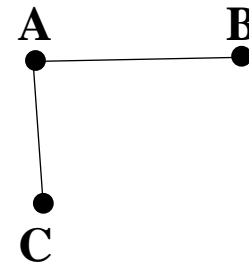
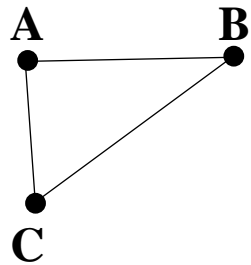


Tree:

It's a connected graph that is efficiently connected, i.e. there are no unnecessary edges to retain connectedness of the vertices.



Properties of Trees:

1. It contains no circuits.



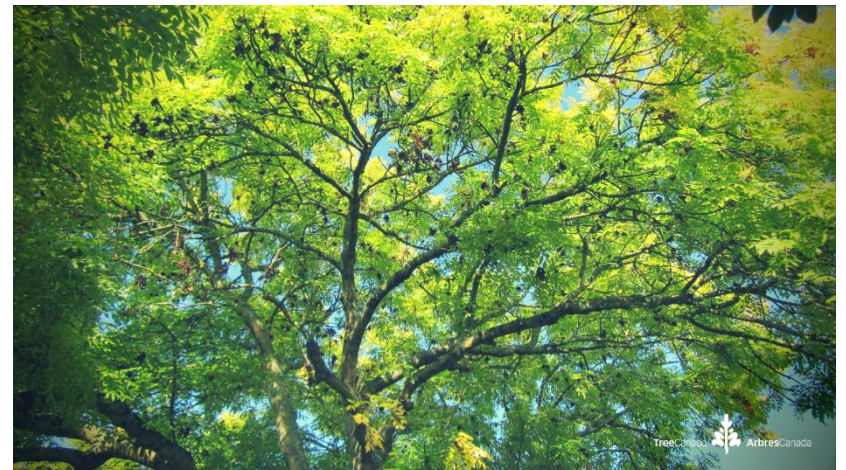
I think that I shall never see a poem
lovely as a tree.

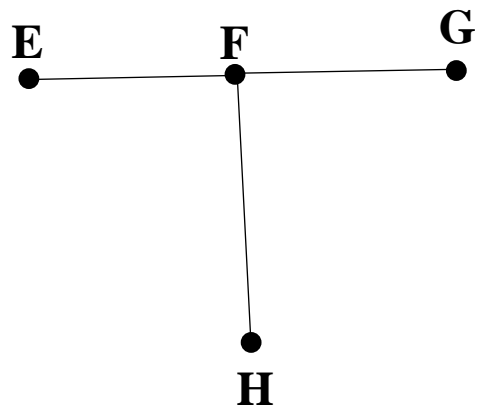
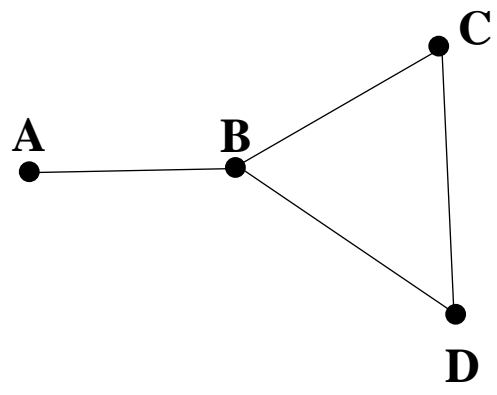
— Joyce Kilmer —

2. There is exactly one path joining any two vertices.

3. Every edge is a bridge.

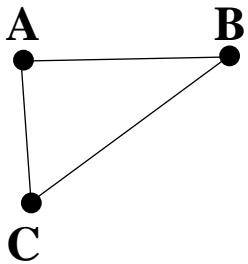
4. A tree with n vertices has $(n-1)$ edges.



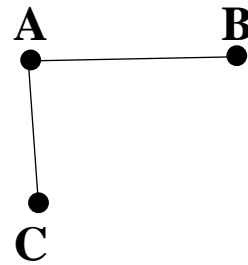


Subgraph:

It's a graph that consists of some of the vertices and edges of an original graph.



Original Graph



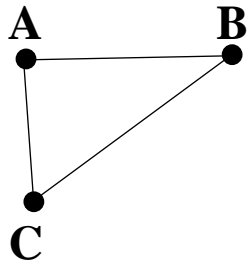
Subgraph



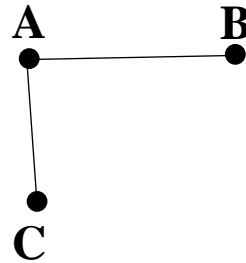
Subgraph

Spanning Tree:

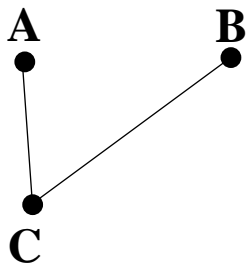
It's a subgraph of a connected graph that contains all of the original vertices and is a tree. The process of producing a spanning tree is called pruning.



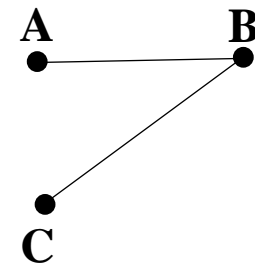
Original Graph



Spanning Tree



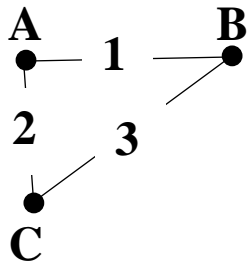
Spanning Tree



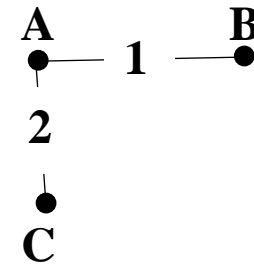
Spanning Tree

Minimum Spanning Tree:

It's a spanning tree for a weighted connected graph that has the smallest possible weight.



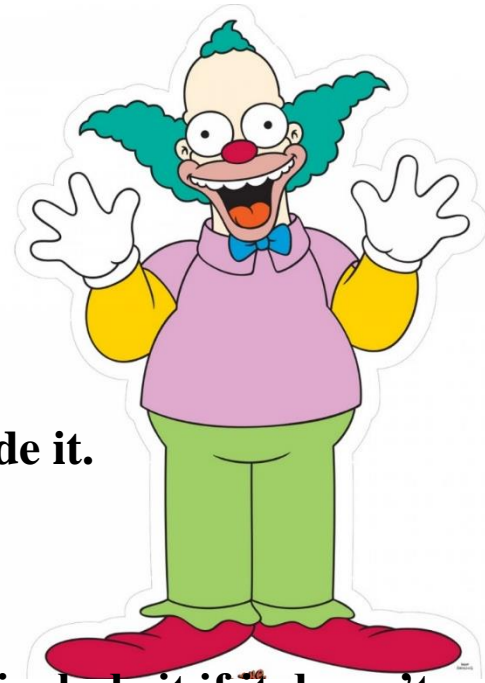
Original Weighted Graph



Minimum Spanning Tree

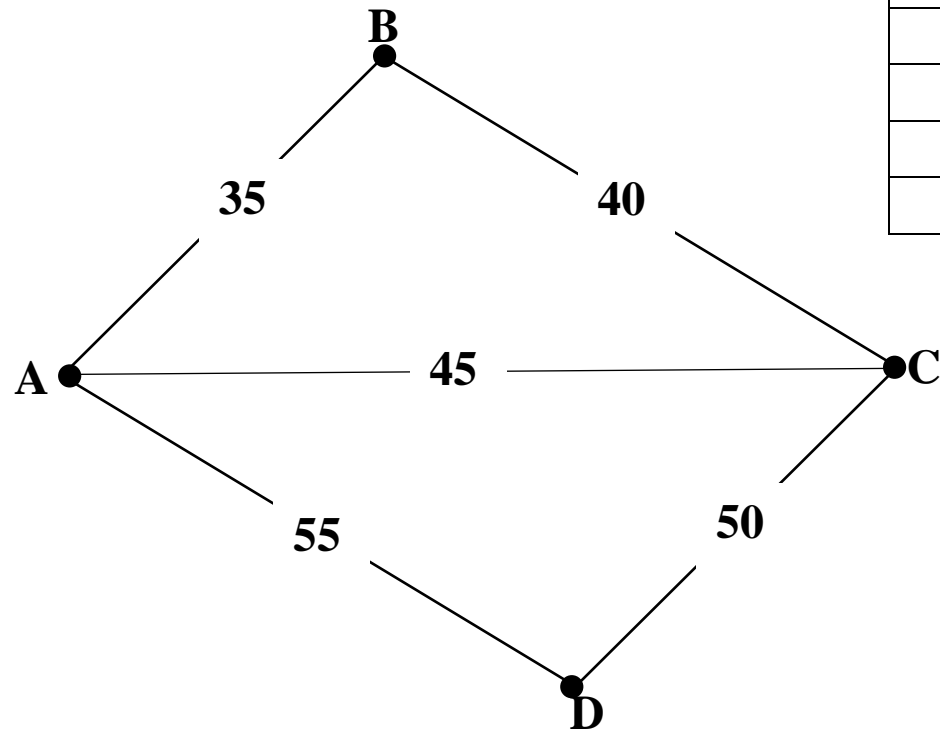
There is an algorithm for finding a minimum spanning tree.

Kruskal's Algorithm:

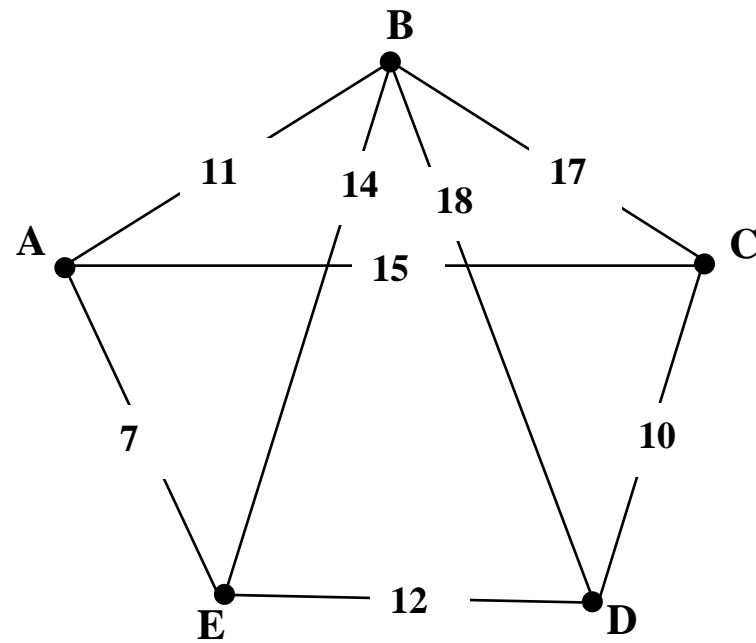


- 1. Find an edge with the smallest weight, and include it.**
- 2. Find an edge with the next-smallest weight, and include it if it doesn't produce a circuit.**
- 3. Find an edge with the next-smallest weight, and include it if it doesn't produce a circuit.**
- 4. Continue the process until all the vertices are included and connected with no circuits.**

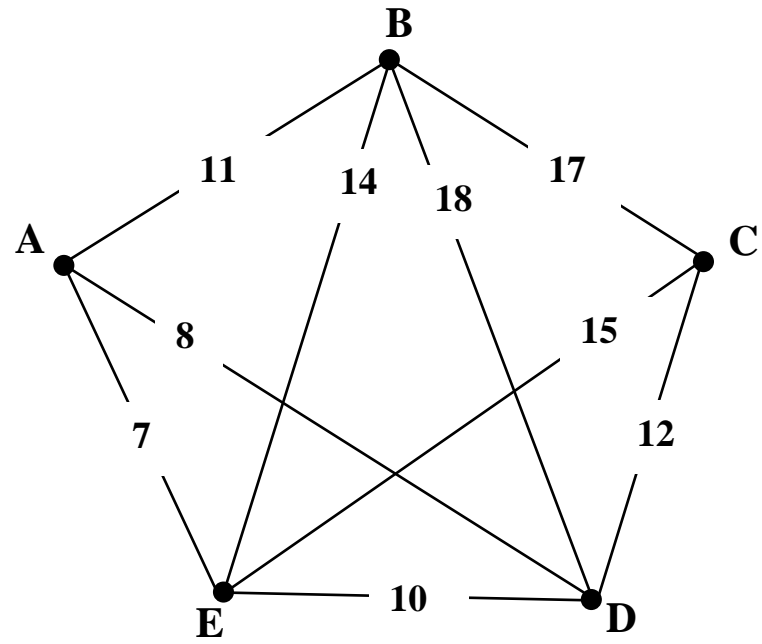
Apply Kruskal's Algorithm to the following weighted graphs.



Weights	
35	
40	
45	
50	
55	



Weights	
7	
10	
11	
12	
14	
15	
17	
18	



Weights	
7	
8	
10	
11	
12	
14	
15	
17	
18	