Finding Probabilities Using Counting Techniques:

In the case of the equally likely assumption, $P(E)=\dfrac{n(E)}{n(S)}$**.**

Examples:

1. Allen, Bob, Carl, and David will be seated in a row of 4 chairs at random.

a) How many different ways can they be seated?

b) How many different ways can Carl be seated in the first chair and Allen in the fourth chair?

c) What's the probability that Carl will be seated in the first chair and Allen in the fourth chair?

2. A group consists of 4 men and 5 women. Three people will be selected at random to attend a conference.

a) How many different selections of 3 people from the group are possible?

b) What's the probability that the 3 people selected are all women?

c) What's the probability that the 3 people selected are all men?

d) What's the probability that the 3 people selected will consist of 2 women and 1 man?

3. Three cards will be randomly selected from a 52-card deck without replacement.

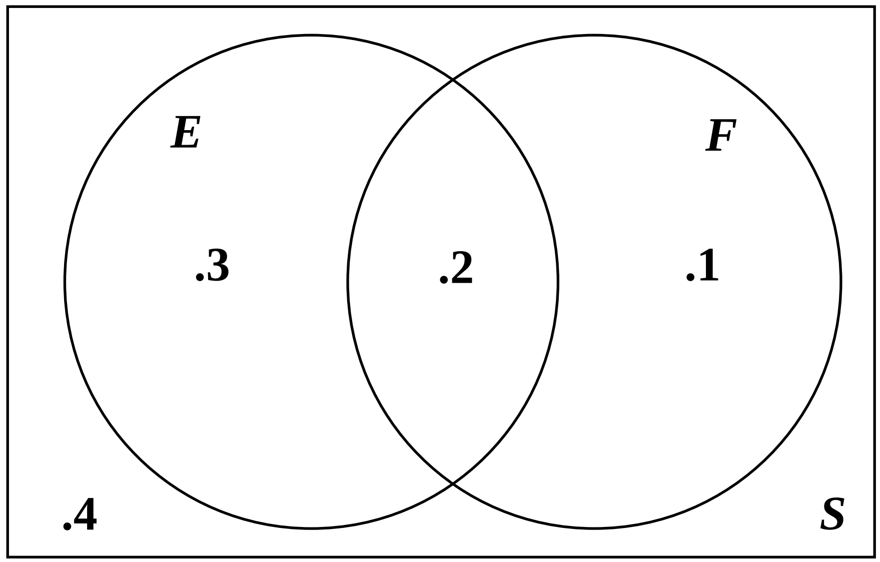
a) What's the probability that it will consist of all hearts?

b) What's the probability that it will consist of exactly 2 aces?

c) What's the probability that it will consist of 2 aces and a king?

Probability Diagrams and Probability Formulas:

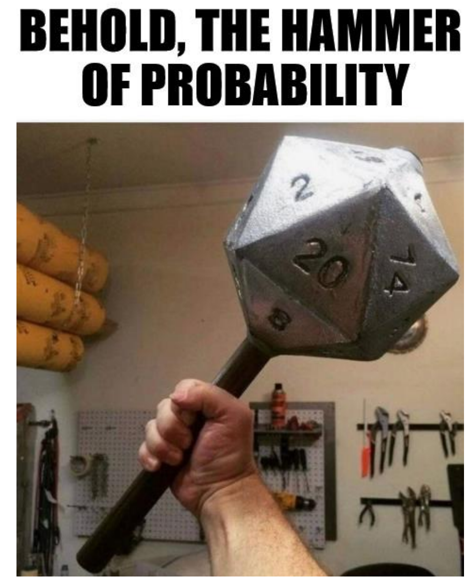
A probability diagram is like a specialized Venn Diagram in which the probabilities of different events in the sample space are labelled.



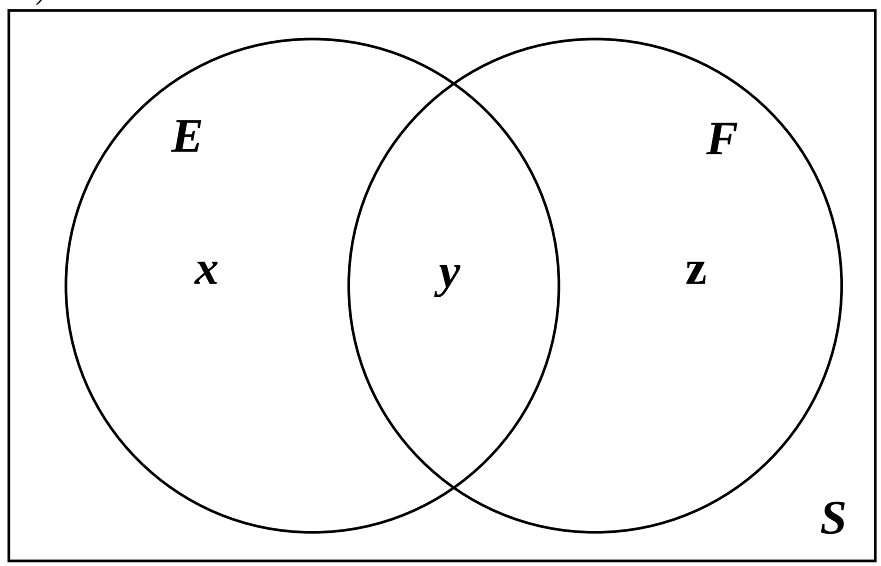
The sum of all the probabilities that make up all the disjoint regions of *S* must be 1.

$P(E)=$ $P(F)=$ $P(E \text{ and } F)=P(E\cap F)=$

$P(E \text{ or } F)=P(E\cup F)=$ $P(\text{not } E)=P(E')=$ $P\left((E\cup F)'\right)=$



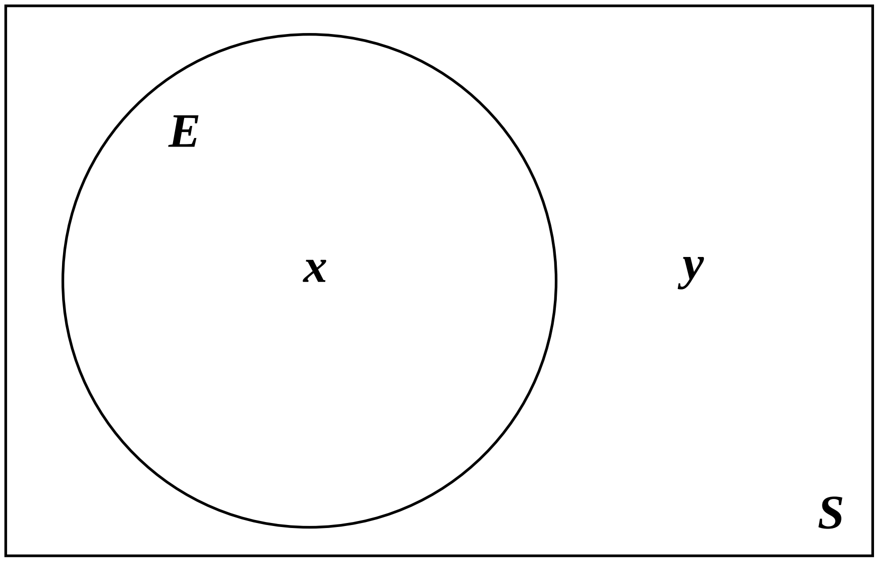
A formula for $P(E \cup F)$**:**



$$P(E \cup F) = x + y + z = x + y + y + z - y$$
$$= (x + y) + (y + z) - y$$
$$= P(E) + P(F) - P(E \cap F)$$

If $E \cap F = \phi$**, then it's impossible for both events to occur, and they are called mutually exclusive events. In this case,** $P(E \cup F) = P(E) + P(F)$

Formulas involving $P(E')$:



$$1 = P(S) = x + y = P(E) + P(E')$$

So

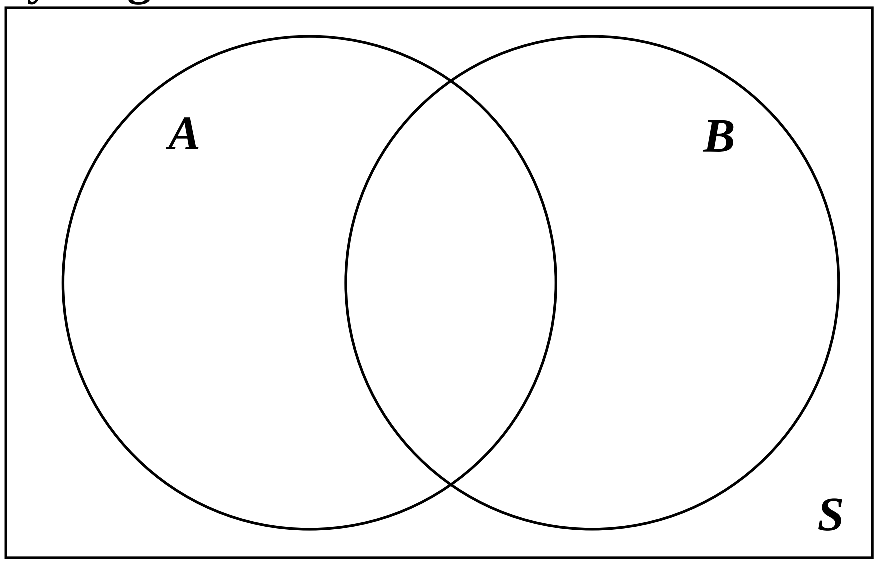
$$P(E) = 1 - P(E')$$

$$P(E') = 1 - P(E)$$

<u>Out of context example:</u>

Suppose $P(A)=.7$**,** $P(B)=.4$**, and** $P(A\cap B)=.3$**.**

Complete the probability diagram:



Behold, my other hammer of probability

Find

$P(A\cup B)$

$P(A')$

$P\left((A\cup B)'\right)$

$P(A\cap B')$

$P(B\cap A')$

$P\left((A\cap B)'\right)$

$P(A\cup A')$

In context examples:

1. A card is randomly selected from a standard 52-card deck.

$P(\text{ace or a heart})$

$P(\text{ace or king})$

$P(\text{face card or a club})$

2. A survey of North Harris students had the following results.

	Pepperoni	Sausage	Mushroom	Total
Freshman	**25**	**15**	**5**	**45**
Sophomore	**30**	**20**	**5**	**55**
Total	**55**	**35**	**10**	**100**

A student from the survey is *selected at random*.

a) $P(\text{sausage or mushroom})$

b) $P(\text{freshman or pepperoni})$