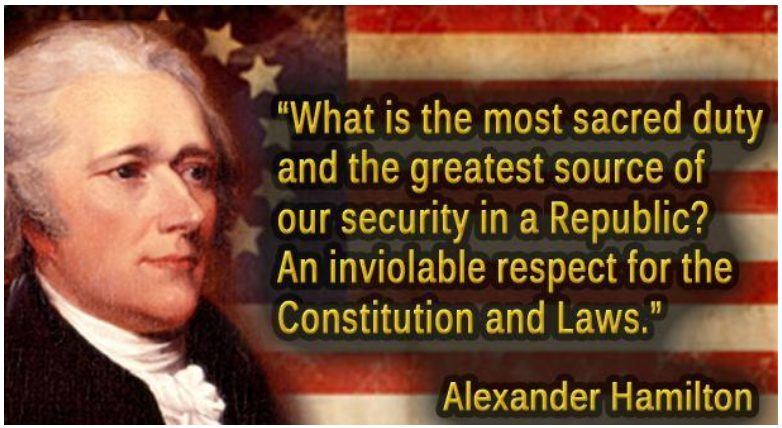
Fairness and Paradoxes in Apportionment Methods:

Fairness: **The only fairness criterion for apportionment methods is that a group should either receive its lower quota or its upper quota of items. An apportionment method that guarantees this is said to satisfy quota.**

The only apportionment method that we've discussed that always satisfies quota is Hamilton's Method.

So Hamilton's Method is the only fair apportionment method.



Paradoxes: **In the history of using Hamilton's Method, some strange things have occurred that have been called apportionment paradoxes.**

The Alabama Paradox:

The total number of items to be allocated is increased, but a group is apportioned fewer items.

Example:

State	A	B	C	D	Total
Population	504	456	404	61	1,425

57 items are to be apportioned. The standard divisor is $\frac{1{,}425}{57} = 25$**.**

State	A	B	C	D	Total
Population	504	456	404	61	1,425
Standard Quota	20.16	18.24	16.16	2.44	57
Lower Quota	20	18	16	2	56
Hamilton Apportionment					

Suppose that the number of items to be allocated increases from 57 to 58.

The new standard divisor is $\frac{1{,}425}{58} = 24.568...$**.**

State	A	B	C	D	Total
Population	504	456	404	61	1,425
Standard Quota	20.51	18.56	16.44	2.48	58
Lower Quota	20	18	16	2	56
Hamilton Apportionment					

Did the Alabama Paradox occur?

The Population Paradox:

One group loses items to another group even though the population of the first group grew at a higher percentage than the second group.

Example:

State	A	B	C	Total
Original Population	53	99	224	376
New Population	68	125	257	450
Percent Change in Population	$\frac{68-53}{53}=.2830$ $=28.30\%$	$\frac{125-99}{99}=.2626$ $=26.26\%$	$\frac{257-224}{224}=.1473$ $=14.73\%$	

The number of items to be allocated is 24.

Original Hamilton:

The standard divisor is $\frac{376}{24} = 15.\overline{6}$**.**

State	A	B	C	Total
Original Population	53	99	224	376
Standard Quota	3.38	6.32	14.30	24
Lower Quota	3	6	14	23
Hamilton Apportionment				24

New Hamilton:

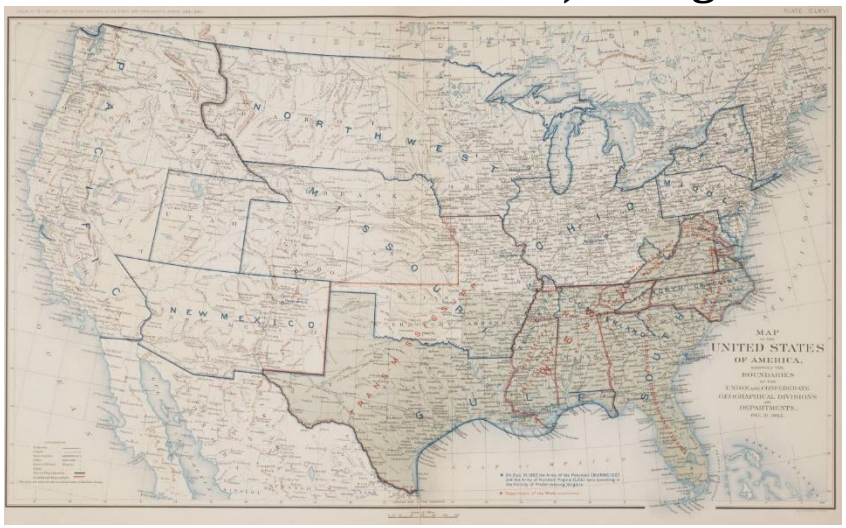
The standard divisor is $\frac{450}{24} = 18.75$**.**

State	A	B	C	Total
New Population	68	125	257	450
Standard Quota	3.63	6.67	13.71	24
Lower Quota	3	6	13	22
Hamilton Apportionment				24

Did the Population Paradox occur?

The New States Paradox:

The addition of a new group(and a proportional number of new items) changes the apportionments of the original groups.



Example:

State	A	B	C	Total
Population	209	769	2,022	3,000

60 items are to be allocated. The standard divisor is $\frac{3{,}000}{60} = 50$**.**

State	A	B	C	Total
Population	209	769	2,022	3,000
Standard Quota	4.18	15.38	40.44	60
Lower Quota	4	15	40	59
Original Hamilton Apportionment				60

A new state, D, is added with a population of 260, and a proportional number of items, 5, is also added.

State	A	B	C	D	Total
Population	209	769	2,022	260	3,260

The new standard divisor is $\frac{3260}{65} = 50.1538....$

State	A	B	C	D	Total
Population	209	769	2,022	260	3,260
Standard Quota	4.17	15.33	40.32	5.18	65
Lower Quota	4	15	40	5	64
New Hamilton Apportionment					65

Did the New States Paradox occur?

One last Example:

A country has five states, and its house of representatives is apportioned by the Hamilton Method. Complete the apportionments for house sizes of 81 and 82.

State	A	B	C	D	E	Total
Population	5,576,330	1,387,342	3,334,241	7,512,860	310,968	18,121,741
Original Quotas	24.925	6.201	14.903	33.581	1.390	81
Original Hamilton						81
New Quotas	25.233	6.278	15.087	33.995	1.407	82
New Hamilton						82

Did the Alabama Paradox Occur?

Balinski and Young's Imposibility Theorem(1980):

Any apportionment method that doesn't violate the quota rule must produce paradoxes, and any apportionment rule that doesn't produce paradoxes must violate the quota rule.