

Rational Zero/Root Theorem:

For $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ a polynomial function with integer coefficients, if the reduced rational number $\frac{p}{q}$ is a zero of $f(x)$, then p must be a factor of a_0 and q must be a factor of a_n .

Examples: List all the possible rational zeros of the following polynomial functions.

1. $f(x) = x^5 - 3x^2 + 1$

2. $f(x) = 10x^5 + 3x^4 - 2x^2 - 6$

Fundamental Theorem of Algebra

Degree of Polynomial = Number of Roots

The Fundamental Theorem of Algebra:

Every polynomial function of degree $n \geq 1$ has n zeros in the set of complex numbers.

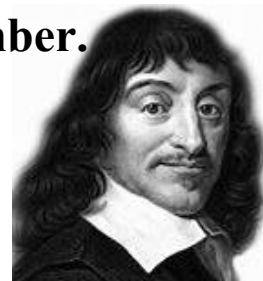
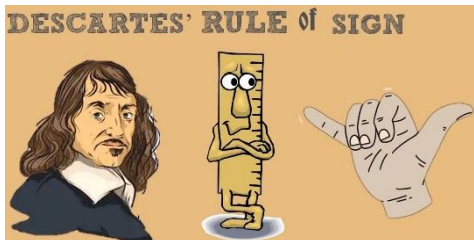
Repeated zeros are counted separately.

If the coefficients in the polynomial function are real numbers, then the imaginary zeros occur in conjugate pairs, i.e. if $a + bi$ is a zero, then so is $a - bi$.

Descartes' Rule of Signs:

For $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ a polynomial function with real coefficients, then

1. The number of positive zeros is either the number of sign changes in the coefficients of $f(x)$, or less than the number of sign changes by an even number.
2. The number of negative zeros is either the number of sign changes in the coefficients of $f(-x)$, or less than the number of sign changes by an even number.

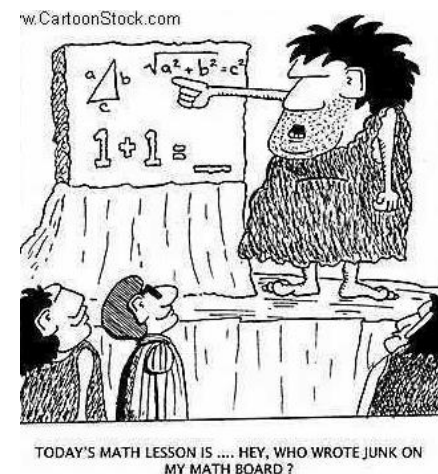


Examples: Find the possible numbers of positive zeros, negative zeros, and imaginary zeros for the following polynomial functions.

1. $f(x) = x^5 - 3x^2 + 1$

coefficients of $f(x)$	1	-3	1
coefficients of $f(-x)$	-1	-3	1

# of positives	# of negatives	# of imaginaries



2. $f(x) = 10x^5 + 3x^4 + 2x^2 - 6$

coefficients of $f(x)$	10	3	2	-6
coefficients of $f(-x)$	-10	3	2	-6

Diagram illustrating the sign change rule for finding the number of positive and negative real roots of a polynomial function.

For $f(x) = x^3 - 4x^2 + 3x + 8$:

- Signs: $+$ to $-$ (change), $-$ to $+$ (change), $+$ to $+$ (no variation).
- Number of sign changes: 2.
- Conclusion: 2 POSITIVE REAL ROOTS.

For $f(-x) = -x^3 - 4x^2 - 3x + 8$:

- Signs: $-$ to $-$ (no variation), $-$ to $-$ (no variation), $-$ to $+$ (change).
- Number of sign changes: 1.
- Conclusion: 1 NEGATIVE REAL ROOT.

Let's try the concept of sign change...

# of positives	# of negatives	# of imaginaries

Find all the zeros of the following polynomial functions:

1. $f(x) = x^3 - x^2 + x - 1$ {Factor, if possible.}



2. $f(x) = x^3 - 3x + 2$ {Check easy values like ± 1 .}

3. $f(x) = 3x^4 - 4x^3 + x^2 + 6x - 2$

{Check easy values like ± 1 .}

4. $f(x) = x^3 - 4x^2 + 2x + 4$

