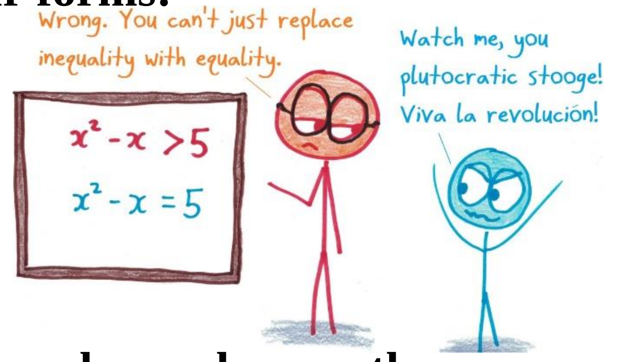
<u>Linear Inequalities in Two Variables:</u>

Any inequality that can be expressed in one of the following four forms:



$$Ax + By > C$$
$$Ax + By < C$$
$$Ax + By \geq C$$
$$Ax + By \leq C$$

where *A*, *B*, and *C* are numbers with *A* and *B* not both zero, and *x* and *y* are the variables, is called a linear inequality in two variables.

Linear inequalities in two variables will always have infinitely many solution pairs, so instead of trying to list all of them out, we'll represent them as shaded regions in the *xy*-plane.

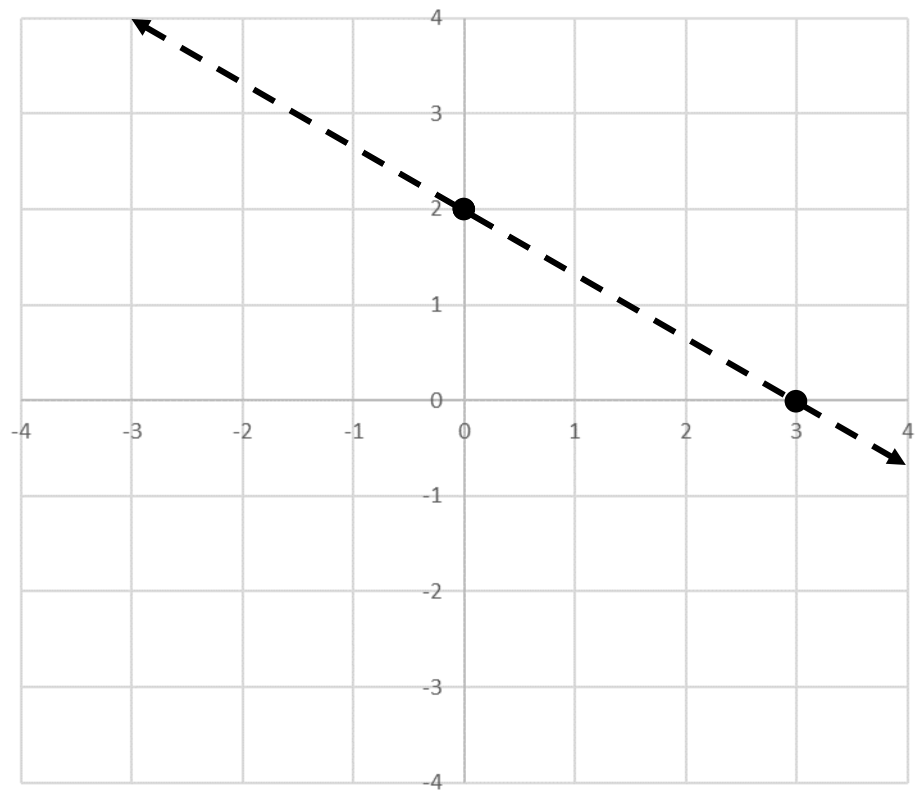
The procedure for doing this is fairly straight-forward. First you graph the line $Ax + By = C$. If the original inequality includes equality, then this line is graphed as a solid line to indicate its inclusion in the solution region. If the original inequality does not included equality, then this line is graphed as a dashed line to indicate its exclusion from the solution region. One side of the line will be shaded to indicate its

inclusion in the solution region, and the determination of the proper side is done by selecting a test point that's not on the previously graphed line. The most popular test point is $(0,0)$, unless it happens to lie on the previously graphed line, in which case you'll have to choose a different test point. If the coordinates of the test point satisfy the original inequality, then that's the side to shade. If the coordinates of the test point don't satisfy the original inequality, then you need to shade the other side of the line.

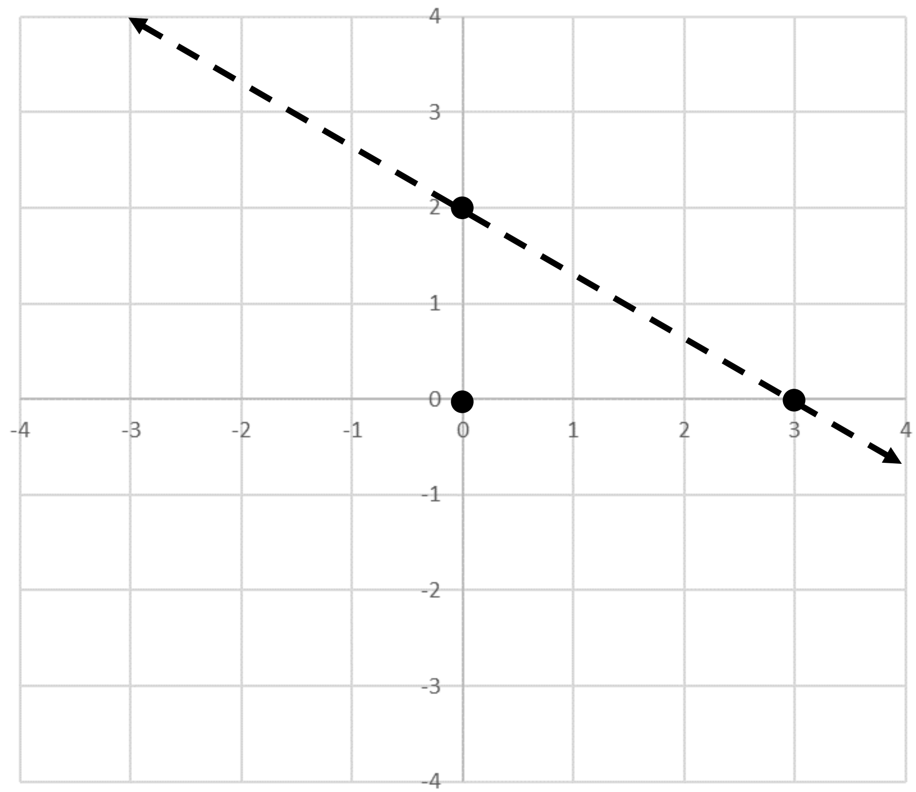
Examples:

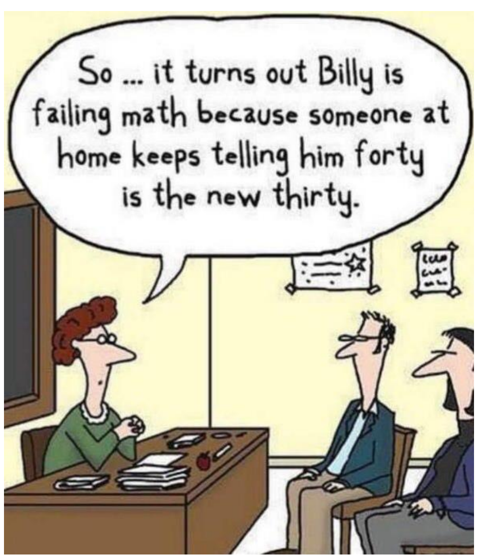
1. Solve the linear inequality $2x+3y<6$.

First, we'll graph the line $2x+3y=6$ using the *x* and *y* intercepts of 3 and 2, and make it dashed.



Now I'll choose a test point that is not on the line, $(0,0)$, and plug its coordinates into the original inequality to see if it makes it true or false.

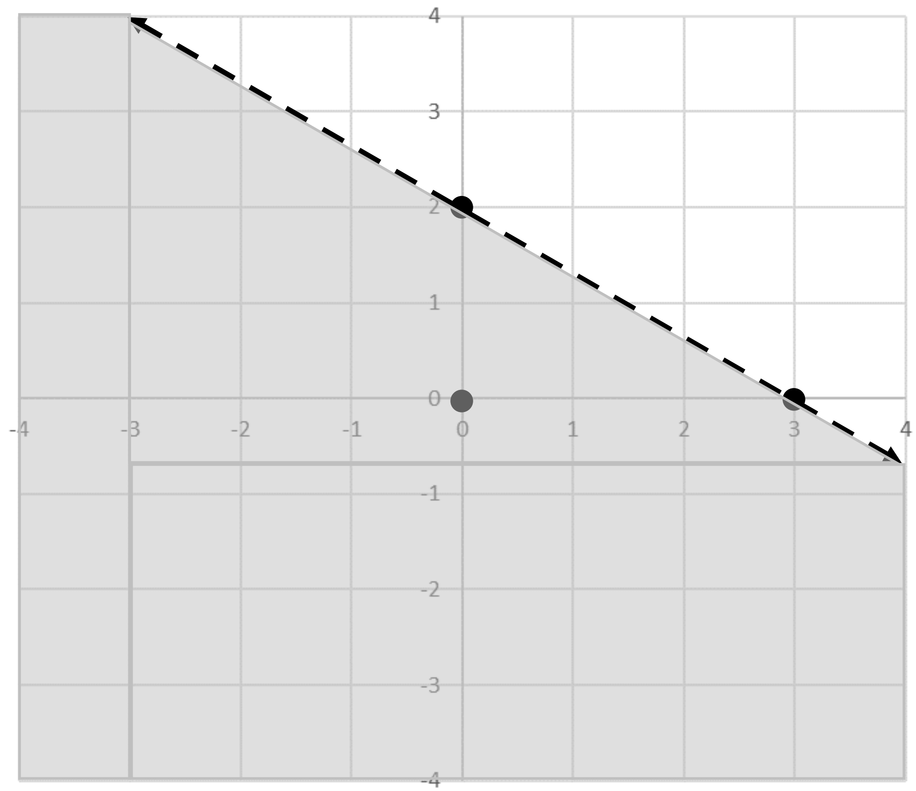




$$2(0)+3(0)\overset{?}{<}6$$

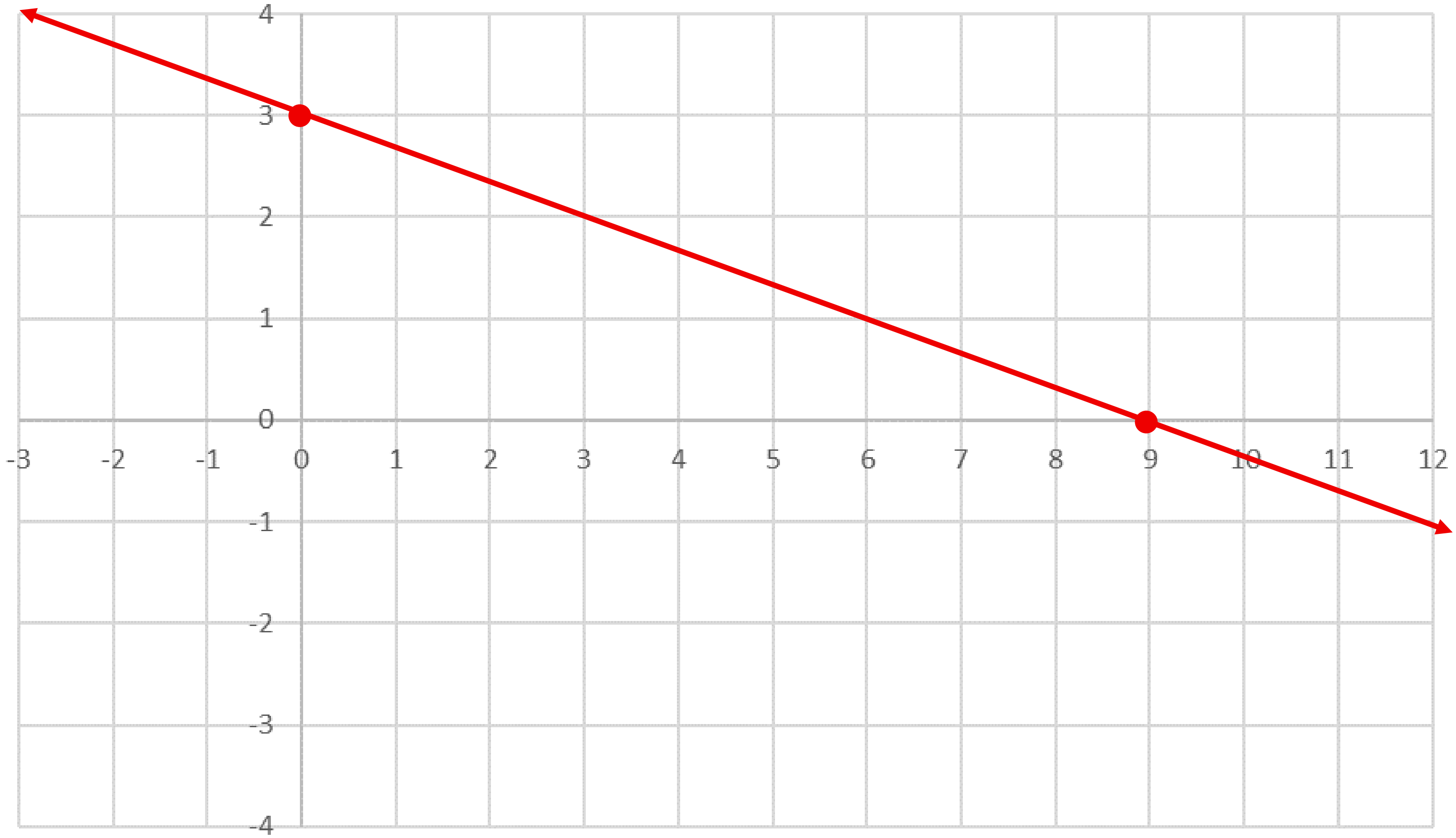
$$0\overset{?}{<}6 \text{ True!}$$

Since the test point satisfies the inequality, we'll shade the side with the test point.

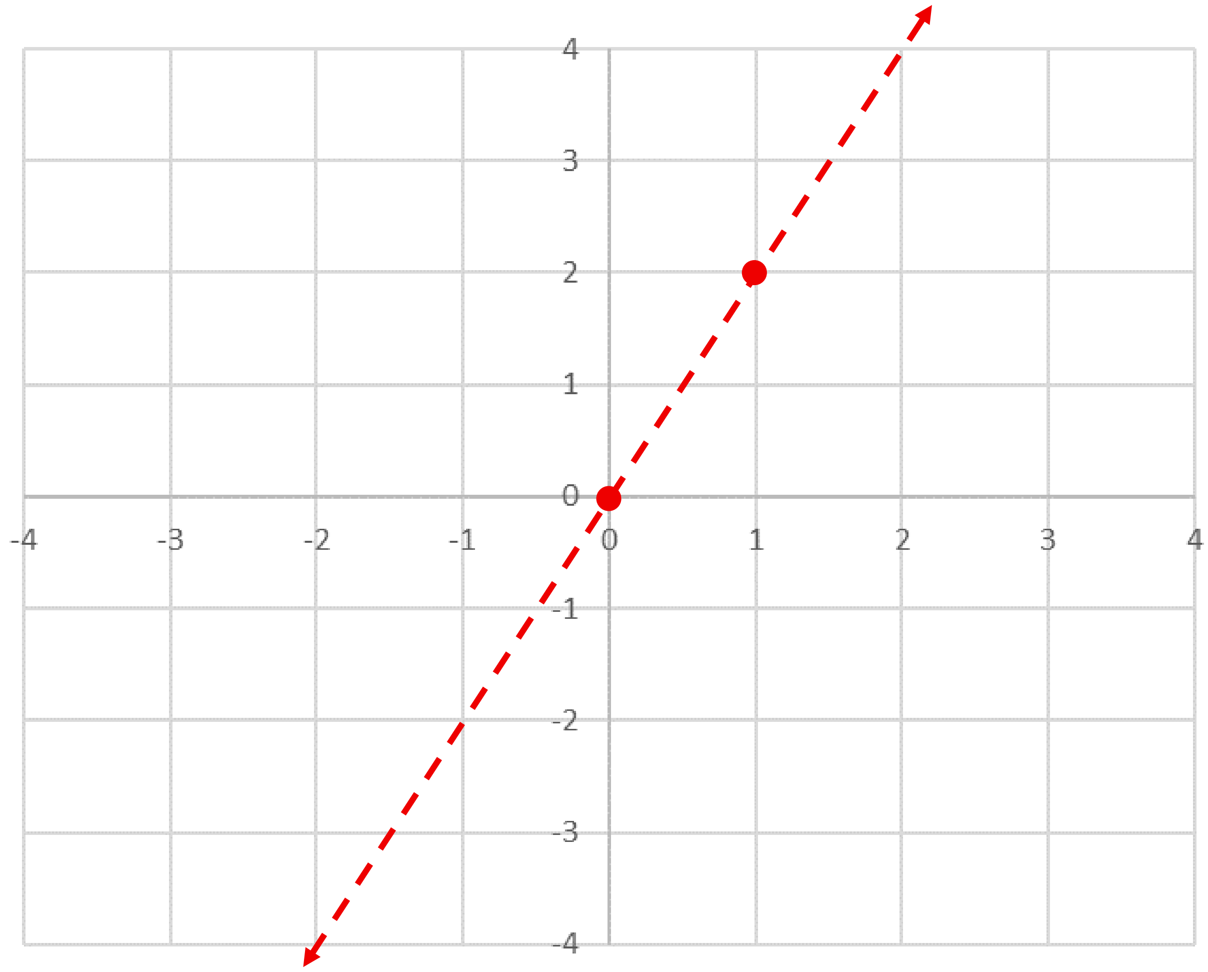


So here's the graphical representation of the infinitely many solution pairs of the inequality $2x+3y<6$. This solution region, also called the *feasible region*, is said to be *unbounded* since it can't be contained inside a circle.

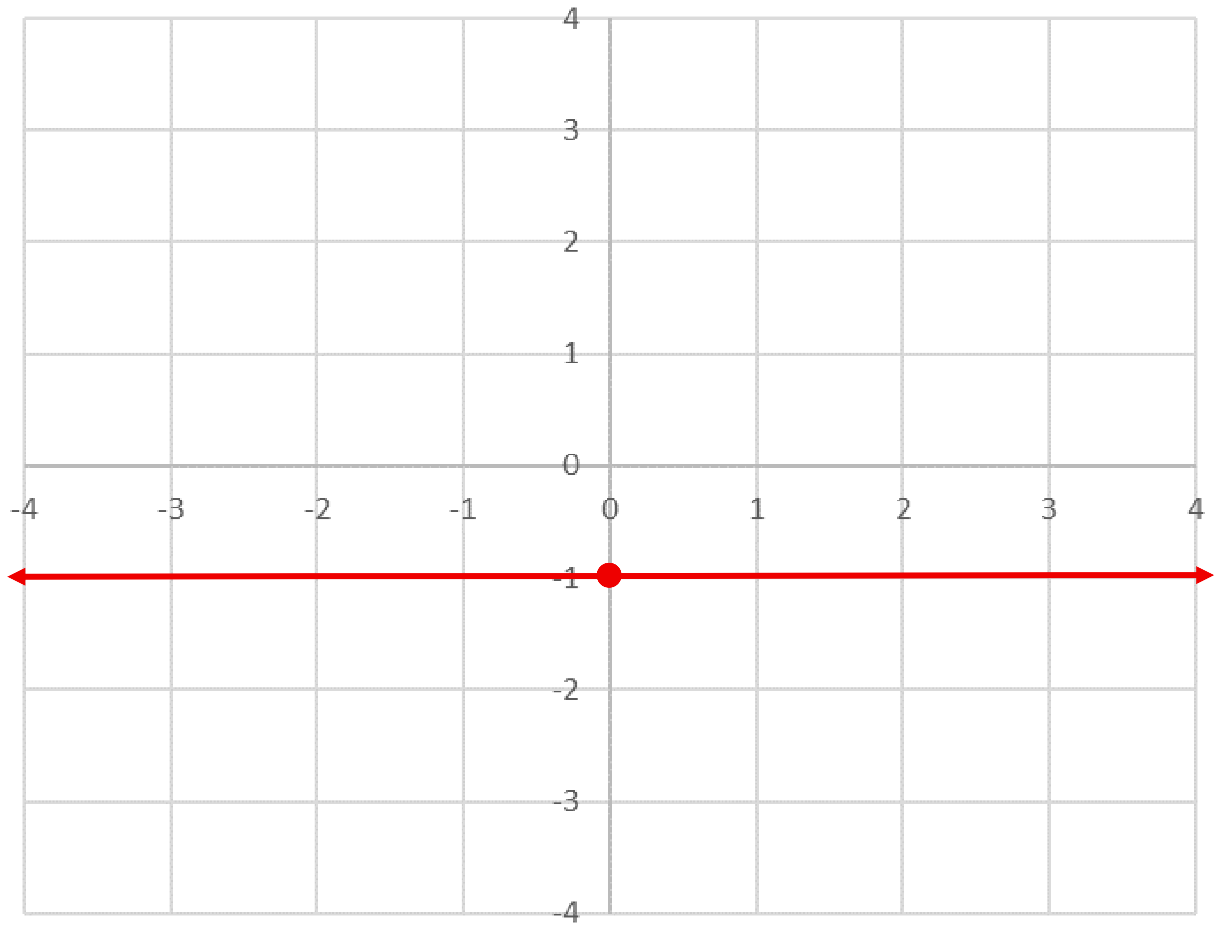
2. Solve the linear inequality $x + 3y \geq 9$.



3. Solve the linear inequality $2x - y > 0$.

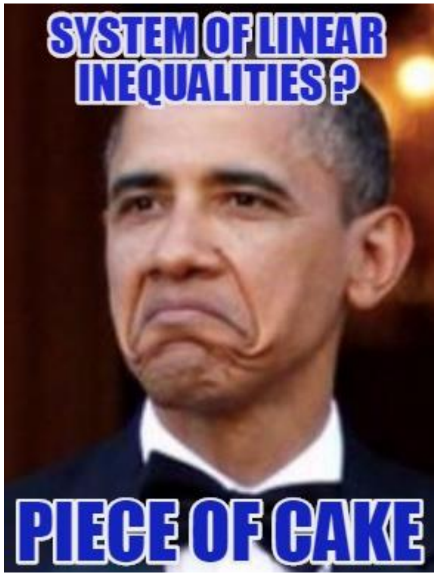


4. Solve the linear inequality $y \leq -1$.



Systems of Linear Inequalities in Two Variables:

Now we'll have a list of linear inequalities, and we need to find all the pairs of numbers that satisfy all of the inequalities in the list. First, we'll graph all of the solution lines pretending that each inequality is actually an equation. The solution line will either be solid or dashed, depending on whether equality is included in the original inequality. We'll choose test points not on the solution lines, but instead of shading the correct side, we'll indicate it with arrows. It's also important that we identify and label any points of intersection among the solution lines that are part of the solution region. Such points are referred to as *corner points*. When this is completed, we'll shade the region that all the arrows point toward.

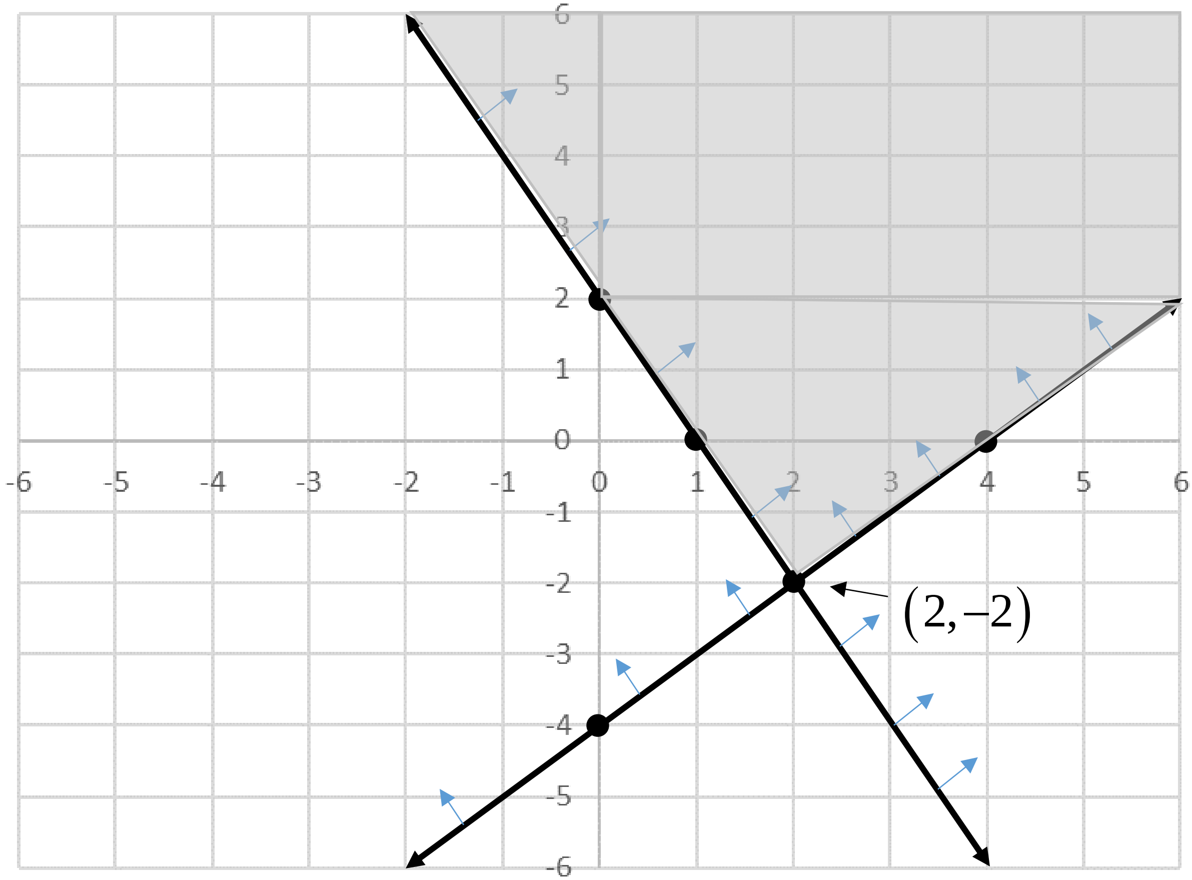


Examples:

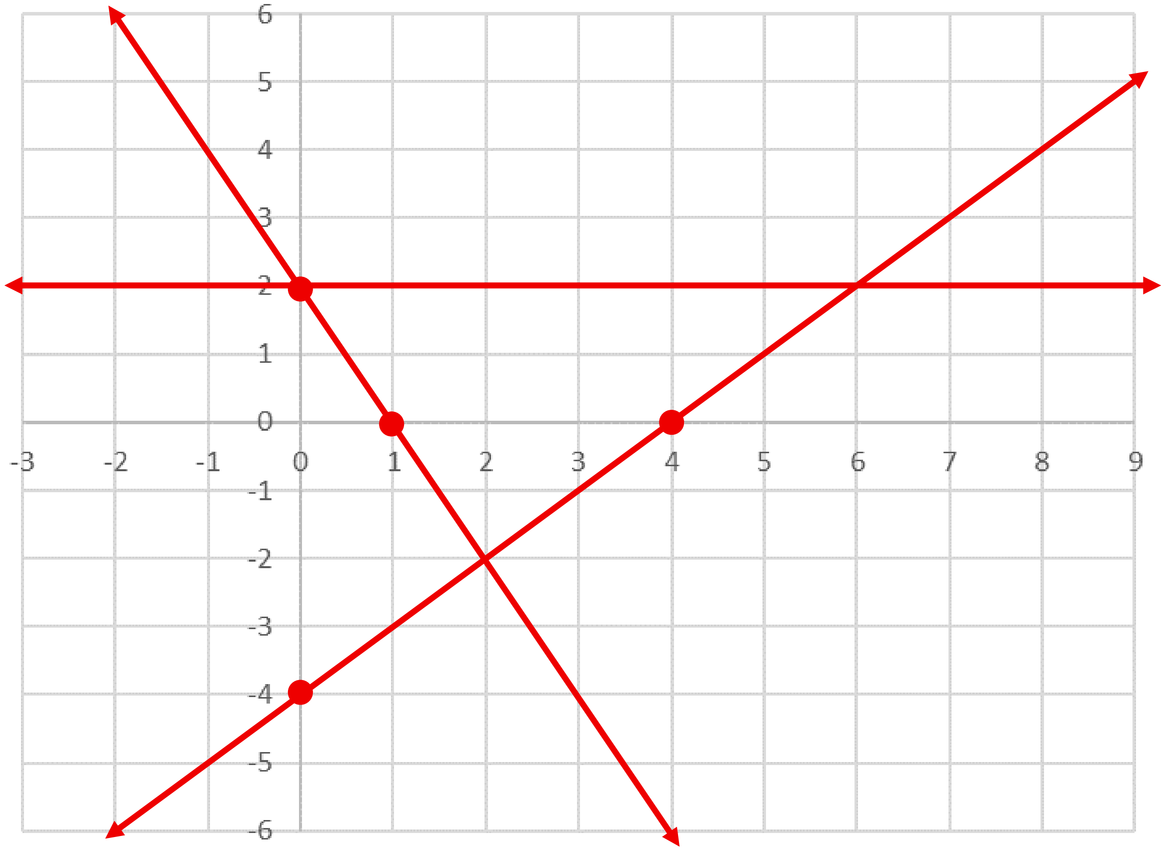
1. Solve the system $\begin{aligned} x - y &\le 4 \\ 2x + y &\ge 2 \end{aligned}$.

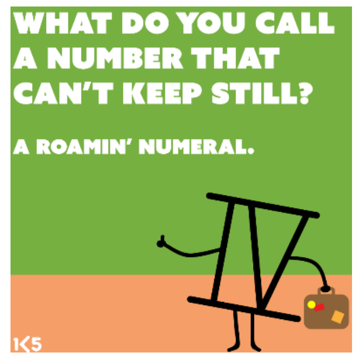
	x-int	y-int	test point	
$x - y = 4$	4	-4	$(0,0)$	true
$2x + y = 2$	1	2	$(0,0)$	false

The feasible region is unbounded and has one corner point.



2. Solve the system $\begin{aligned} x - y &\le 4. \\ 2x + y &\ge 2 \\ y &\le 2 \end{aligned}$



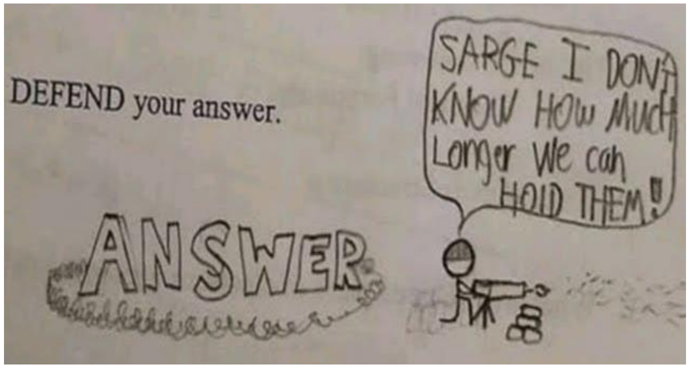


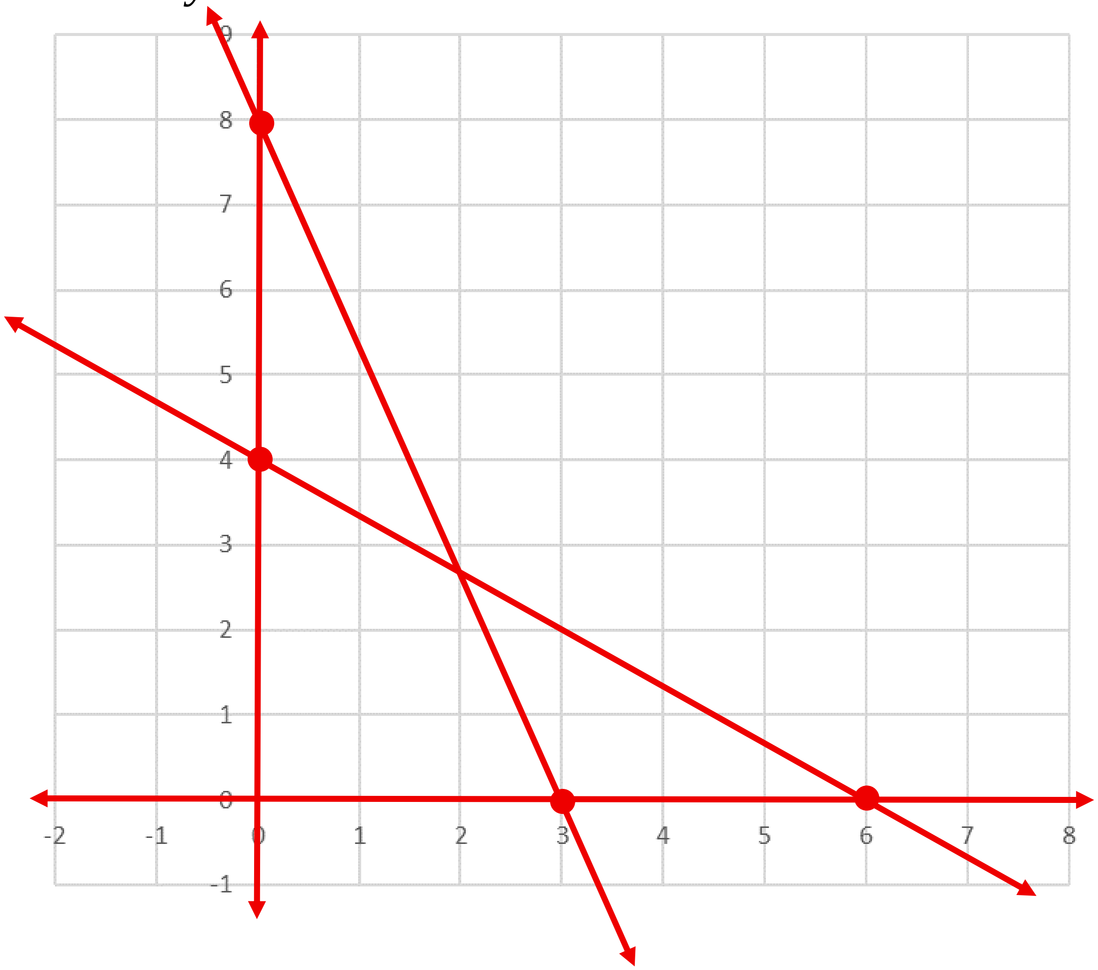
3. **Solve the system** $2x+3y\le 12$.

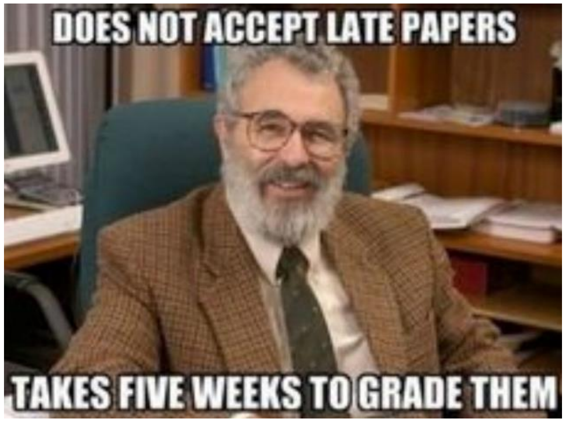
$$8x+3y\le 24$$

$$x\ge 0$$

$$y\ge 0$$





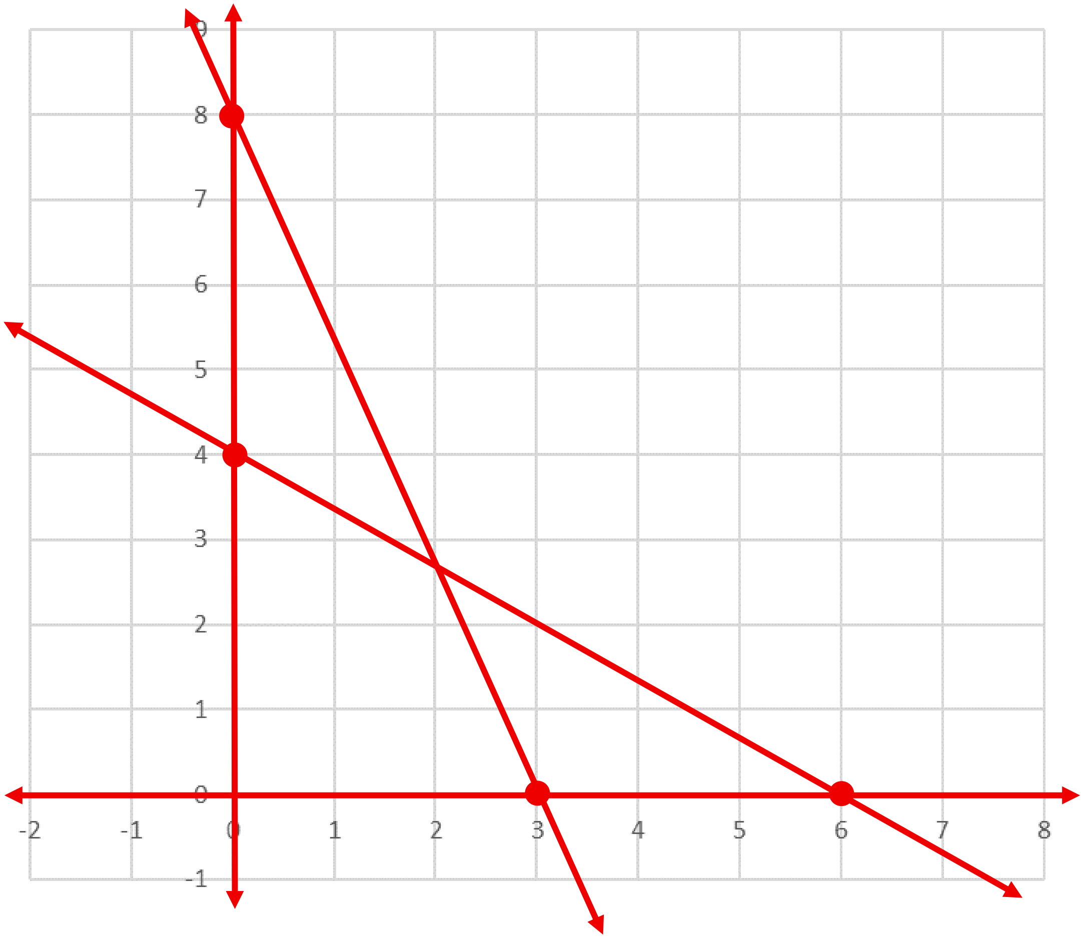


4. Solve the system $2x+3y\geq 12$ **.**

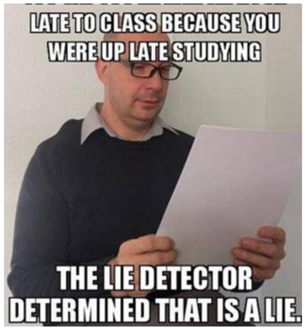
$$8x+3y\geq 24$$

$$x\geq 0$$

$$y\geq 0$$

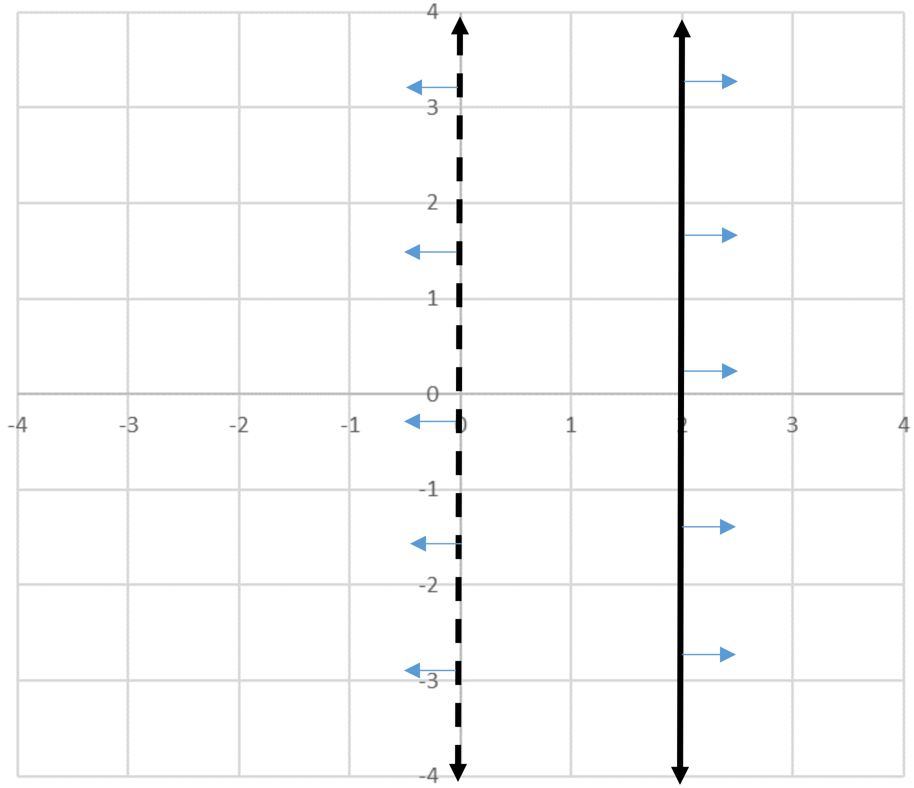


Although a single linear inequality in two variables will always have infinitely many solution pairs, this is not the case for a system.



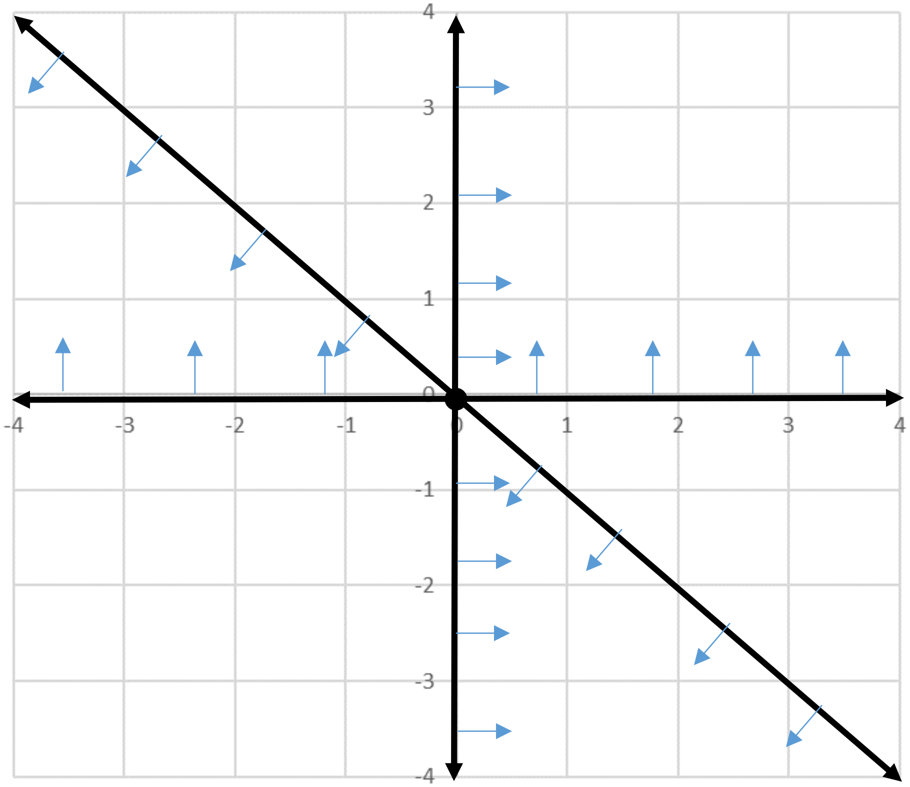
Examples: 1. $x \geq 2$

$x < 0$



So this system has no solutions.

2. $$\begin{aligned} x &\ge 0 \\ y &\ge 0 \\ x + y &\le 0 \end{aligned}$$



Here, the feasible region consists of a single point, $(0,0)$, so the only solution pair for the system is $x = 0, y = 0$.