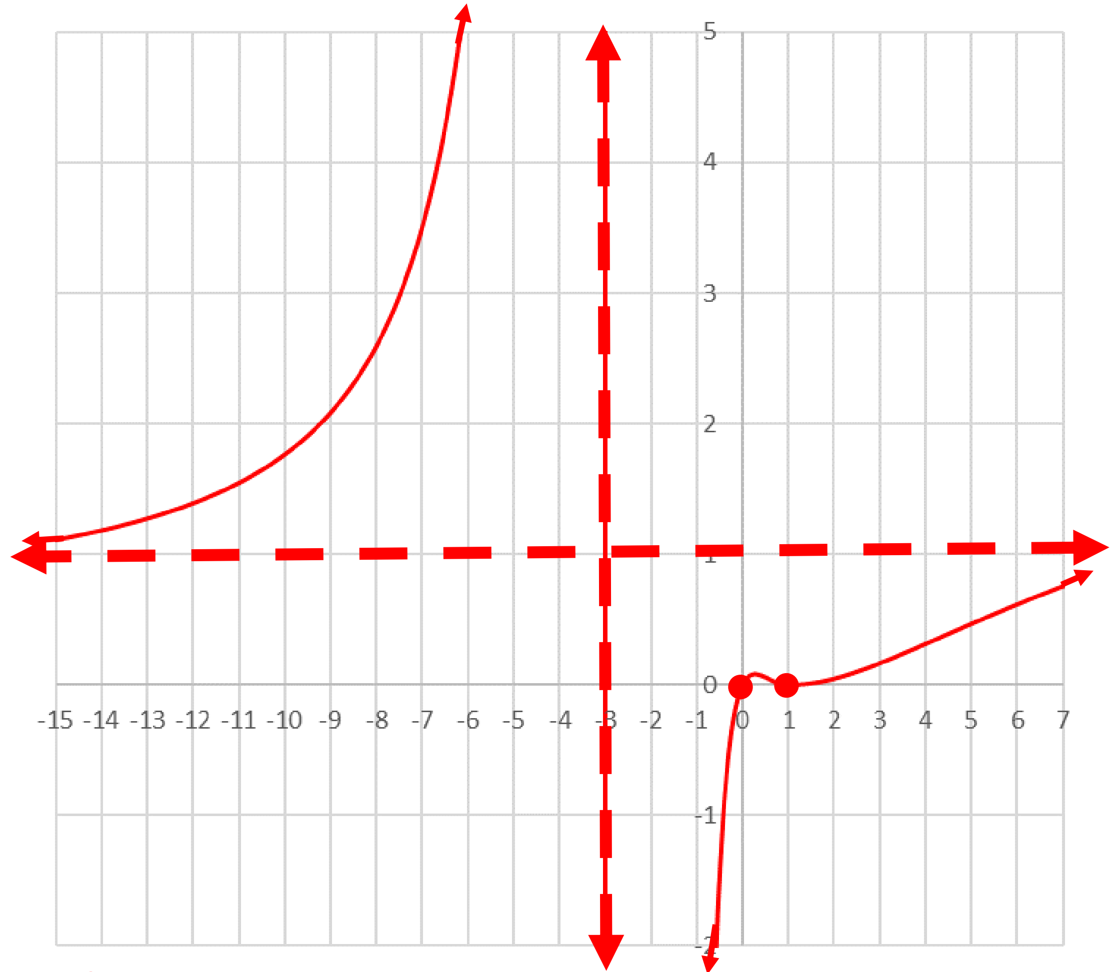
***x*-int(s):** -2

***y*-int:** -4

Sign Chart:



3. $f(x)=\dfrac{x(x-1)^2}{(x+3)^3}$



Asymptotes:

H.A.: $y=1$

V.A.: $x=-3$

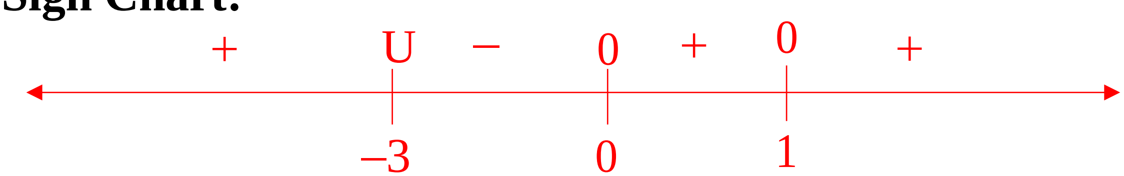
S.A.: none

Intercepts:

***x*-int(s):** 0,1

***y*-int:** 0

Sign Chart:



4. $f(x)=\dfrac{x^2+5x+6}{x-3}=\dfrac{(x+2)(x+3)}{x-3}$

Asymptotes:

H.A.: none

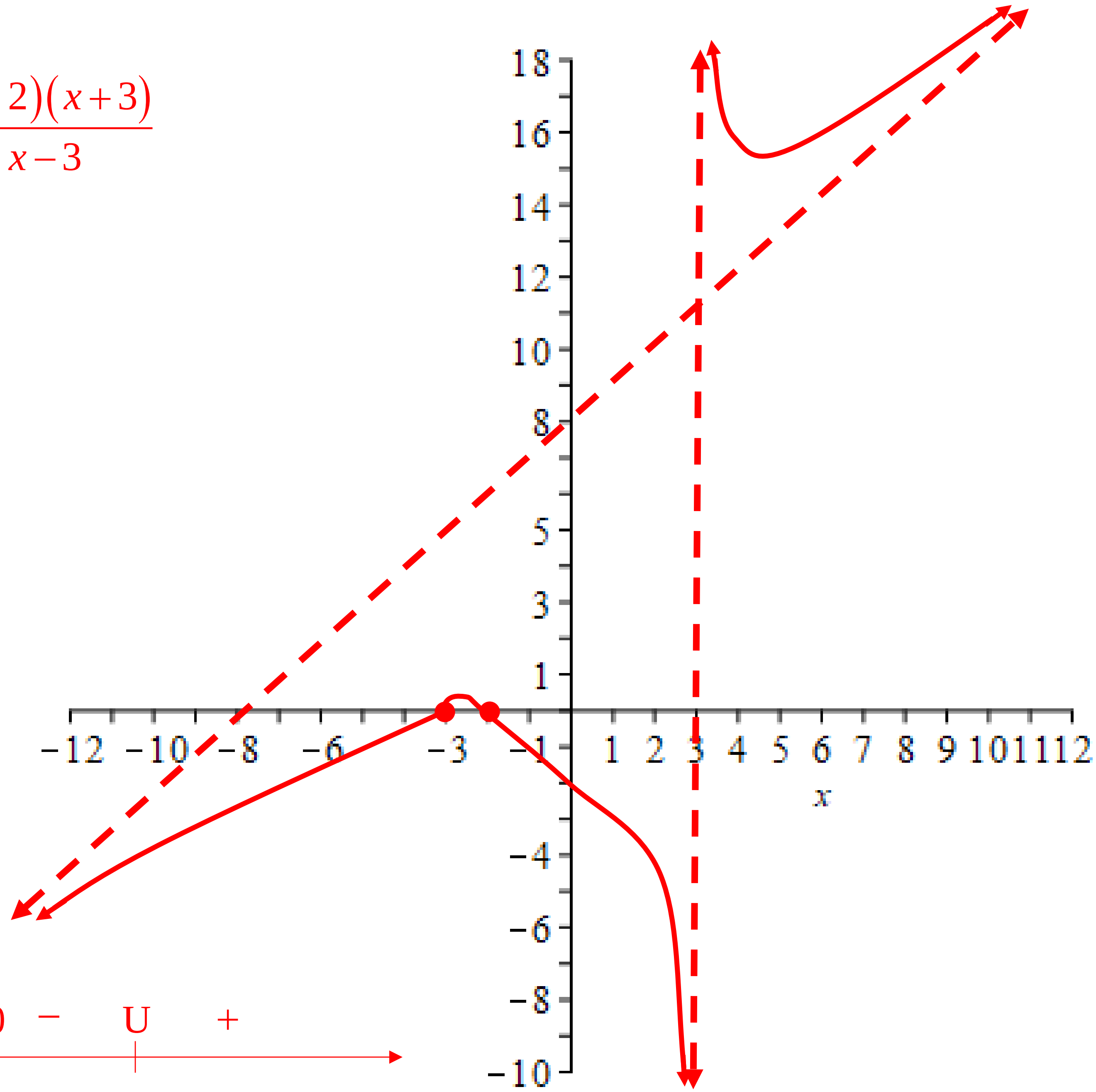
V.A.: $x=3$

S.A.: $y=x+8$

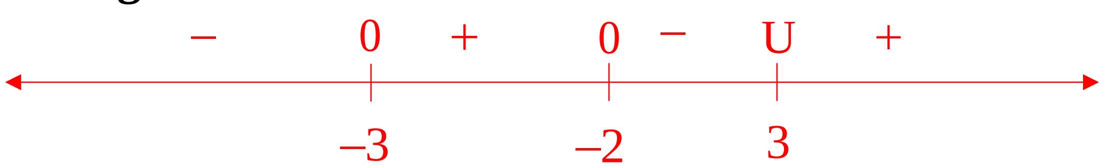
Intercepts:

***x*-int(s):** $-2, -3$

***y*-int:** -2



Sign Chart:



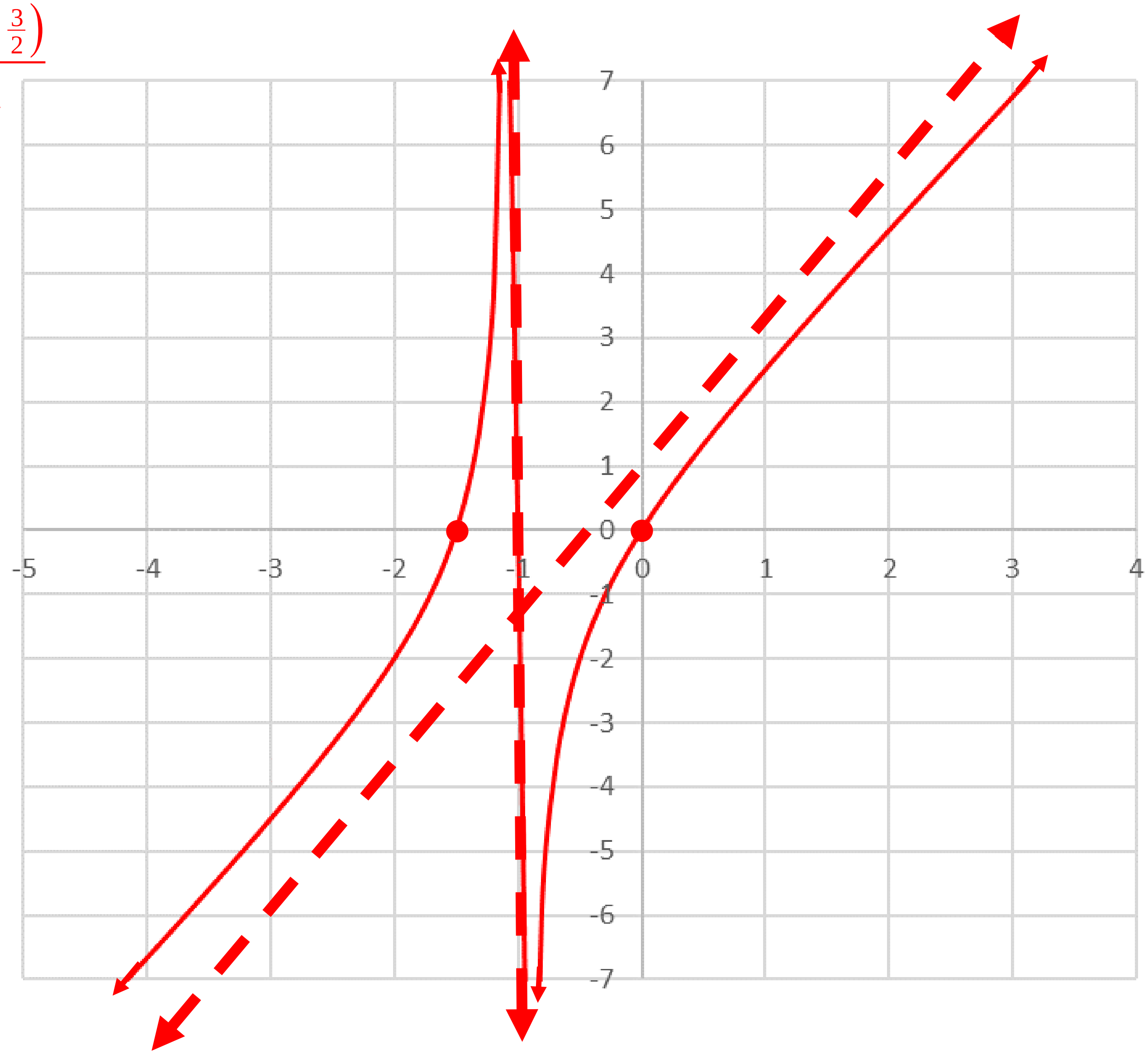
5. $f(x) = \dfrac{2x^2 + 3x}{x+1} = \dfrac{2x\left(x + \frac{3}{2}\right)}{x+1}$

Asymptotes:

H.A.: none

V.A.: $x = -1$

S.A.: $y = 2x + 1$

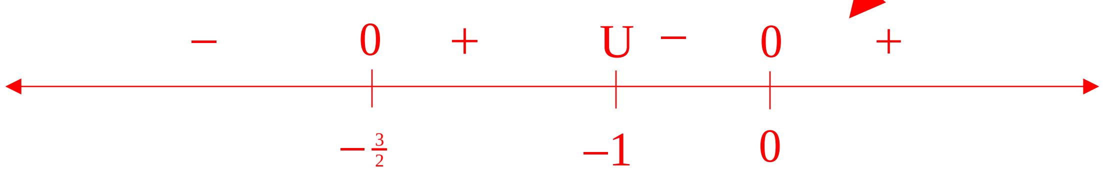


Intercepts:

***x*-int(s):** $0, -\frac{3}{2}$

***y*-int:** 0

Sign Chart:



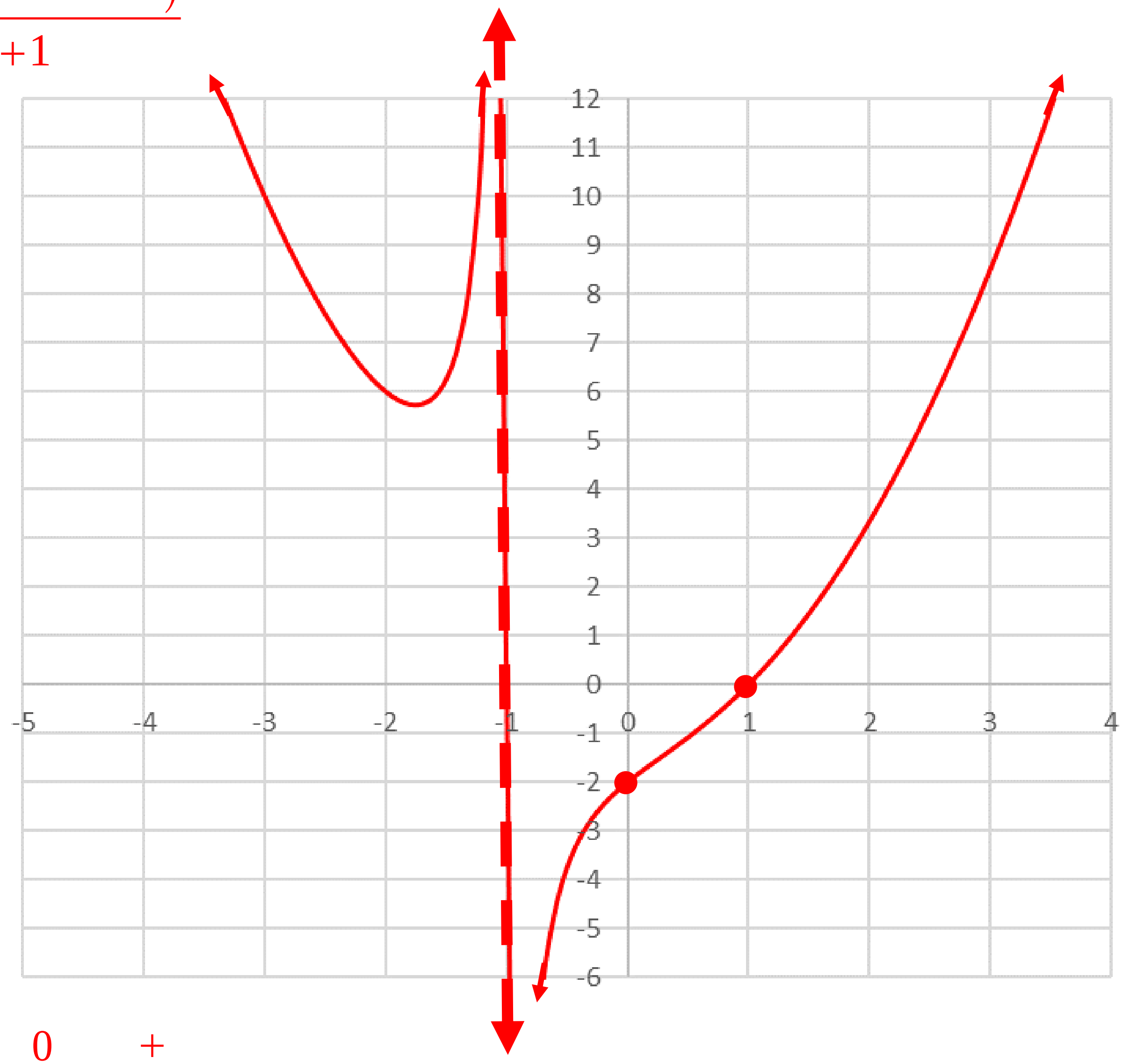
6. $f(x)=\dfrac{x^3+x^2-2}{x+1}=\dfrac{(x-1)(x^2+2x+2)}{x+1}$

Asymptotes:

H.A.: none, end-behavior of x^2

V.A.: $x=-1$

S.A.: none



Intercepts:

***x*-int(s):** 1

***y*-int:** -2

Sign Chart: