Relations and Functions:

A Relation is a set of ordered pairs of numbers.

Examples:

$R = \{(1,2),(2,4),(3,5)\}$

$S = \{(1,2),(1,3),(2,6)\}$

Domain of a Relation:

The set of first numbers

Domain of R?

$\{1,2,3\}$

Domain of S?

$\{1,2\}$

Range of a Relation:

The set of second numbers

Range of R?

$\{2,4,5\}$

Range of S?

$\{2,3,6\}$

Function:

A function is a relation in which each number in the domain is associated with exactly one number in the range.

Is R a function?

Yes

Is S a function?

No, the domain value 1 is associated with two different range values, 2 and 3.

Relations and Functions from equations:

Sometimes relations are represented by an equation. The *x*-values correspond to domain values, and the *y*-values correspond to range values. If for each domain value *x*, it's possible to uniquely solve for the corresponding range value, *y*, then the relation represented by the equation is a function.

Determine if the following relations are functions:

$$x + y = 6$$

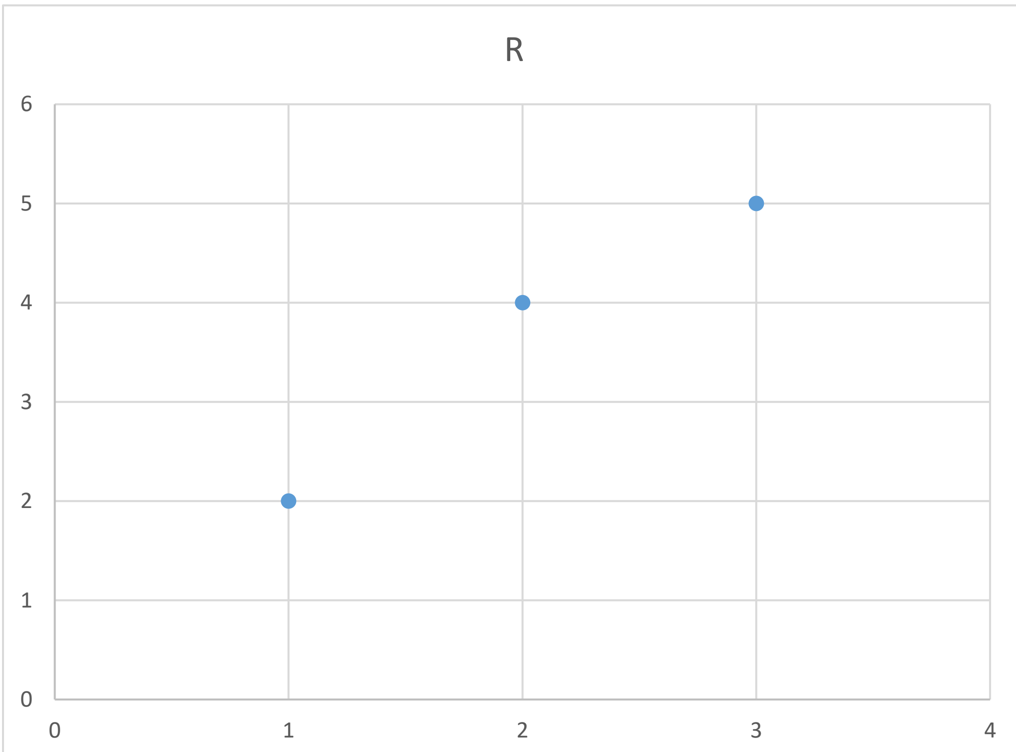
$y = 6 - x \Rightarrow$ each domain value x is associated with exactly one range value y, so the relation is a function.

$$x = y^2$$

The domain value 1 is associated with the range values of 1 and -1, so the relation is not a function.

Relations and Functions from graphs:

Sometimes relations are represented as graphs. The *x*-coordinates are the domain values, and the *y*-coordinates are the range values.

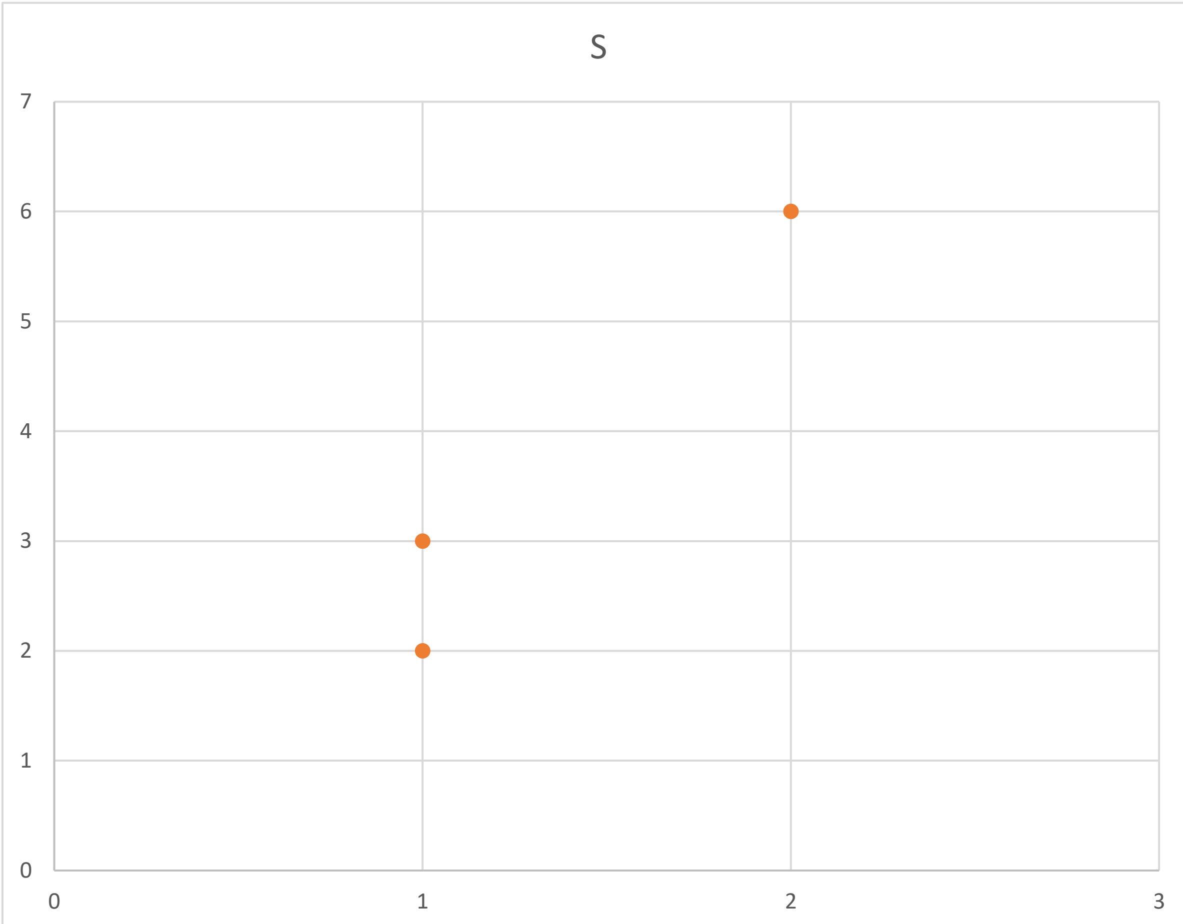


Domain? $\{1,2,3\}$

Range? $\{2,4,5\}$

Function? Yes

Vertical Line Test? Since no vertical line touches or crosses the graph in more than one point, it's the graph of a function.

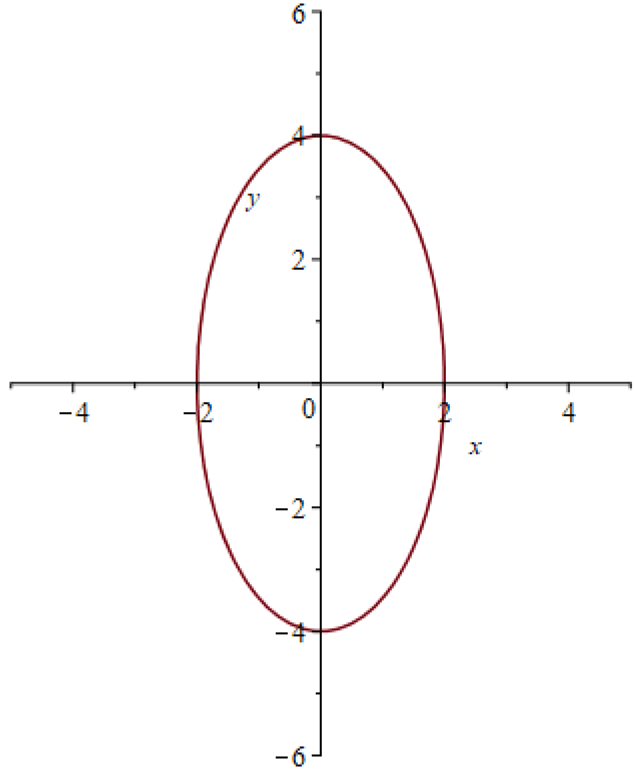


Domain? $\{1,2\}$

Range? $\{2,3,6\}$

Function? No, the vertical line $x = 1$ hits two points on the graph.

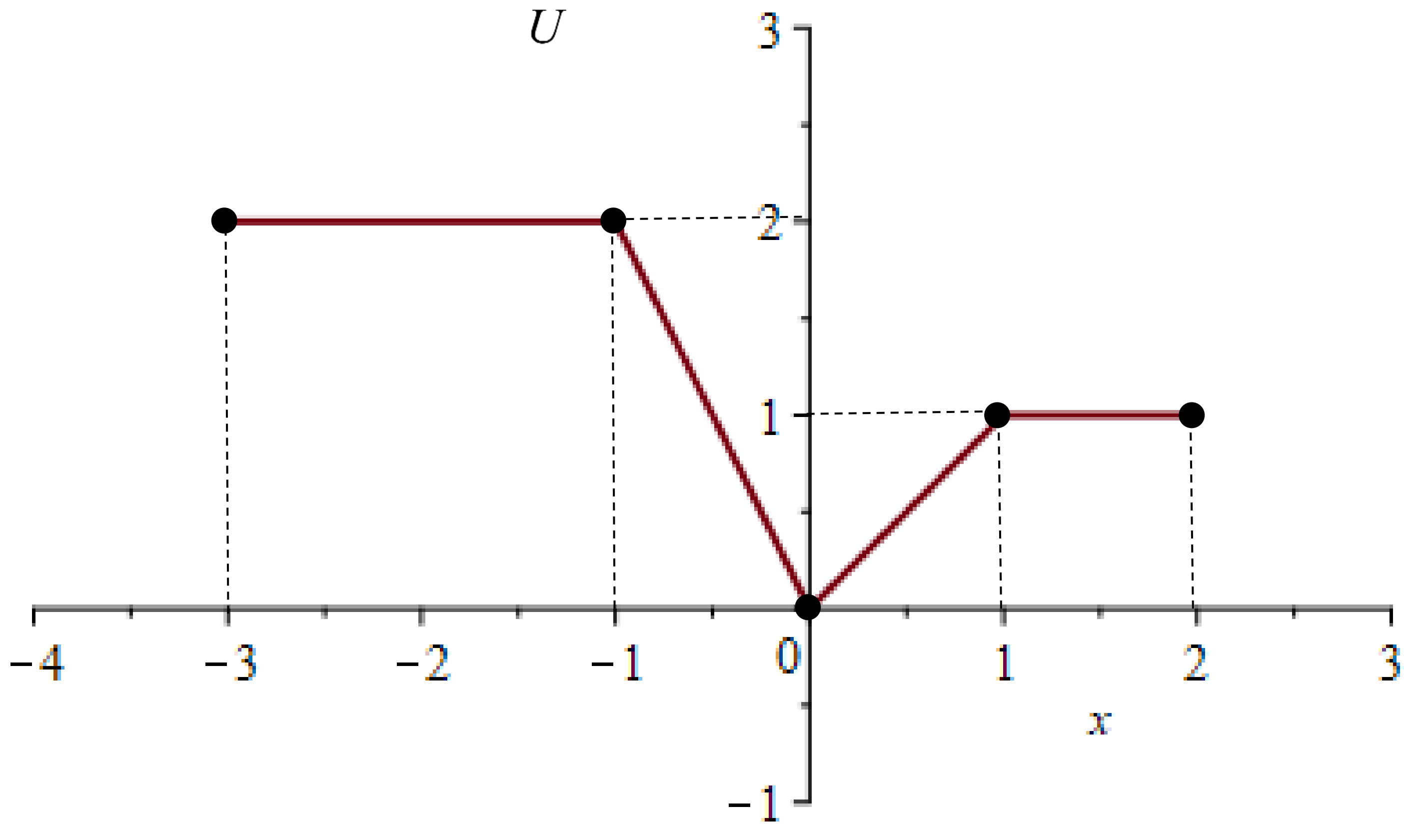
T



Domain? $[-2,2]$

Range? $[-4,4]$

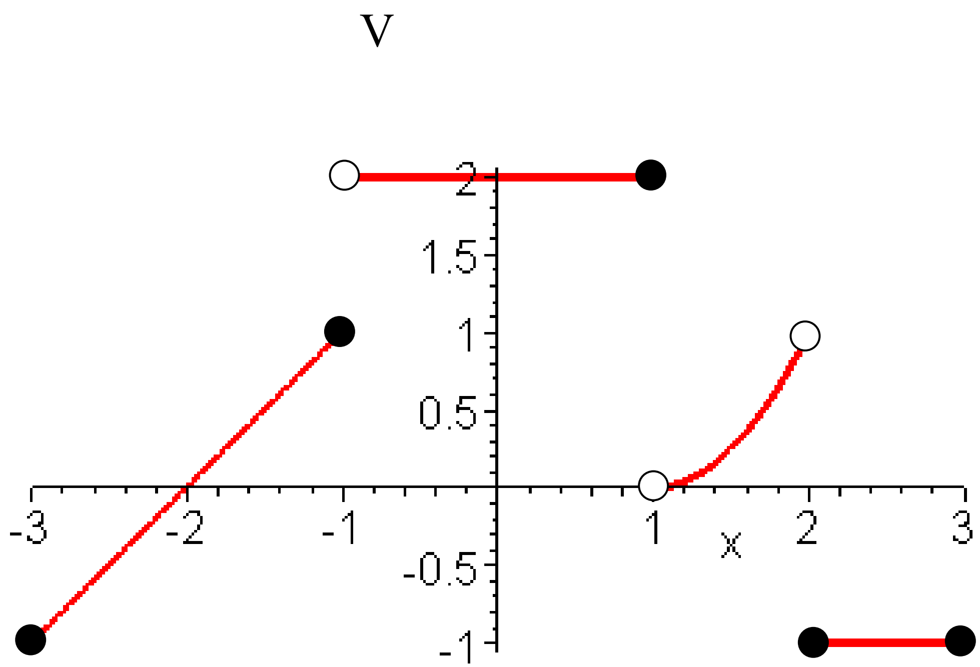
Function? No, the vertical line $x = 0$ crosses the graph in two points.



Domain? [−3,2]

Range? [0,2]

Function? Yes



Domain? $[-3,3]$

Range? $[-1,1]\cup\{2\}$

Function? Yes

Function Notation:

When a relation is a function, there is a special notation for connecting domain values and range values called function notation: $f(x)$. It's frequently used to define a function in terms of a formula.

$$f(x) = x^2 - 1$$

$$f(-1) = (-1)^2 - 1 = \boxed{0}, f(0) = 0^2 - 1 = \boxed{-1}, f(2) = 2^2 - 1 = \boxed{3}, f\left(\tfrac{1}{2}\right) = \left(\tfrac{1}{2}\right)^2 - 1 = \boxed{-\tfrac{3}{4}}, f(a) = \boxed{a^2 - 1}$$

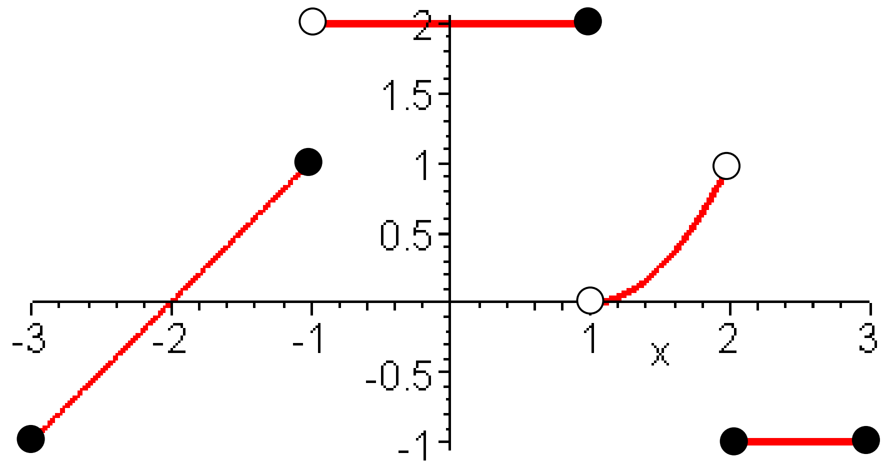
$$g(x) = \begin{cases} x & ; x \le -1 \\ 2x + 1 & ; x > -1 \end{cases}$$

$$g(-2) = -2, g(-1) = -1, g(1) = 2 \cdot 1 + 1 = \boxed{3}, g\left(\tfrac{1}{2}\right) = 2 \cdot \tfrac{1}{2} + 1 = \boxed{2},$$

$$g(x-1) = \begin{cases} x - 1; x - 1 \le -1 \\ 2(x-1) + 1; x - 1 > -1 \end{cases} = \boxed{\begin{cases} x - 1; x \le 0 \\ 2x - 1; x > 0 \end{cases}}$$

Use the g formula that corresponds to the inequality that is satisfied by the given domain value.

Function values from graphs: f



$f(-4)$, what's the y-coordinate of the point with x-coordinate -4?

undefined

$f(-3)$, what's the y-coordinate of the point with x-coordinate -3?

-1

$f(-2)$, what's the y-coordinate of the point with x-coordinate -2?

0

$f(-1) = 1, f(0) = 2, f(1) = 2, f\left(\frac{5}{2}\right) = -1$

x-intercepts: -2 **y-intercept:** 2

Solve the equations:

$f(x)=1,$ what are the x-coordinates of the points with y-coordinate of 1?

$$x=\boxed{-1}$$

$f(x)=\frac{3}{2},$ what are the x-coordinates of the points with y-coordinate of $\frac{3}{2}$?

$$\boxed{\text{no solution}}$$

$f(x)=2,$ what are the x-coordinates of the points with y-coordinate of 2?

$$-1<x\le 1\Rightarrow \boxed{(-1,1]}$$

Solve the inequalities:

$f(x)>1,$ what are the x-coordinates of the points with y-coordinates greater than 1?

$$-1<x\le 1\Rightarrow \boxed{(-1,1]}$$

$0\le f(x)<1,$ what are the x-coordinates of the points with y-coordinates satisfying $0\le y<1$?

$$-2\le x<-1 \text{ and } 1<x<2\Rightarrow \boxed{[-2,-1)\cup(1,2)}$$

Domains from function formulas:

When functions are defined using function notation and a formula, the domain is assumed to be all real values of *x* so that the formula produces a real number. The things to avoid are division by zero or an even root of a negative number.

$$f(x)=\frac{1}{x^2+3x+2},x^2+3x+2=0\Rightarrow(x+1)(x+2)=0\Rightarrow x=-1,-2$$

so the domain is $\boxed{(-\infty,-2)\cup(-2,-1)\cup(-1,\infty)}$

$$g(x)=\sqrt{1-2x},1-2x\geq 0\Rightarrow 2x\leq 1\Rightarrow x\leq\frac{1}{2}$$

so the domain is $\boxed{\left(-\infty,\frac{1}{2}\right]}$

$$h(x)=\frac{\sqrt{x-2}}{x-3},x-2\geq 0,x\neq 3\Rightarrow x\geq 2,x\neq 3$$

so the domain is $\boxed{[2,3)\cup(3,\infty)}$