Review of Radical Equations:

1. Isolate a radical on one side of the equation.

2. Raise both sides to a power that eliminates the isolated radical.

3. Repeat steps 1 and 2, if needed.

4. Solve the new radical-free equation.

5. Check your solution(s) in the original equation.

Examples:

1. $\sqrt{5x+2}=7$

Square both sides to eliminate the isolated radical. $5x+2=49 \Rightarrow 5x=47 \Rightarrow x=\boxed{\dfrac{47}{5}}$

2. $\sqrt{3x}-4=6$

Add 4 on both sides to isolate the radical. Then square both sides to eliminate it.

$\left(\sqrt{3x}\right)^2=(10)^2 \Rightarrow 3x=100 \Rightarrow x=\boxed{\dfrac{100}{3}}$

3. $\sqrt[3]{x}=-2$

Cube both sides to eliminate the isolated radical. $x=\boxed{-8}$

4. $\sqrt{x-3}=-4$

Square roots must be nonnegative, so this equation has $\boxed{\text{no solution}}$.

5. $x-5=\sqrt{x+7}$

Square both sides to eliminate the isolated radical. Solve the resulting equation by factoring.

$$x^2-10x+25=x+7 \Rightarrow x^2-11x+18=0 \Rightarrow (x-2)(x-9)=0 \Rightarrow x=2,9$$

Checking: 2 doesn't satisfy the original equation, $-3 \neq \sqrt{9}$, but $4=\sqrt{16}$, so $x=\boxed{9}$.

6. $\sqrt{2x+7}-2=x$

Add 2 to both sides to isolate the radical. Square both sides to eliminate the isolated radical. Solve the resulting equation by factoring.

$$\sqrt{2x+7}=x+2 \Rightarrow \left(\sqrt{2x+7}\right)^2=(x+2)^2 \Rightarrow 2x+7=x^2+4x+4 \Rightarrow x^2+2x-3=0$$

$$\Rightarrow (x+3)(x-1)=0 \Rightarrow x=-3,1$$

Checking: -3 doesn't satisfy the original equation, $\sqrt{1}-2 \neq -3$, but $\sqrt{9}-2=1$, so $x=\boxed{1}$.

7. $\sqrt{5x-3}=\sqrt{2x+3}$

Square both sides to eliminate both radicals.

$5x-3=2x+3 \Rightarrow 3x=6 \Rightarrow x=2$, and Checking: $\sqrt{7}=\sqrt{7}$, so $x=\boxed{2}$

8. $\sqrt{x-9}+\sqrt{x}=1$

Subtract $\sqrt{x}$ on both sides to isolate $\sqrt{x-9}$, and then square both sides to eliminate it.

Isolate the $\sqrt{x}$, and then square both sides to eliminate it.

$\left(\sqrt{x-9}\right)^2=\left(1-\sqrt{x}\right)^2 \Rightarrow x-9=x-2\sqrt{x}+1 \Rightarrow 2\sqrt{x}=10 \Rightarrow \sqrt{x}=5 \Rightarrow x=25$

Checking: 25 doesn't satisfy the original equation, $\sqrt{16}+\sqrt{25}\neq 1$, so $\boxed{\text{no solution}}$.

9. $\sqrt{4x-3} = 2 + \sqrt{2x-5}$

Square both sides to eliminate $\sqrt{4x-3}$. Isolate the $2\sqrt{2x-5}$ and square both sides to eliminate it.

$4x-3 = 2x-5+4\sqrt{2x-5}+4 \Rightarrow 2x-2 = 4\sqrt{2x-5} \Rightarrow x-1 = 2\sqrt{2x-5}$

$\Rightarrow (x-1)^2 = \left(2\sqrt{2x-5}\right)^2 \Rightarrow x^2-2x+1 = 4(2x-5) \Rightarrow x^2-10x+21=0$

$\Rightarrow (x-3)(x-7)=0 \Rightarrow x=3,7$ and Checking: $\sqrt{9} = 2+\sqrt{1},\ \sqrt{25} = 2+\sqrt{9},$ so $x = \boxed{3,7}$

<u>Quadratic-like radical equations:</u>

1. $x^{\frac{2}{3}} + x^{\frac{1}{3}} - 6 = 0$

$$\left(x^{\frac{1}{3}}\right)^2 + x^{\frac{1}{3}} - 6 = 0 \textbf{ or } \left(\sqrt[3]{x}\right)^2 + \sqrt[3]{x} - 6 = 0$$

Factor into $\left(\sqrt[3]{x} + 3\right)\left(\sqrt[3]{x} - 2\right) = 0 \Rightarrow \sqrt[3]{x} = -3$ or $\sqrt[3]{x} = 2 \Rightarrow x = \boxed{-27, 8}$

2. $x^{\frac{1}{2}} - 4x^{\frac{1}{4}} + 3 = 0$

$$\left(x^{\frac{1}{4}}\right)^2 - 4x^{\frac{1}{4}} + 3 = 0 \textbf{ or } \left(\sqrt[4]{x}\right)^2 - 4\sqrt[4]{x} + 3 = 0$$

Factor into $\left(\sqrt[4]{x} - 3\right)\left(\sqrt[4]{x} - 1\right) = 0 \Rightarrow \sqrt[4]{x} = 3$ or $\sqrt[4]{x} = 1 \Rightarrow x = \boxed{81, 1}$

<u>Absolute Value Equations:</u>

The absolute value of a number is its distance from zero on the number line.

1. For $a > 0$**,**

$|something| = a$ **means that** $something = \pm a$**.**

2. For $a < 0$**,**

$|something| = a$ **means that the equation has no solution.**

3. $|something| = 0$ **means that** $something = 0$**.**

Examples:

1. $|x| = 5$

$x = \boxed{\pm 5}$

2. $|x| = -9$

The absolute value can't be negative, so $\boxed{\text{no solution}}$

3. $|3x - 2| = 7$

$$3x - 2 = \pm 7 \Rightarrow 3x = 2 \pm 7 \Rightarrow x = \frac{2 \pm 7}{3} \Rightarrow x = \boxed{3, -\frac{5}{3}}$$

4. $|x|-2=6$

Add 2 on both sides to isolate the absolute value. $|x|=8 \Rightarrow x=\boxed{\pm 8}$

5. $|6x|+8=32$

Subtract 8 on both sides to isolate the absolute value. $|6x|=24 \Rightarrow 6x=\pm 24 \Rightarrow x=\boxed{\pm 4}$

6. $\left|\frac{4-5x}{6}\right|=7$

$$\Rightarrow \frac{4-5x}{6}=\pm 7 \Rightarrow 4-5x=\pm 42 \Rightarrow -5x=-4\pm 42 \Rightarrow x=\frac{-4\pm 42}{-5} \Rightarrow x=\boxed{\frac{46}{5},-\frac{38}{5}}$$

7. $2|2x-7|+11=25$

Subtract 11 and divide by 2 to isolate the absolute value.

$$|2x-7|=7 \Rightarrow 2x-7=\pm 7 \Rightarrow 2x=7\pm 7 \Rightarrow x=\frac{7\pm 7}{2} \Rightarrow x=\boxed{0,7}$$

8. $|x-6|=-8$

An absolute value must be nonnegative, so there is $\boxed{\text{no solution}}$.

9. $|2x-8|=|x+3|$ **{If $|a|=|b|$, then either $a=b$ or $a=-b$.}**

$$2x-8=x+3 \text{ or } 2x-8=-(x+3) \Rightarrow x=11 \text{ or } 3x=5 \Rightarrow x=\boxed{11,\frac{5}{3}}$$

10. $|x-15|=|x+8|$

$$x-15=x+8 \text{ or } x-15=-(x+8) \Rightarrow -15=8 \text{ or } 2x=7 \Rightarrow x=\boxed{\frac{7}{2}}$$