

Properties of Logarithms:

For M and N positive numbers and r a real number,

Product Rule:

Expansion



$$\log_b(MN) = \log_b M + \log_b N$$



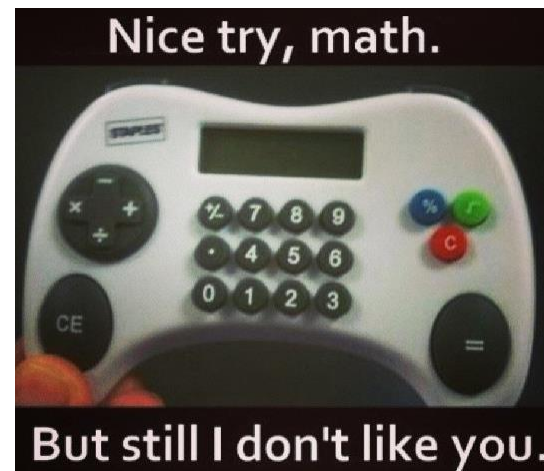
Compression

Expand and simplify:

$$\log_5(25x)$$


Compress(or Condense) and simplify:

$$\log_6 9 + \log_6 4$$




Quotient Rule:

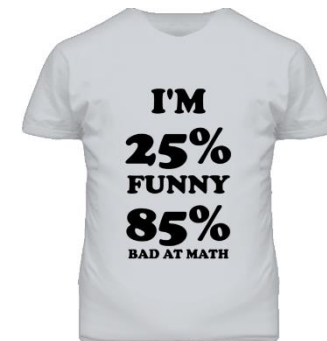
Expansion



$$\log_b \left(\frac{M}{N} \right) = \log_b M - \log_b N$$



Compression

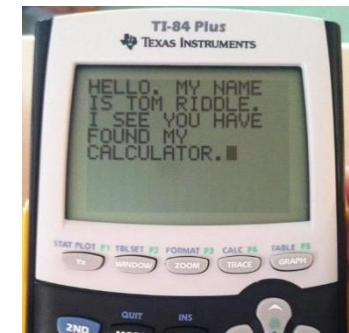


Expand and simplify:

$$\log_3 \left(\frac{x}{9} \right)$$

Compress and simplify:

$$\log_3 2 - \log_3 6$$



Power Rule:

Expansion



$$\log_b(M^r) = r \log_b M$$



Compression

Expand and simplify:

$$\log_7(7x^5)$$

Compress:

$$2\log_3 x - 4\log_3 y$$

Expand: $\log_2 \left[\frac{x^3(x+2)}{(x+3)^2} \right]$

$$\ln(ab) = \ln(a) + \ln(b)$$

$$\ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b)$$

$$\ln(a^b) = b \ln(a)$$

Compress: $3\log_5(3x+1) - 2\log_5(2x-1) - \log_5 x$

$$\log_b x + \log_b y = \log_b(x \cdot y)$$

$$\log_b x - \log_b y = \log_b\left(\frac{x}{y}\right)$$

$$z \cdot \log_b x = \log_b(x^z)$$

Change of Base Formula:

Suppose that $y = \log_b x$. Then $b^y = x$ and therefore $\log_a (b^y) = \log_a x$. From the Power

Rule, you get $y \log_a b = \log_a x$, and solving for y yields $y = \frac{\log_a x}{\log_a b}$. So

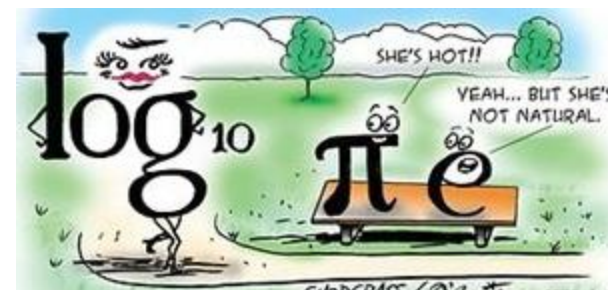
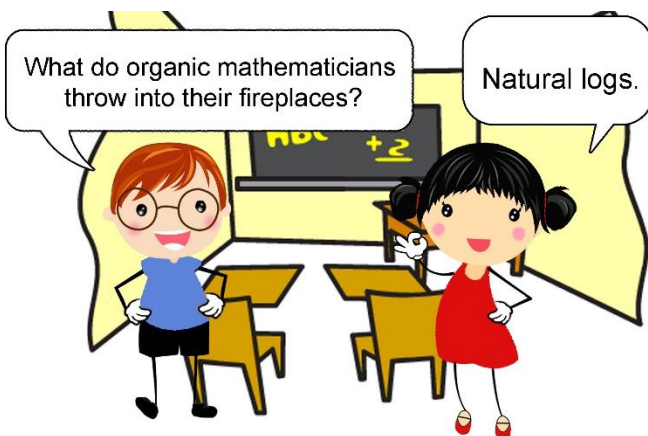
$$\log_b x = \frac{\log_a x}{\log_a b}.$$

Calculators have a logarithm key for base 10, \log , called the common logarithm. They also have a logarithm key for base e , \ln , called the natural logarithm. $e = 2.7182818...$

$$\log_b x = \frac{\log x}{\log b}$$

Or

$$\log_b x = \frac{\ln x}{\ln b}$$



Example:

Calculate $\log_3 5$ to 3 decimal places.

$$\log_3 5 = \frac{\log 5}{\log 3}$$

Or

$$\log_3 5 = \frac{\ln 5}{\ln 3}$$

