Polynomial and Rational Inequalities:

To solve a polynomial or rational inequality, just do the following steps:

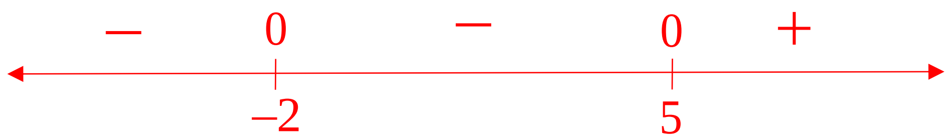
1. Get zero on one side.

2. Create the sign chart for the other side.

3. Read the solution from the sign chart.

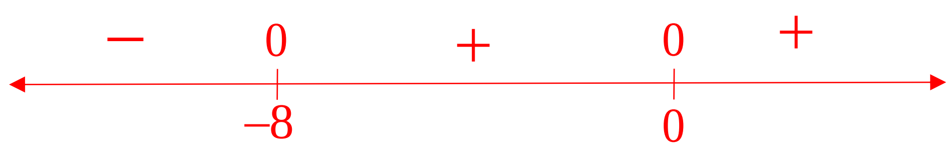
Examples:

1. $(x-5)(x+2)^2 > 0$



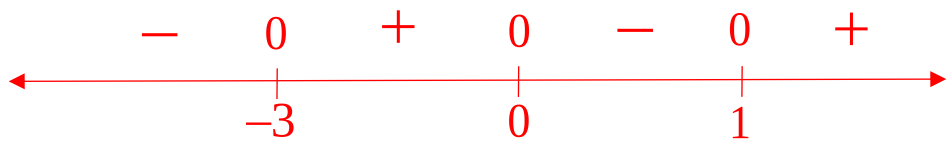
The solution in interval notation is $(5,\infty)$.

2. $x^3 + 8x^2 < 0 \Rightarrow x^2(x+8) < 0$



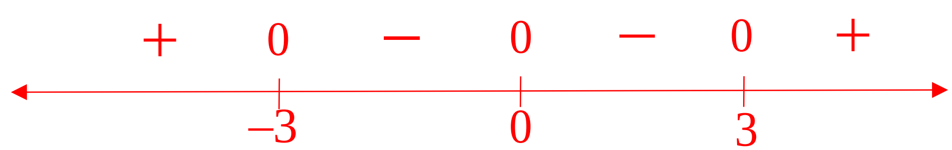
$(-\infty,-8)$

3. $x^3 + 2x^2 - 3x \geq 0 \Rightarrow x(x+3)(x-1) \geq 0$



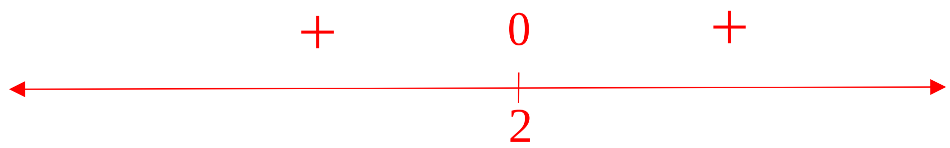
$$[-3,0] \cup [1,\infty)$$

4. $x^4 \leq 9x^2 \Rightarrow x^4 - 9x^2 \leq 0 \Rightarrow x^2(x-3)(x+3) \leq 0$



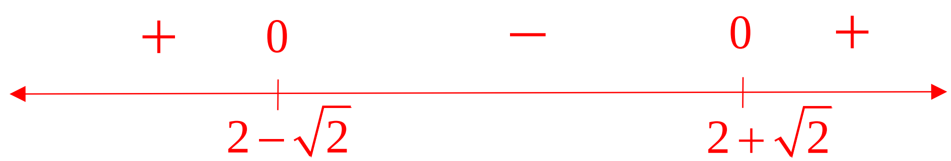
$$[-3,3]$$

5. $x^2+4\le 4x \Rightarrow x^2-4x+4\le 0 \Rightarrow (x-2)^2\le 0$



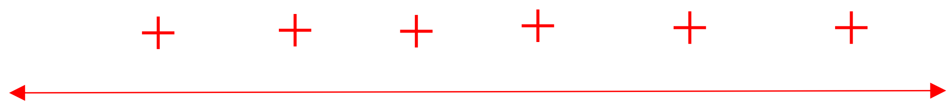
$\{2\}$

6. $x^2-4x\le -2 \Rightarrow x^2-4x+2\le 0$



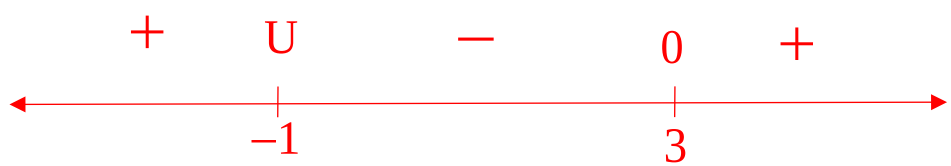
$\left[2-\sqrt{2},2+\sqrt{2}\right]$

7. $\underbrace{x^2-4x+5}_{\text{no real zeros}}<0$



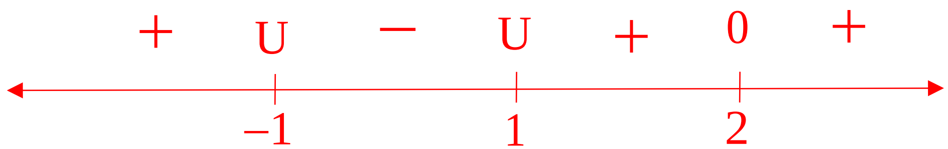
no solution

8. $\dfrac{x-3}{x+1}\geq 0$



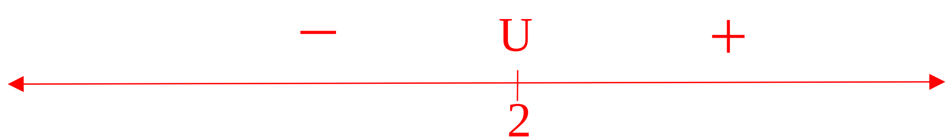
The solution in interval notation is $(-\infty,-1)\cup[3,\infty)$.

9. $\dfrac{(x-2)^2}{x^2-1} \geq 0 \Rightarrow \dfrac{(x-2)^2}{(x-1)(x+1)} \geq 0$



$(-\infty,-1) \cup (1,\infty)$

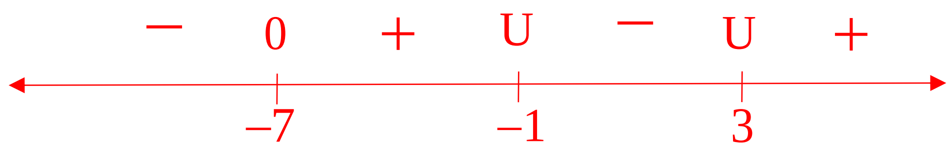
10. $\dfrac{x+4}{x-2} \leq 1 \Rightarrow \dfrac{x+4}{x-2} - \dfrac{x-2}{x-2} \leq 0 \Rightarrow \dfrac{6}{x-2} \leq 0$



$(-\infty,2)$

11. $\dfrac{5}{x-3} > \dfrac{3}{x+1} \Rightarrow \dfrac{5}{x-3} - \dfrac{3}{x+1} > 0 \Rightarrow \dfrac{5(x+1)}{(x-3)(x+1)} - \dfrac{3(x-3)}{(x+1)(x-3)} > 0$

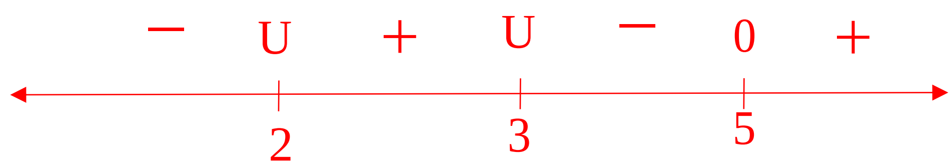
$$\Rightarrow \frac{2(x+7)}{(x-3)(x+1)} > 0$$



$(-7,-1)\cup(3,\infty)$

12. $\dfrac{1}{x-2} < \dfrac{2}{3x-9} \Rightarrow \dfrac{1}{x-2} - \dfrac{2}{3x-9} < 0 \Rightarrow \dfrac{3x-9}{(x-2)(3x-9)} - \dfrac{2(x-2)}{(x-2)(3x-9)} < 0$

$$\Rightarrow \frac{x-5}{3(x-2)(x-3)} < 0$$



$(-\infty,2)\cup(3,5)$