Finding Probabilities Using Counting Techniques:

In the case of the equally likely assumption, $P(E)=\dfrac{n(E)}{n(S)}$**.**

Examples:

1. Allen, Bob, Carl, and David will be seated in a row of 4 chairs at random.

a) How many different ways can they be seated?

$$n(S)=\underset{\text{first}}{\underline{4}}\cdot\underset{\text{second}}{\underline{3}}\cdot\underset{\text{third}}{\underline{2}}\cdot\underset{\text{fourth}}{\underline{1}}=\boxed{24}$$

b) How many different ways can Carl be seated in the first chair and Allen in the fourth chair?

$$n(E)=\underset{\text{first}}{\underline{1}}\cdot\underset{\text{second}}{\underline{2}}\cdot\underset{\text{third}}{\underline{1}}\cdot\underset{\text{fourth}}{\underline{1}}=\boxed{2}$$

c) What's the probability that Carl will be seated in the first chair and Allen in the fourth chair?

$$P(E)=\frac{n(E)}{n(S)}=\frac{2}{24}=\frac{1}{12}$$

2. A group consists of 4 men and 5 women. Three people will be selected at random to attend a conference.

a) How many different selections of 3 people from the group are possible?

$${}_9C_3 = \boxed{84}$$

b) What's the probability that the 3 people selected are all women?

$$\frac{{}_5C_3}{{}_9C_3} = \frac{10}{84} = \frac{5}{42}$$

c) What's the probability that the 3 people selected are all men?

$$\frac{{}_4C_3}{{}_9C_3} = \frac{4}{84} = \frac{1}{21}$$

d) What's the probability that the 3 people selected will consist of 2 women and 1 man?

$$\frac{{}_5C_2 \cdot {}_4C_1}{{}_9C_3} = \frac{10 \cdot 4}{84} = \frac{40}{84} = \frac{10}{21}$$

3. Three cards will be randomly selected from a 52-card deck without replacement.

a) What's the probability that it will consist of all hearts?

$$\frac{{}_{13}C_3}{{}_{52}C_3} = \frac{286}{22{,}100} = \frac{11}{850}$$

b) What's the probability that it will consist of exactly 2 aces?

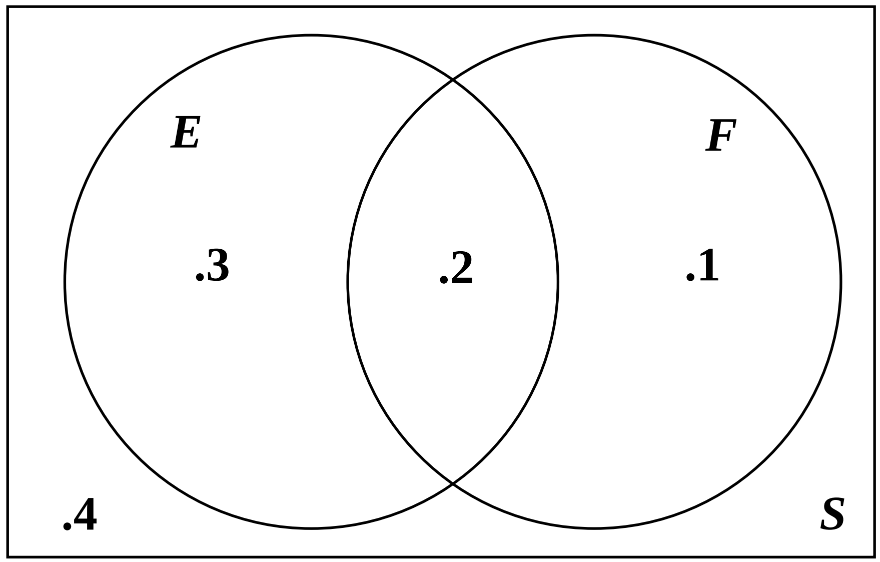
$$\frac{{}_{4}C_2 \cdot {}_{48}C_1}{{}_{52}C_3} = \frac{6 \cdot 48}{22{,}100} = \frac{288}{22{,}100} = \frac{72}{5{,}525}$$

c) What's the probability that it will consist of 2 aces and a king?

$$\frac{{}_{4}C_2 \cdot {}_{4}C_1}{{}_{52}C_3} = \frac{6 \cdot 4}{22{,}100} = \frac{24}{22{,}100} = \frac{6}{5{,}525}$$

<u>Probability Diagrams and Probability Formulas:</u>

A probability diagram is like a specialized Venn Diagram in which the probabilities of different events in the sample space are labelled.



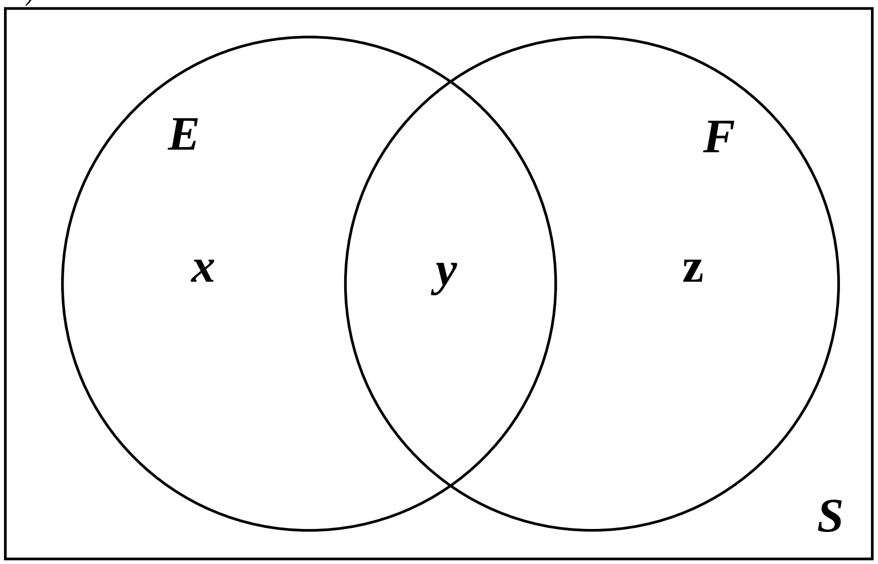
The sum of all the probabilities that make up all the disjoint regions of *S* must be 1.

$P(E) = .3 + .2 = \boxed{.5}$ $\qquad$ $P(F) = .2 + .1 = \boxed{.3}$ $\qquad$ $P(E \text{ and } F) = P(E \cap F) = .2$

$P(E \text{ or } F) = P(E \cup F) = .3 + .2 + .1 = \boxed{.6}$ $\qquad$ $P(\text{not } E) = P(E') = .1 + .4 = \boxed{.5}$

$P\left((E \cup F)'\right) = .4$

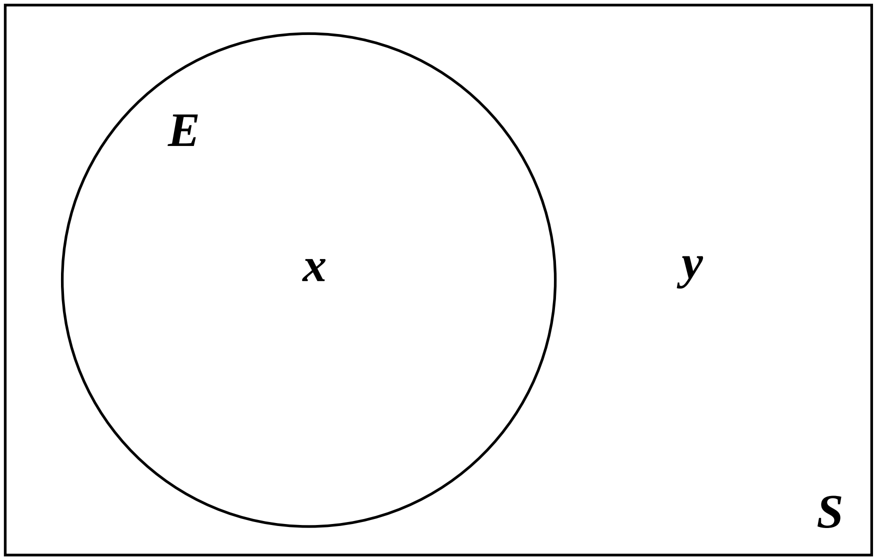
A formula for $P(E \cup F)$**:**



$$P(E \cup F) = x + y + z = x + y + y + z - y$$
$$= (x + y) + (y + z) - y$$
$$= P(E) + P(F) - P(E \cap F)$$

If $E \cap F = \phi$**, then it's impossible for both events to occur, and they are called mutually exclusive events. In this case,** $P(E \cup F) = P(E) + P(F)$

Formulas involving $P(E')$:



$$1 = P(S) = x + y = P(E) + P(E')$$

So

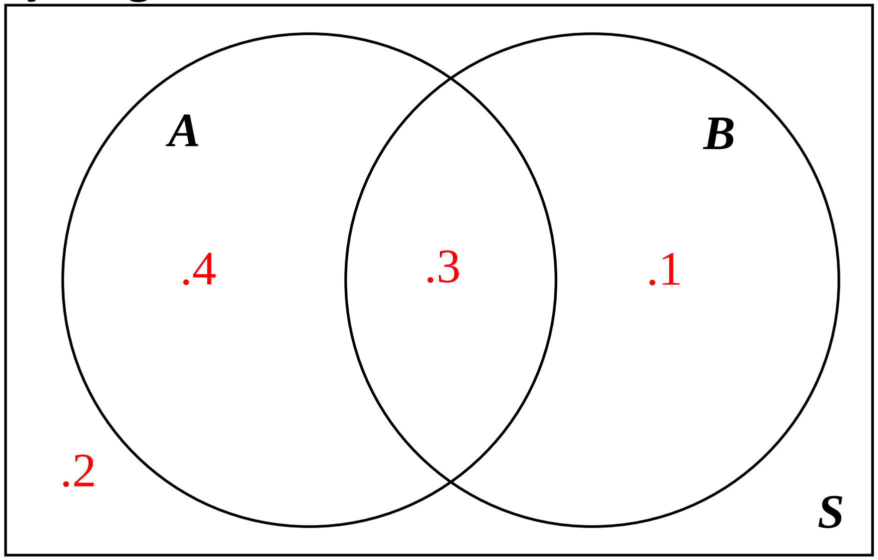
$$P(E) = 1 - P(E')$$

$$P(E') = 1 - P(E)$$

<u>Out of context example:</u>

Suppose $P(A) = .7$**,** $P(B) = .4$**, and** $P(A \cap B) = .3$**.**

Complete the probability diagram:



Find

$P(A \cup B)$

.4 + .3 + .1 = .8

Or

.7 + .4 - .3 = .8

$P(A')$

.1 + .2 = .3

Or

1 - .7 = .3

$P\left((A \cup B)'\right)$

.2

$P(A \cap B')$

.4

$P(B \cap A')$

.1

$P\left((A \cap B)'\right)$

.4 + .1 + .2 = .7 Or 1 - .3 = .7

$P(A \cup A') = 1$

<u>In context examples:</u>

1. A card is randomly selected from a standard 52-card deck.

$$P(\text{ace or a heart})$$

$$= P(\text{ace}) + P(\text{heart}) - P(\text{ace and heart})$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \boxed{\frac{4}{13}}$$

$$P(\text{ace or king})$$

$$= P(\text{ace}) + P(\text{king})$$

$$= \frac{4}{52} + \frac{4}{52} = \frac{8}{52} = \boxed{\frac{2}{13}}$$

$$P(\text{face card or a club})$$

$$= P(\text{face card}) + P(\text{club}) - P(\text{face card and club})$$

$$= \frac{12}{52} + \frac{13}{52} - \frac{3}{52} = \frac{22}{52} = \boxed{\frac{11}{26}}$$

2. A survey of North Harris students had the following results.

	Pepperoni	Sausage	Mushroom	Total
Freshman	**25**	**15**	**5**	**45**
Sophomore	**30**	**20**	**5**	**55**
Total	**55**	**35**	**10**	**100**

A student from the survey is *selected at random*.

a) $P(\text{sausage or mushroom})$

$$= P(\text{sausage}) + P(\text{mushroom})$$

$$= \frac{35}{100} + \frac{10}{100} = \frac{45}{100} = \boxed{\frac{9}{20}}$$

b) $P(\text{freshman or pepperoni})$

$$= P(\text{freshman}) + P(\text{pepperoni}) - P(\text{freshman and pepperoni})$$

$$= \frac{45}{100} + \frac{55}{100} - \frac{25}{100} = \frac{75}{100} = \boxed{\frac{3}{4}}$$