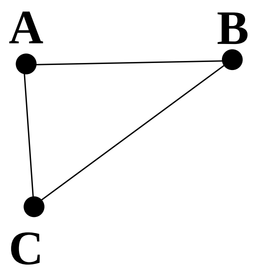
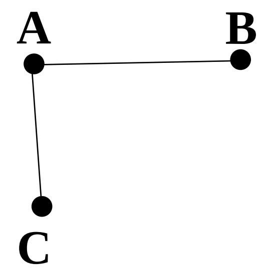
Tree:

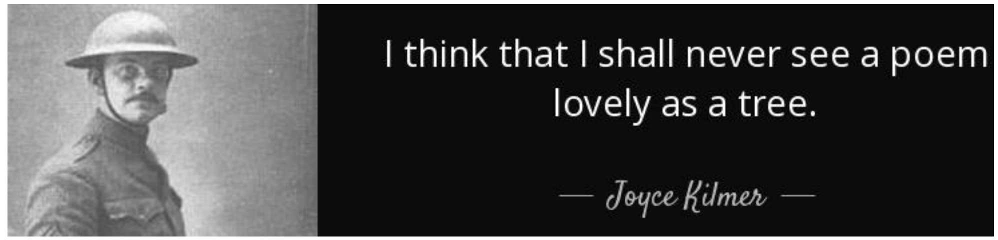
It's a connected graph that is efficiently connected, i.e. there are no unnecessary edges.





Properties of Trees:

1. It contains no circuits.

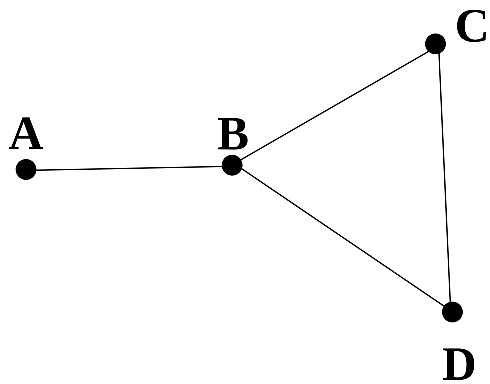


2. There is exactly one path joining any two vertices.

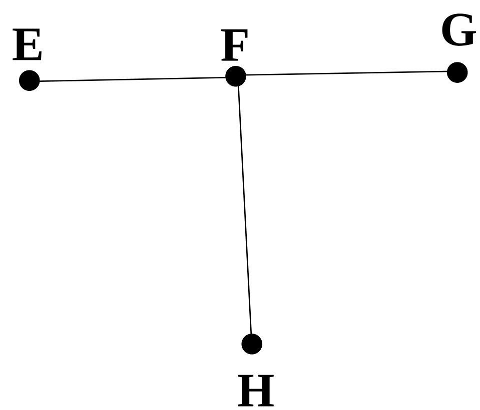
3. Every edge is a bridge.

4. A tree with *n* vertices has (*n*-1) edges.

A connected graph that satisfies any one of these four properties must be a tree.



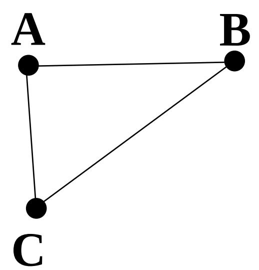
Not a tree-it contains a circuit, there's more than one path from B to C, $\overline{CD}$ is not a bridge, and it doesn't have three edges.



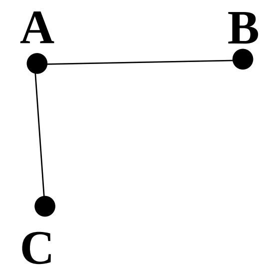
Is a tree-it certainly satisfies all four of the properties.

Subgraph:

It's a graph that consists of some of the vertices and edges of an original graph.



Original Graph

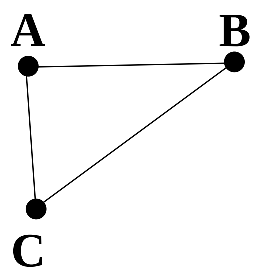


Subgraph

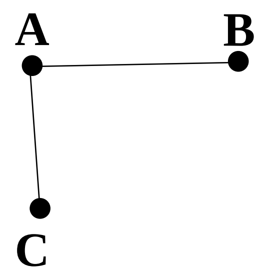
Subgraph

Spanning Tree:

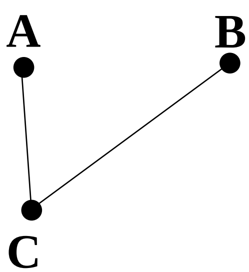
It's a subgraph of a connected graph that contains all of the original vertices and is a tree. The process of producing a spanning tree is called pruning.



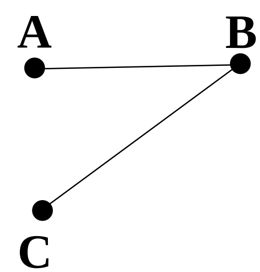
Original Graph



Spanning Tree



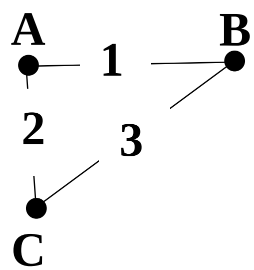
Spanning Tree



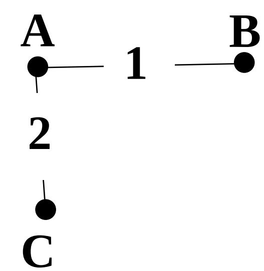
Spanning Tree

Minimum Spanning Tree:

It's a spanning tree for a weighted connected graph that has the smallest possible weight.



Original Weighted Graph



Minimum Spanning Tree

There is an algorithm for finding a minimum spanning tree.

Kruskal's Algorithm:

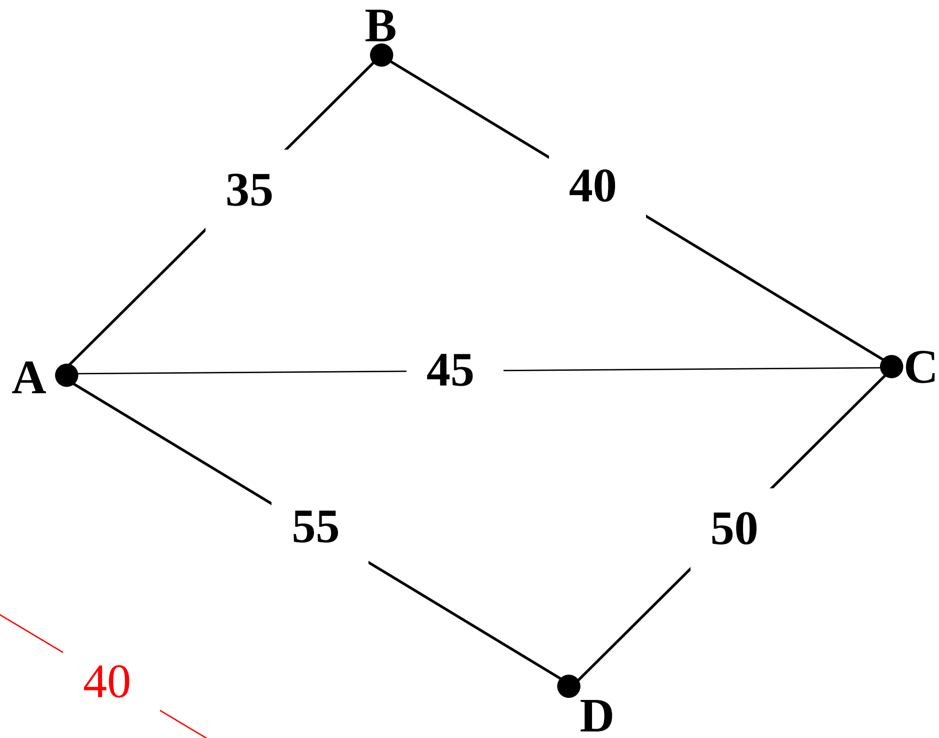
1. Find an edge with the smallest weight, and include it.

2. Find an edge with the next-smallest weight, and include it if it doesn't produce a circuit.

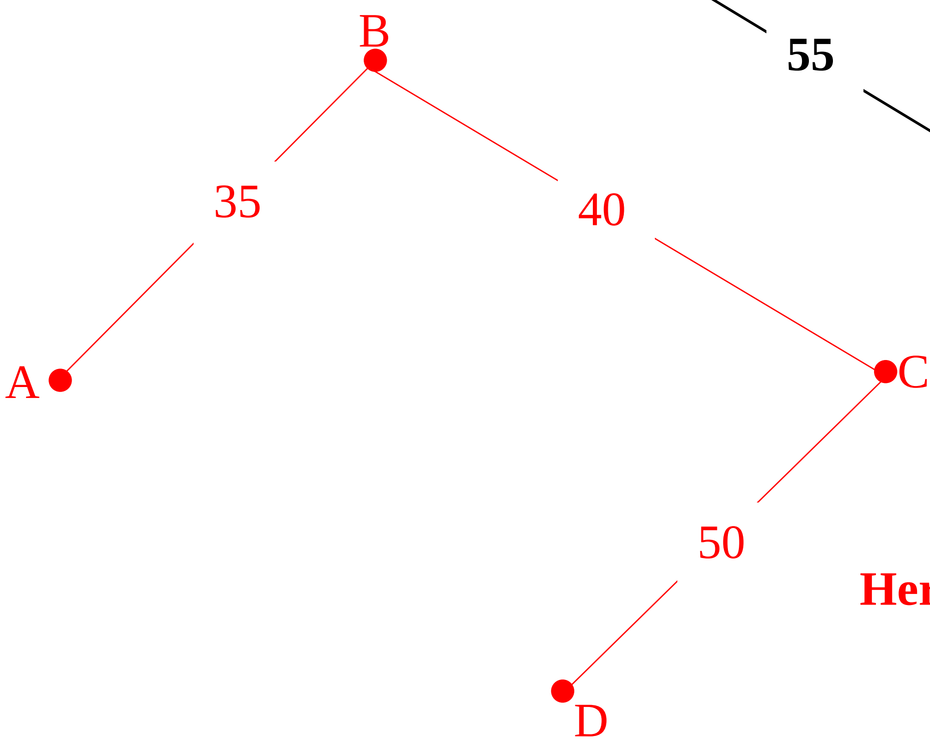
3. Find an edge with the next-smallest weight, and include it if it doesn't produce a circuit.

4. Continue the process until all the vertices are included and connected with no circuits.

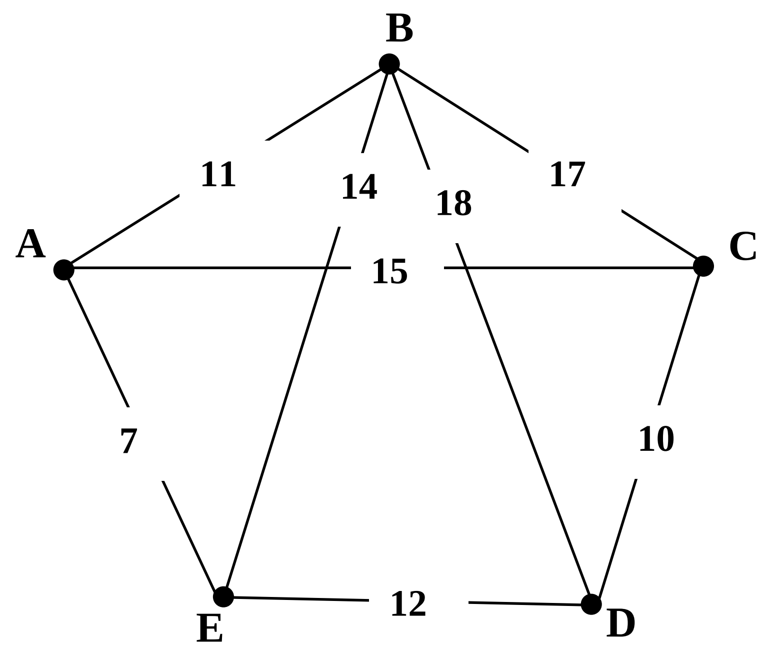
Apply Kruskal's Algorithm to the following weighted graphs.



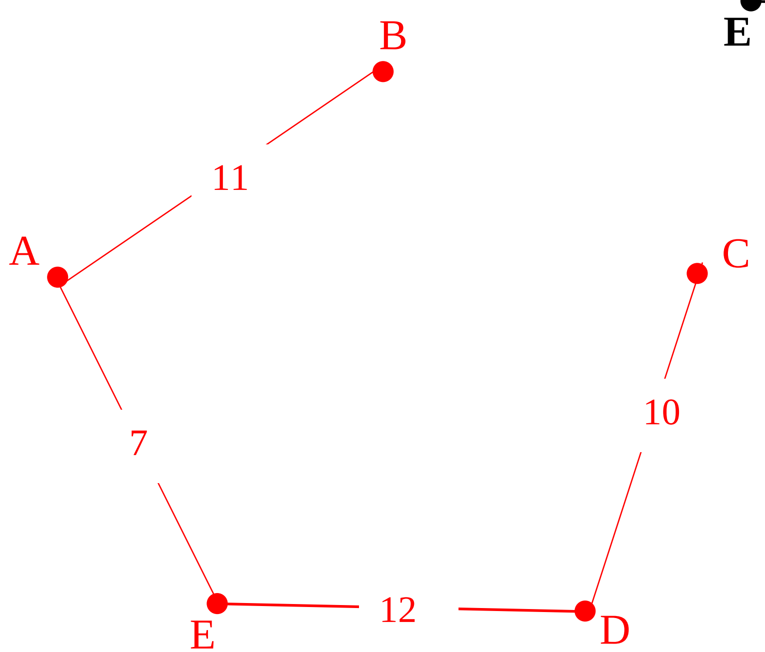
Weights	
35	✓
40	✓
45	*X*
50	✓
55	



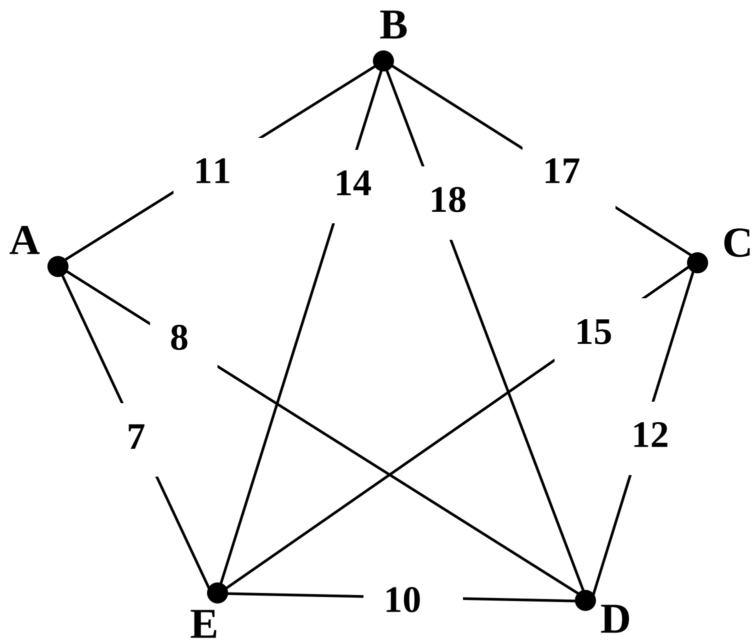
Here is a minimum spanning tree.



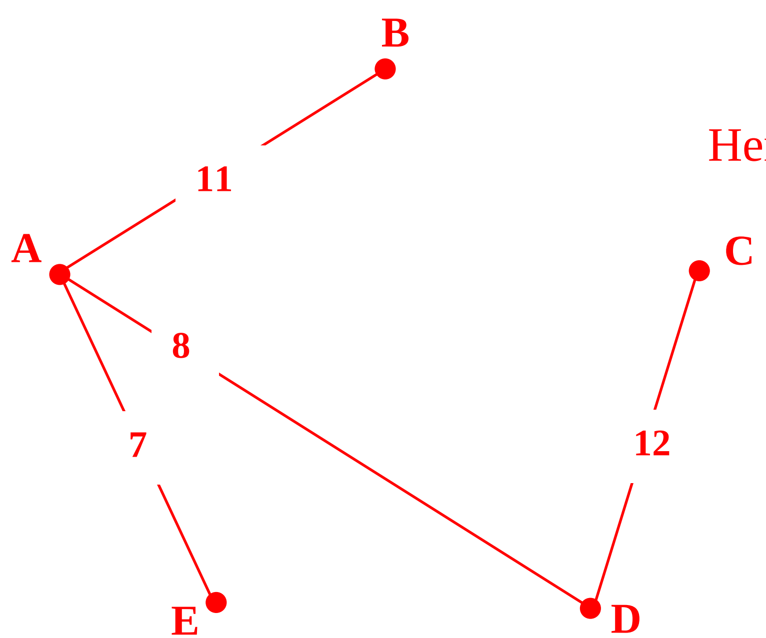
Weights	
7	✓
10	✓
11	✓
12	✓
14	
15	
17	
18	



Here is a minimum spanning tree.



Weights	
7	✓
8	✓
10	*X*
11	✓
12	✓
14	
15	
17	
18	



Here is a minimum spanning tree.