## Properties of Logarithms:

**For *M* and *N* positive numbers and *r* a real number,**

## Product Rule:

**Expansion**

$$log_b(MN) = log_b M + log_b N$$

**Compression**

**Why?** $b^{log_b(MN)} = MN$     **and** $b^{(log_b M + log_b N)} = b^{log_b M} \cdot b^{log_b N} = MN$

**Expand and simplify:**

$$log_5(25x)$$

$$log_5(25x) = log_5 25 + log_5 x = \boxed{2 + log_5 x}$$

**Compress(or Condense) and simplify:**

$$log_6 9 + log_6 4$$

$$log_6 9 + log_6 4 = log_6 36 = \boxed{2}$$

## Quotient Rule:

**Expansion**

$$log_b\left(\frac{M}{N}\right) = log_b M - log_b N$$

**Compression**

**Why?** $b^{log_b\left(\frac{M}{N}\right)} = \dfrac{M}{N}$          **and** $b^{(log_b M - log_b N)} = \dfrac{b^{log_b M}}{b^{log_b N}} = \dfrac{M}{N}$

**Expand and simplify:**

$$log_3\left(\frac{x}{9}\right)$$

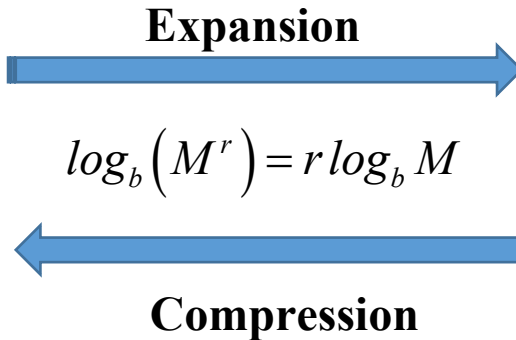$$log_3\left(\frac{x}{9}\right) = log_3 x - log_3 9 = \boxed{log_3 x - 2}$$

**Compress and simplify:**

$$log_3 2 - log_3 6$$

$$log_3 2 - log_3 6 = log_3\left(\tfrac{1}{3}\right) = \boxed{-1}$$

## Power Rule:

$$\text{Expansion}$$

$$\log_b\left(M^r\right) = r\log_b M$$

$$\text{Compression}$$

**Why?** $b^{\log_b\left(M^r\right)} = M^r$  **and** $b^{r\log_b M} = \left(b^{\log_b M}\right)^r = M^r$

## Expand and simplify:

$$\log_7\left(7x^5\right)$$

$$\log_7\left(7x^5\right) = \log_7 7 + 5\log_7 x = \boxed{1 + 5\log_7 x}$$

## Compress:

$$2\log_3 x - 4\log_3 y$$

$$2\log_3 x - 4\log_3 y = \log_3 x^2 - \log_3 y^4 = \boxed{\log_3\left(\frac{x^2}{y^4}\right)}$$

**Expand:** $log_2 \left[ \dfrac{x^3(x+2)}{(x+3)^2} \right]$

$$log_2 \left[ \frac{x^3(x+2)}{(x+3)^2} \right] = log_2 x^3 + log_2(x+2) - log_2\left[(x+3)^2\right]$$

$$= \boxed{3\,log_2 x + log_2(x+2) - 2\,log_2(x+3)}$$

**Compress:** $3\,log_5(3x+1) - 2\,log_5(2x-1) - log_5 x$

$$3\,log_5(3x+1) - 2\,log_5(2x-1) - log_5 x = log_5\left[(3x+1)^3\right] - log_5\left[(2x-1)^2\right] - log_5 x$$

$$= \boxed{log_5\left( \frac{(3x+1)^3}{x(2x-1)^2} \right)}$$

## _Change of Base Formula:_

**Suppose that** $y = \log_b x$. **Then** $b^y = x$ **and therefore** $\log_a (b^y) = \log_a x$. **From the Power**

**Rule, you get** $y \log_a b = \log_a x$, **and solving for _y_ yields** $y = \dfrac{\log_a x}{\log_a b}$. **So**

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

**Calculators have a logarithm key for base 10, log, called the common logarithm. They also have a logarithm key for base _e_, ln, called the natural logarithm.** $e = 2.7182818...$

$$\log_b x = \frac{\log x}{\log b}$$

**Or**

$$\log_b x = \frac{\ln x}{\ln b}$$

**Example:**

**Calculate $\log_3 5$ to 3 decimal places.**

$$\log_3 5 = \frac{\log 5}{\log 3}$$

**Or**

$$\log_3 5 = \frac{\ln 5}{\ln 3}$$

<span style="color:red">1.465</span>