**Review of Solving Quadratic Equations:**

**1. Factoring**

1. Get zero on one side.
2. Factor the other side.
3. Set the factors containing the variable equal to zero, and solve.

**2. Square root method**

If $(\text{something})^2 = \text{number}$, then $\text{something} = \pm\sqrt{\text{number}}$.

**3. Completing the square**

If $x^2 + bx = c$, then add $\left(\frac{b}{2}\right)^2$ on both sides to get a perfect square on the left. Finish solving using the square root method.

**4. Quadratic formula**

If $ax^2 + bx + c = 0; a \neq 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

**Factoring examples:**

**1.** $(2x+3)(x-2)=0$

Zero is on one side, and the other side is factored, so set the factors equal to zero and solve. $2x+3=0$ or $x-2=0 \Rightarrow x=\boxed{-\frac{3}{2},2}$.

**2.** $2x^2-6x=0$

Zero is on one side, but the other side needs to be factored. Factor out the GCF of $2x$.
$2x(x-3)=0 \Rightarrow 2x=0$ or $x-3=0 \Rightarrow x=\boxed{0,3}$.

**3.** $x^2+6x=-8$

Add 8 on both sides to get zero on one side. Factor the resulting special trinomial by finding factors of 8 that add to 6. $x^2+6x+8=0 \Rightarrow (x+2)(x+4)=0 \Rightarrow x=\boxed{-2,-4}$.

**4.** $4x^2+7=16x$

Subtract $16x$ on both sides to get zero on one side. Factor the resulting general trinomial using the method of your choice(see the Methods of Factoring Trinomials link on the course-page) $4x^2-16x+7=0 \Rightarrow (2x-7)(2x-1)=0 \Rightarrow x=\boxed{\frac{7}{2},\frac{1}{2}}$.

**5.** $3x^3+x^2-12x-4=0$

Zero is on one side, but the other side needs to be factored by grouping and difference of squares.

$\left(3x^3+x^2\right)-\left(12x+4\right)$

$x^2(3x+1)-4(3x+1)=0 \Rightarrow (3x+1)\left(x^2-4\right)=0 \Rightarrow (3x+1)(x-2)(x+2)=0 \Rightarrow x=\boxed{-\frac{1}{3},2,-2}$

**Square root method examples:**

**1.** $x^2 = 9$

$x = \pm\sqrt{9} \Rightarrow x = \boxed{\pm 3}$

**2.** $3x^2 = 21$

Divide by 3 to isolate the squared term. $x^2 = 7 \Rightarrow x = \boxed{\pm\sqrt{7}}$.

**3.** $5x^2 = -20$

Divide by 5 to isolate the squared term. $x^2 = -4 \Rightarrow x = \pm\sqrt{-4} \Rightarrow x = \boxed{\pm 2i}$.

**4.** $(x+1)^2 = 8$

$$x+1=\pm\sqrt{8} \Rightarrow x=-1\pm\sqrt{8} \Rightarrow x=\boxed{-1\pm 2\sqrt{2}}$$

**5.** $(2x-1)^2 = -5$

$$2x-1=\pm\sqrt{-5} \Rightarrow 2x=1\pm\sqrt{5}i \Rightarrow x=\boxed{\frac{1}{2}\pm\frac{\sqrt{5}}{2}i}$$

**Completing the square examples:**

**1.** $x^2 + 8x = -15$

Add $\left(\frac{8}{2}\right)^2 = 16$ on both sides.

$x^2 + 8x + 16 = 1 \Rightarrow (x+4)^2 = 1 \Rightarrow x + 4 = \pm\sqrt{1} \Rightarrow x = -4 \pm 1 \Rightarrow x = \boxed{-5, -3}$

**2.** $x^2 = 22 + 10x$

Subtract $10x$ on both sides to get $x^2 - 10x = 22$, and then add $\left(\frac{-10}{2}\right)^2 = 25$ on both sides.

$x^2 - 10x + 25 = 47 \Rightarrow (x-5)^2 = 47 \Rightarrow x - 5 = \pm\sqrt{47} \Rightarrow x = \boxed{5 \pm \sqrt{47}}$.

**3.** $x^2 + 6x + 13 = 0$

Subtract 13 on both sides, and then add $\left(\frac{6}{2}\right)^2 = 9$ on both sides.

$x^2 + 6x + 9 = -4 \Rightarrow (x+3)^2 = \pm\sqrt{-4} \Rightarrow x + 3 = \pm 2i \Rightarrow x = \boxed{-3 \pm 2i}$.

**4.** $2x^2 - 5x - 3 = 0$

Add 3 on both sides, and then divide by 2. $x^2 - \frac{5}{2}x = \frac{3}{2}$. Add $\left(\frac{-\frac{5}{2}}{2}\right)^2 = \frac{25}{16}$ on both sides.

$x^2 - \frac{5}{2}x + \frac{25}{16} = \frac{3}{2} + \frac{25}{16} \Rightarrow \left(x - \frac{5}{4}\right)^2 = \frac{49}{16} \Rightarrow x - \frac{5}{4} = \pm\frac{7}{4} \Rightarrow x = \frac{5}{4} \pm \frac{7}{4} \Rightarrow x = \boxed{3, -\frac{1}{2}}$.

**Quadratic formula examples:**

**1.** $x^2+4x-5=0$ $\quad a=1, b=4, c=-5$

$$x=\frac{-4\pm\sqrt{4^2-4(1)(-5)}}{2(1)}\Rightarrow x=\frac{-4\pm\sqrt{36}}{2}\Rightarrow x=\frac{-4\pm 6}{2}\Rightarrow x=\boxed{1,-5}$$

**2.** $3x^2+8x+3=0$ $\quad a=3, b=8, c=3$

$$x=\frac{-8\pm\sqrt{8^2-4(3)(3)}}{2(3)}\Rightarrow x=\frac{-8\pm\sqrt{28}}{6}\Rightarrow x=\frac{-8\pm 2\sqrt{7}}{6}\Rightarrow x=\boxed{\frac{-4\pm\sqrt{7}}{3}}$$

**3.** $x^2+1=x$

Subtract $x$ on both sides. $x^2-x+1=0$ $\quad a=1, b=-1, c=1$

$$x=\frac{1\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}\Rightarrow x=\frac{1\pm\sqrt{-3}}{2}\Rightarrow x=\frac{1\pm\sqrt{3}i}{2}\Rightarrow x=\boxed{\frac{1}{2}\pm\frac{\sqrt{3}}{2}i}$$

**Quadratic-like equation examples:**

**1.** $x^4 - 3x^2 + 2 = 0$

It's quadratic in $x^2$, so we'll factor in terms of $x^2$ rather than $x$. You can factor as difference of squares or use the square root method.

$$\left(x^2\right)^2 - 3x^2 + 2 = 0 \Rightarrow \left(x^2 - 1\right)\left(x^2 - 2\right) = 0 \Rightarrow (x-1)(x+1)\left(x-\sqrt{2}\right)\left(x+\sqrt{2}\right) = 0 \Rightarrow x = \boxed{\pm 1, \pm\sqrt{2}}$$

**2.** $(3x+2)^2 + 7(3x+2) - 8 = 0$

It's quadratic in $(3x+2)$, so we'll factor in terms of $(3x+2)$ rather than $x$.

$\left[(3x+2)+8\right]\left[(3x+2)-1\right] = 0 \Rightarrow (3x+10)(3x+1) = 0 \Rightarrow x = \boxed{-\frac{10}{3}, -\frac{1}{3}}$. Your other option is to multiply out the original equation and then factor.

**3.** $\left(x^2-5x\right)^2+\left(x^2-5x\right)=12$

It's quadratic in $\left(x^2-5x\right)$, so we'll factor in terms of $\left(x^2-5x\right)$ rather than $x$. Subtract 12 on both sides.

$$\left(x^2-5x\right)^2+\left(x^2-5x\right)-12=0\Rightarrow\left[\left(x^2-5x\right)+4\right]\left[\left(x^2-5x\right)-3\right]=0$$

$$\Rightarrow\left(x^2-5x+4\right)\left(x^2-5x-3\right)=0\Rightarrow\left(x^2-5x+4\right)=0 \text{ or } \left(x^2-5x-3\right)=0$$

$\Rightarrow x=1,4$ from factoring the first and $x=\dfrac{5\pm\sqrt{37}}{2}$ from the quadratic formula in the second, so

$$x=\boxed{1,4,\frac{5\pm\sqrt{37}}{2}}$$

**The discriminant:**

The discriminant of the quadratic equation $ax^2 + bx + c = 0$ is $b^2 - 4ac$.

If the quadratic equation has real coefficients, then

If $b^2 - 4ac > 0$, then the equation has 2 real solutions.

If $b^2 - 4ac < 0$, then the equation has 2 imaginary solutions.

If $b^2 - 4ac = 0$, then the equation has 1 real solution.

**Discriminant examples:**

**1.** $4x^2 - 12x + 9 = 0$ $a = 4, b = -12, c = 9$

$b^2 - 4ac = (-12)^2 - 4(4)(9) \boxed{= 0}$, so this quadratic equation has one real solution.

**2.** $x^2 - 2x + 4 = 0$ $a = 1, b = -2, c = 4$

$b^2 - 4ac = (-2)^2 - 4(1)(4) = -12 \boxed{< 0}$, so this quadratic equation has two imaginary solutions.

**3.** $5x^2 - 4x = 10 \Rightarrow 5x^2 - 4x - 10 = 0$ $a = 5, b = -4, c = -10$

$b^2 - 4ac = (-4)^2 - 4(5)(-10) = 216 \boxed{> 0}$, so this quadratic equation has two real solutions.