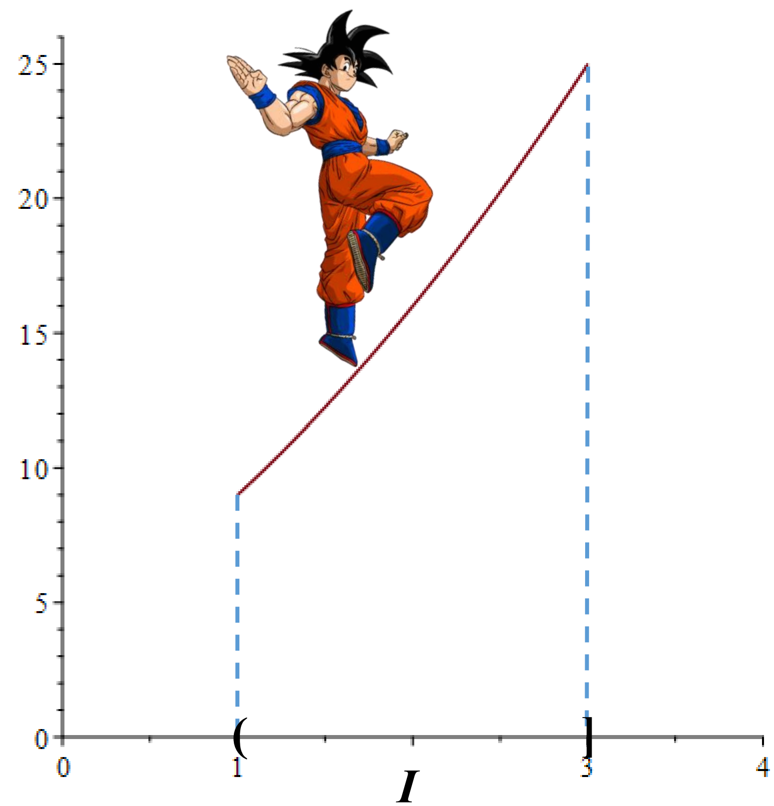
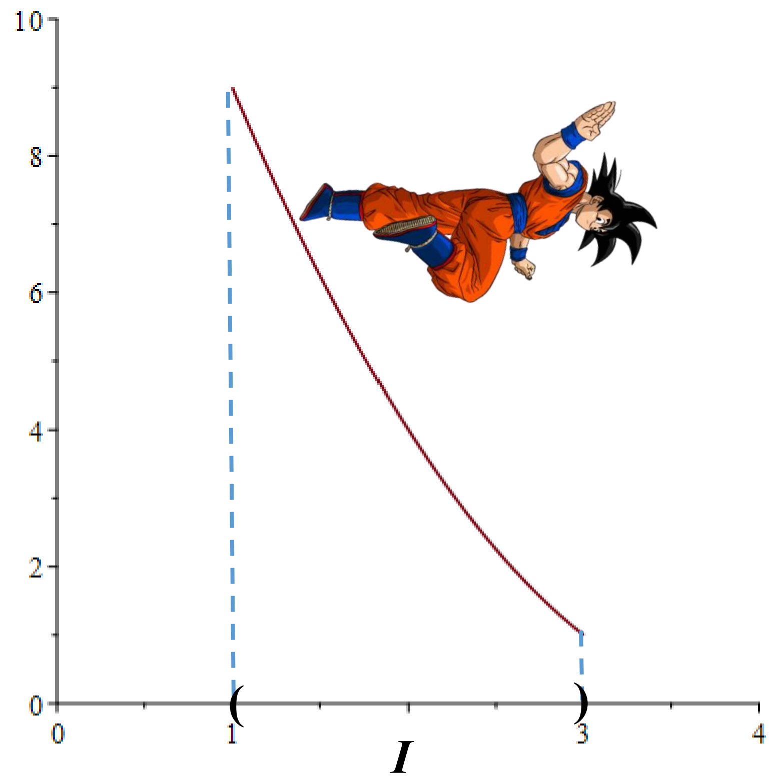
Increasing, Decreasing, and Constant:

A function *f* is increasing on an interval *I*, if for x, y in *I* with $x < y$, then $f(x) < f(y)$.



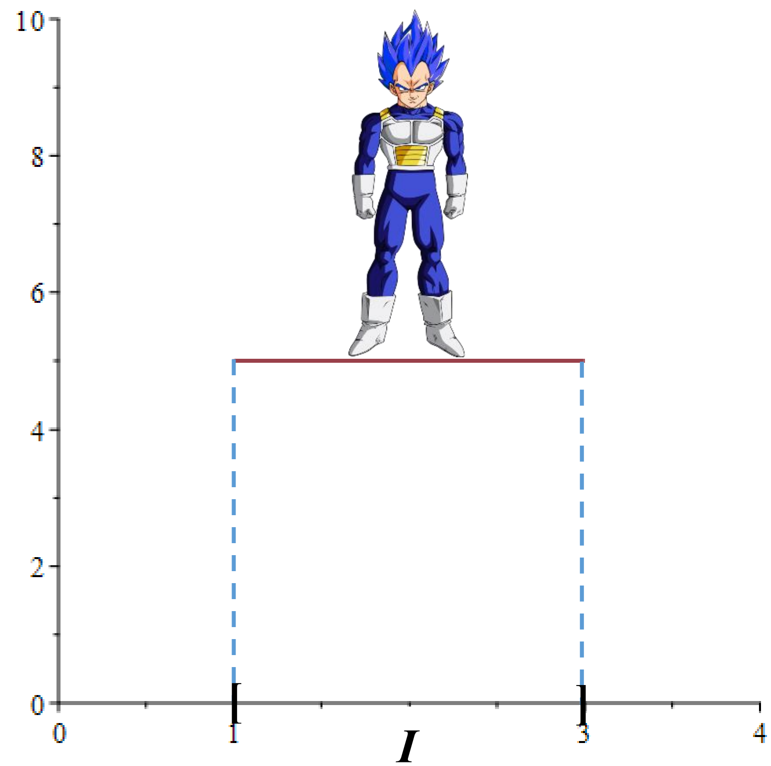
Moving up from left to right!

A function *f* is decreasing on an interval *I*, if for x, y in *I* with $x < y$, then $f(x) > f(y)$.



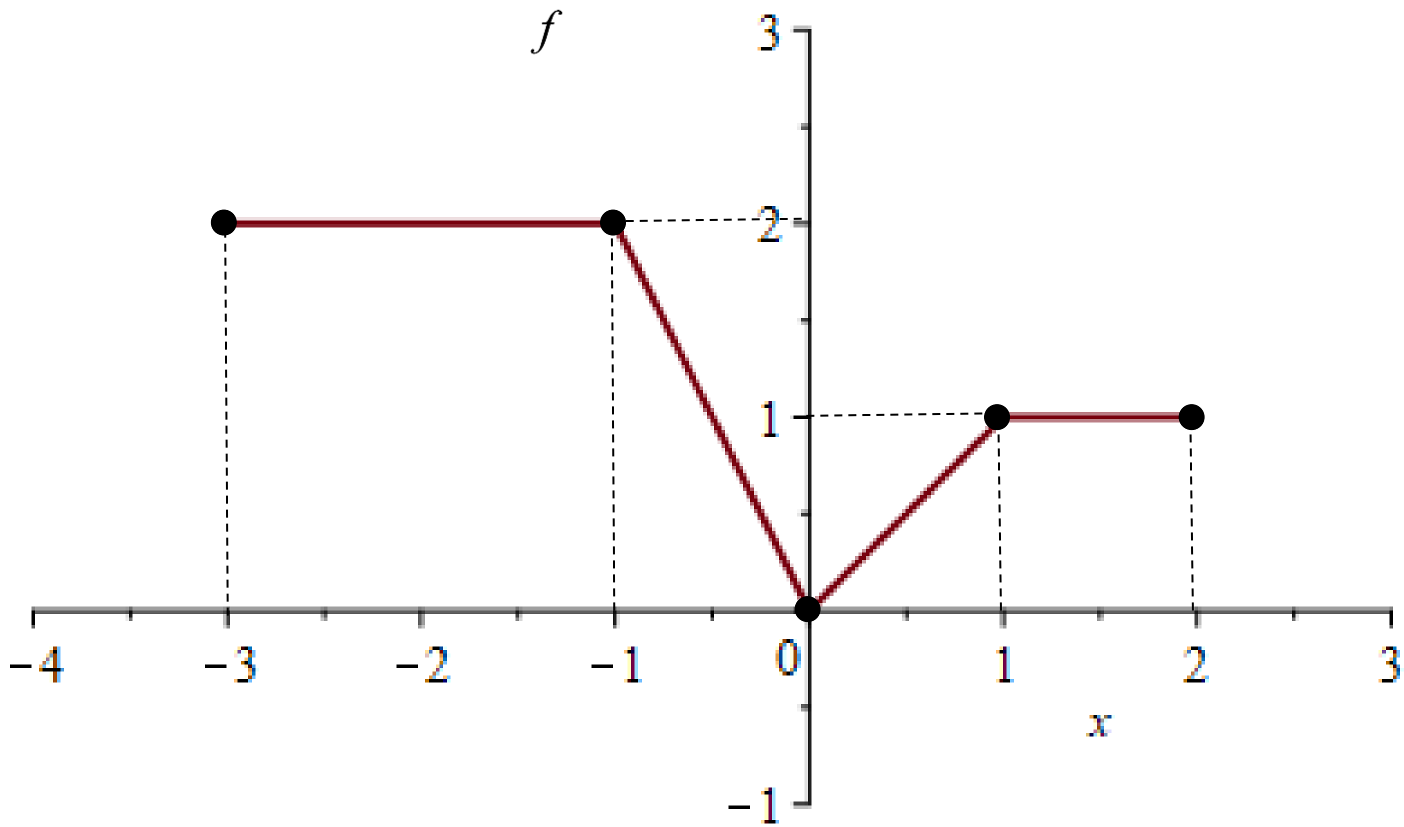
Moving down from left to right!

A function *f* is constant on an interval *I*, if for x, y in *I*, then $f(x) = f(y)$.



Level ground!

Determine the intervals where *f* is increasing, decreasing, and constant.

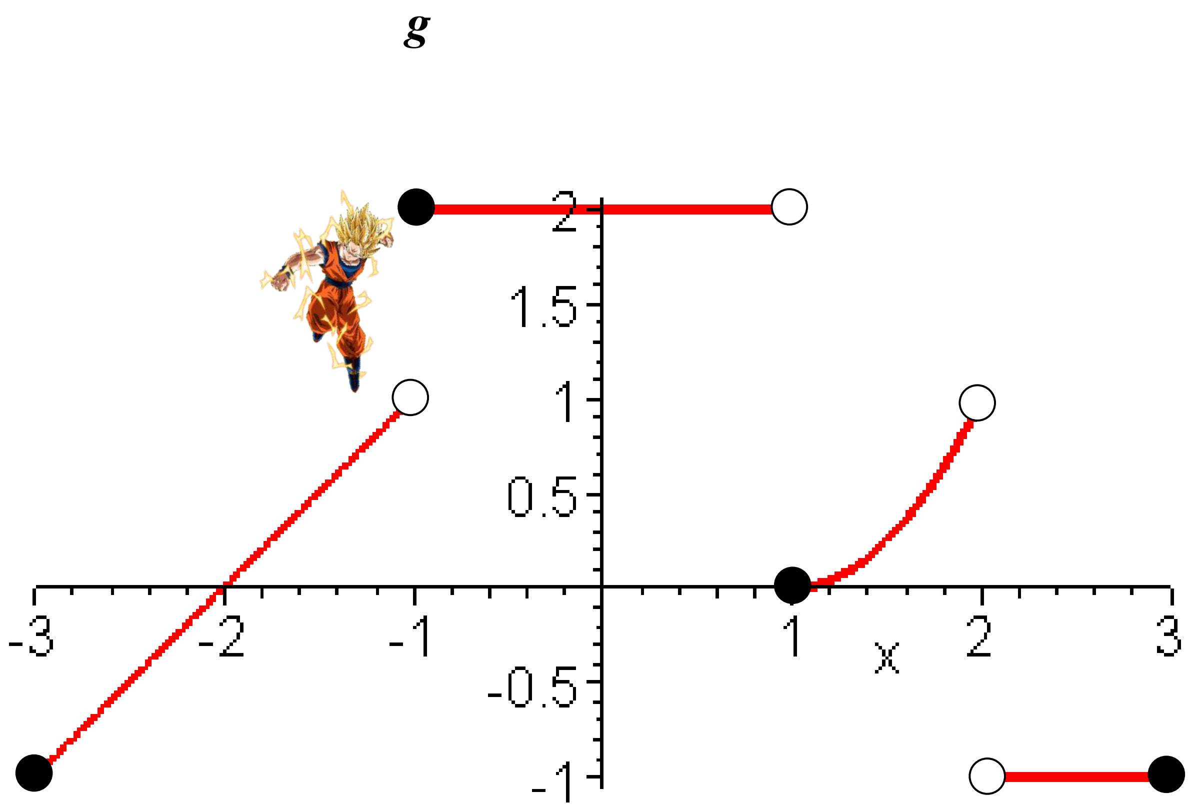


Increasing: $[0,1]$

Decreasing: $[-1,0]$

Constant: $[-3,-1]\cup[1,2]$

Determine the intervals where *g* is increasing, decreasing, and constant.



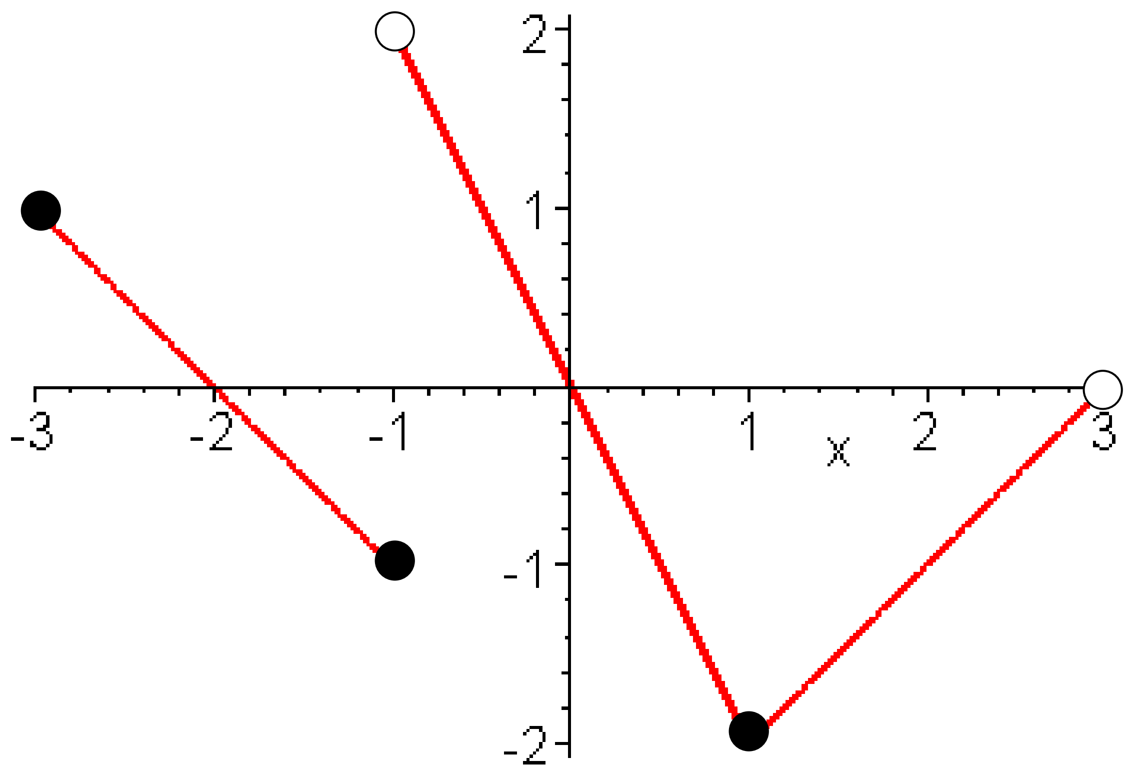
Increasing: $[-3,-1]\cup[1,2)$

Decreasing: nowhere

Constant: $[-1,1)\cup(2,3]$

Determine the intervals where *h* is increasing, decreasing, and constant.

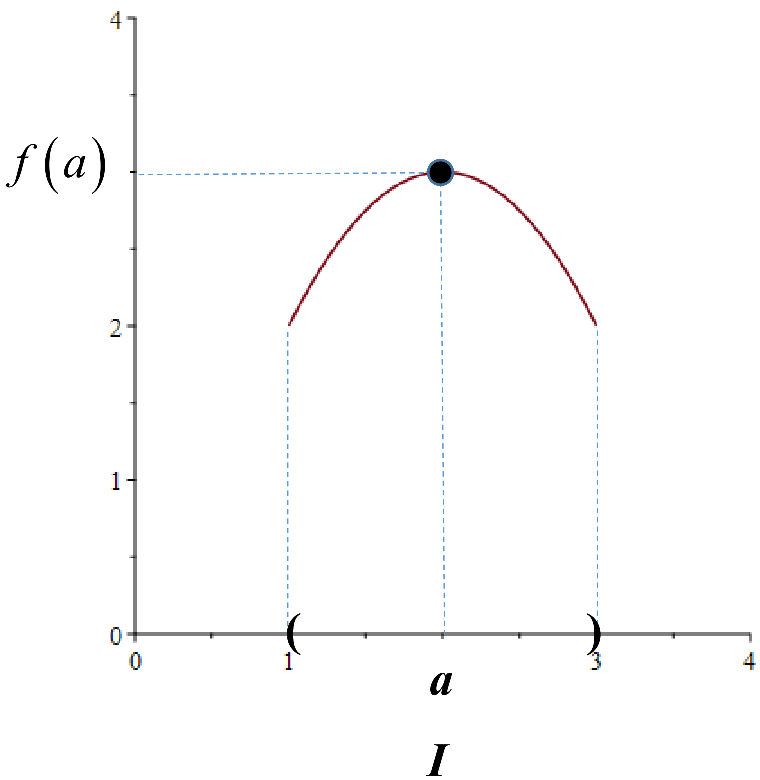
h

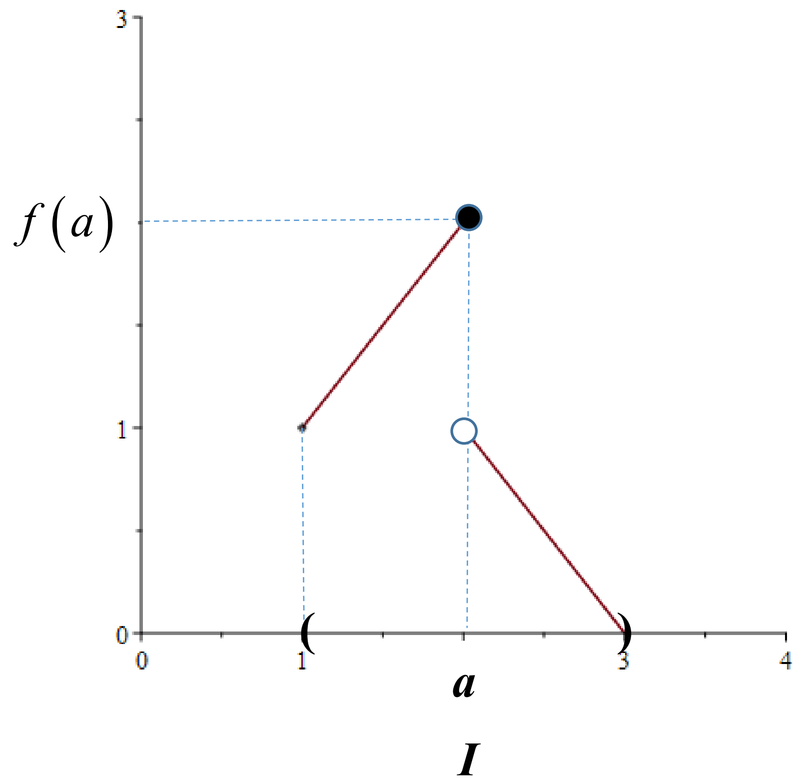


Increasing: $[1,3)$ Decreasing: $[-3,-1]\cup(-1,1]$ Constant: nowhere

Local (or Relative) Extrema:

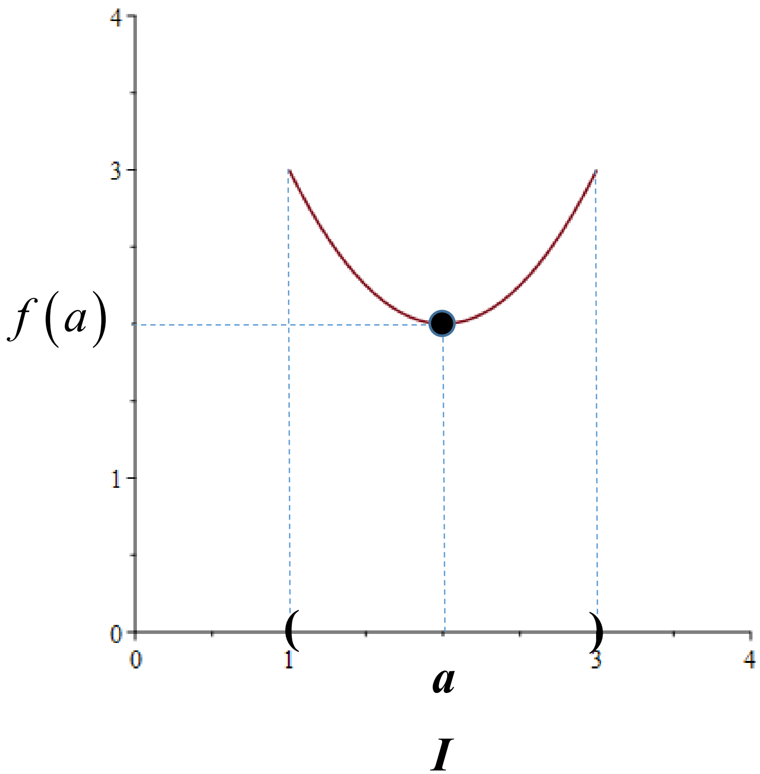
A function f has a local(or relative) maximum at a, if there is an open interval I containing a with $f(x) < f(a)$ for all x in I with $x \neq a$.

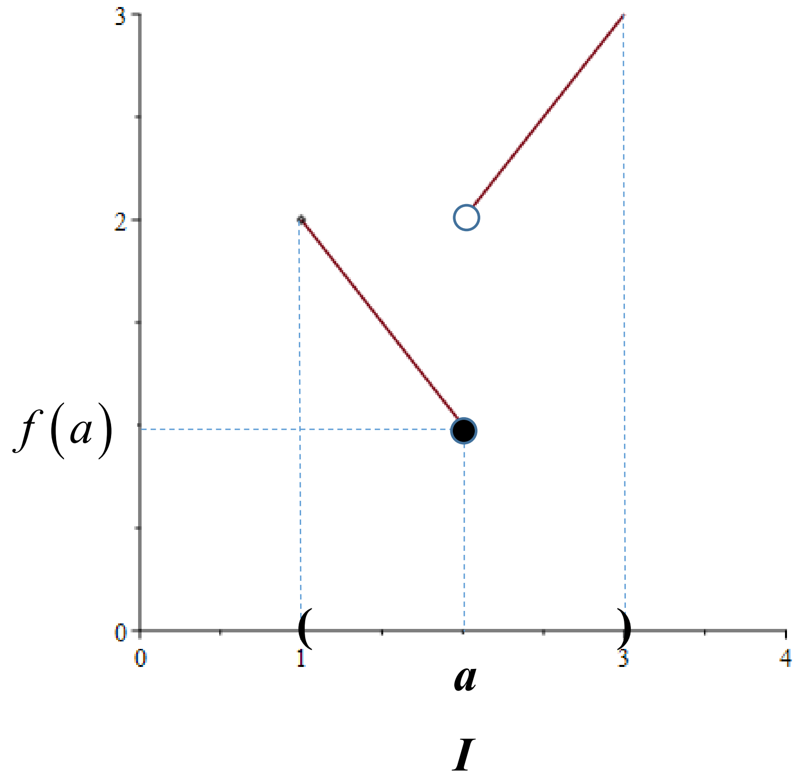




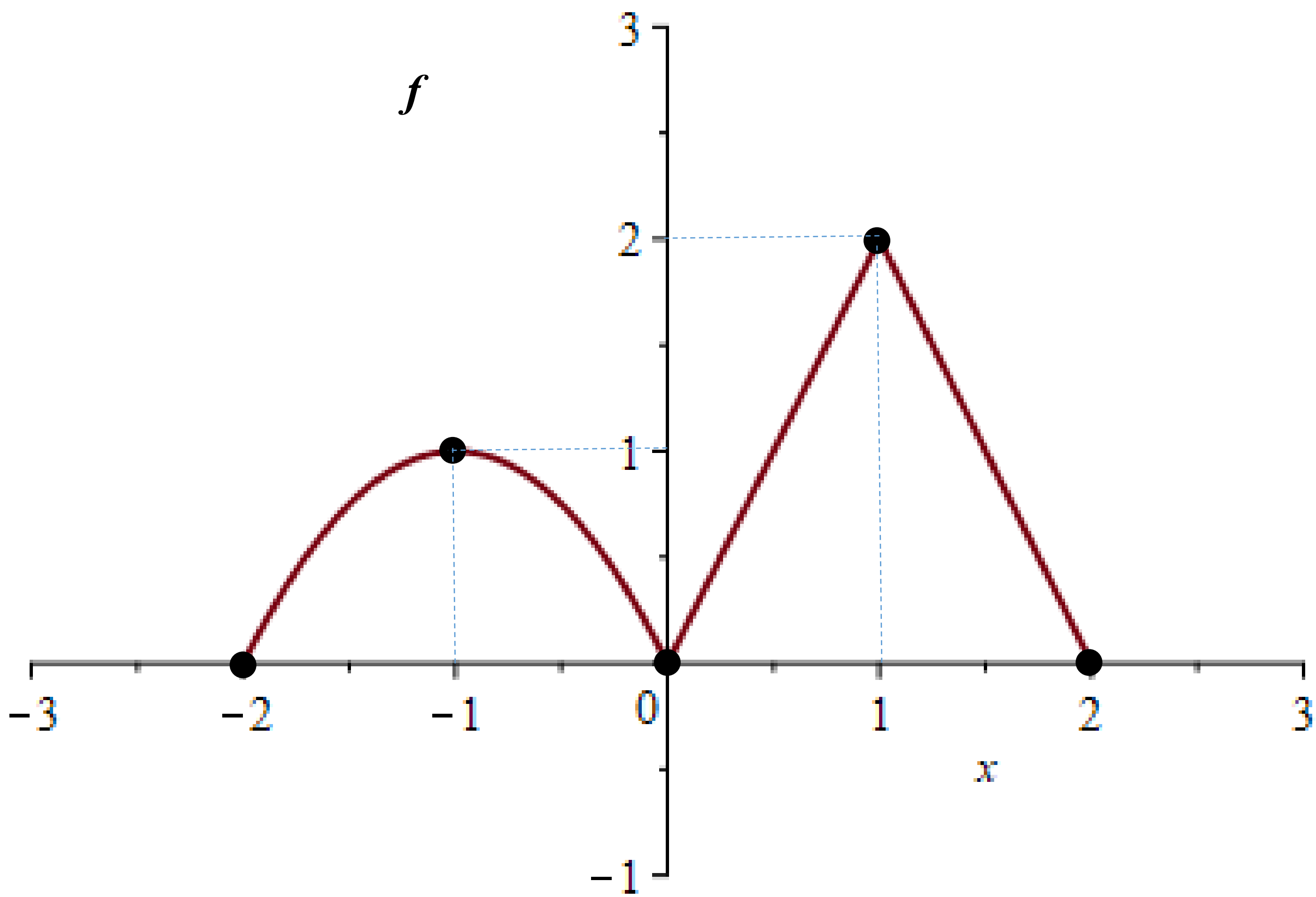
A local maximum corresponds to a high spot in the graph of the function!

A function *f* has a local minimum at *a*, if there is an open interval *I* containing *a* with $f(x) > f(a)$ for all *x* in *I* with $x \neq a$.





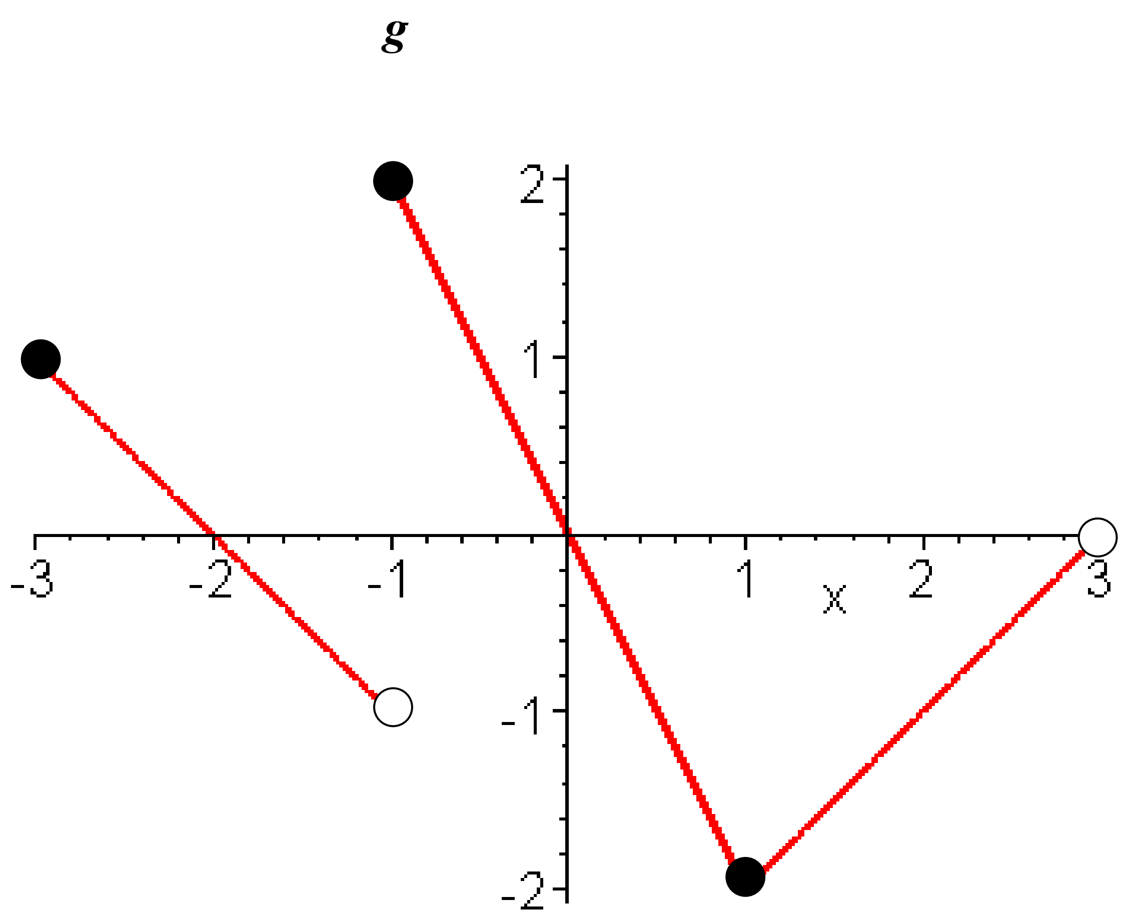
A local minimum corresponds to a low spot in the graph of the function!



Find all the local extrema of the function *f*.

Local Maxima: $x = -1, 1$ Local Minima: $x = 0$

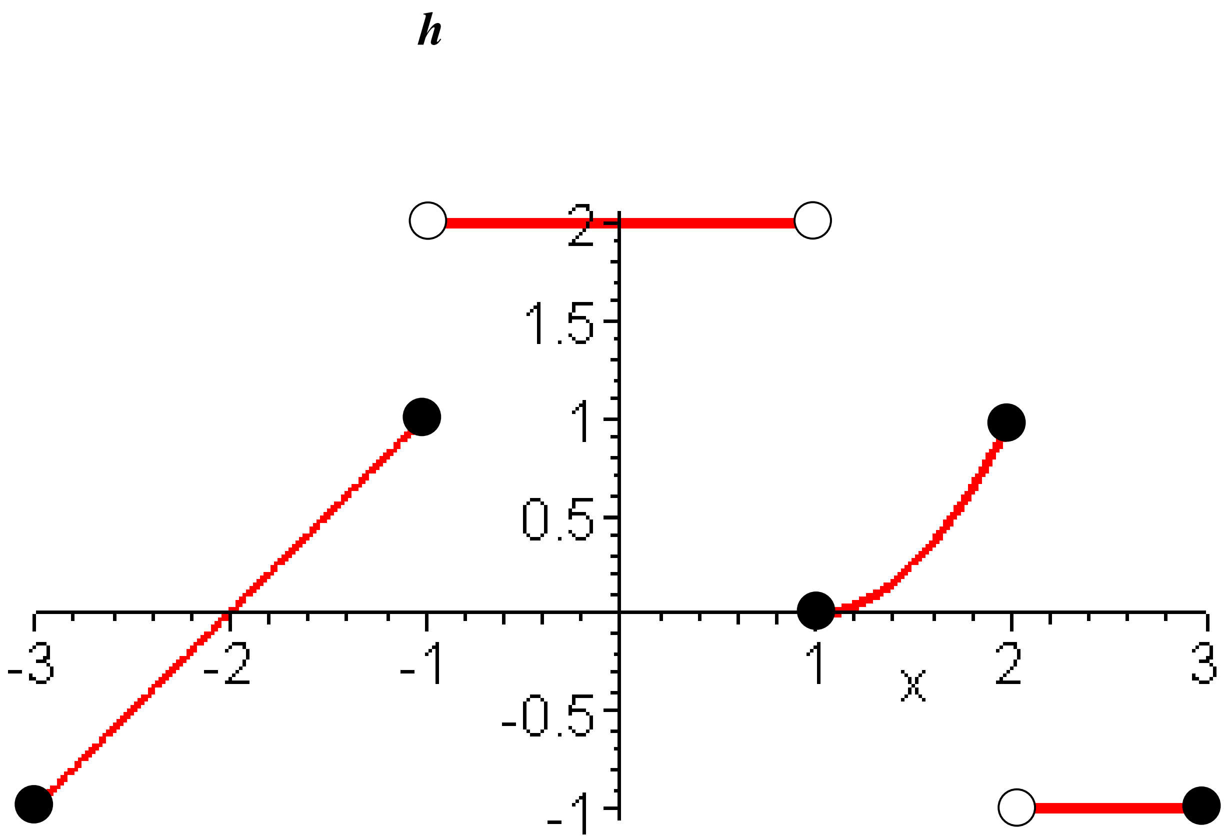
You don't have local minima at $x = -2, 2$ because you don't have portions of the graph on both sides of the corresponding points on the graph.



Find all the local extrema of the function *g*.

Local Maxima: $x = -1$

Local Minima: $x = 1$



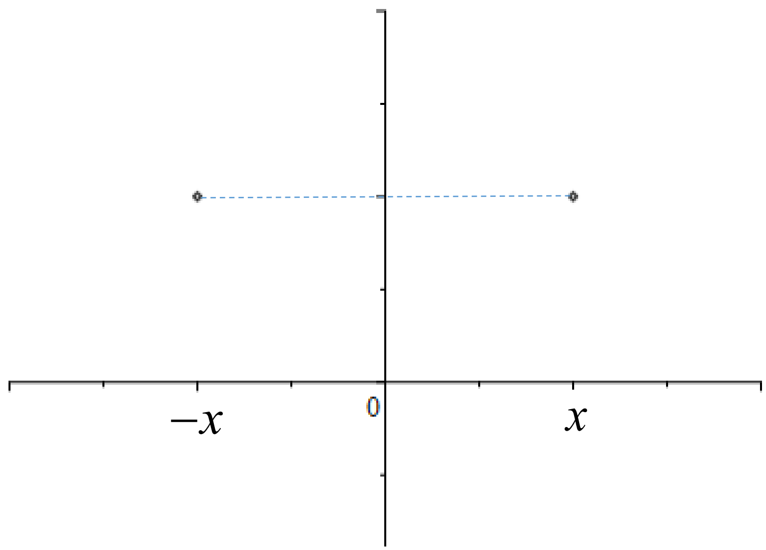
Find all the local extrema of the function *h*.

Local Maxima: $x = 2$

Local Minima: $x = 1$

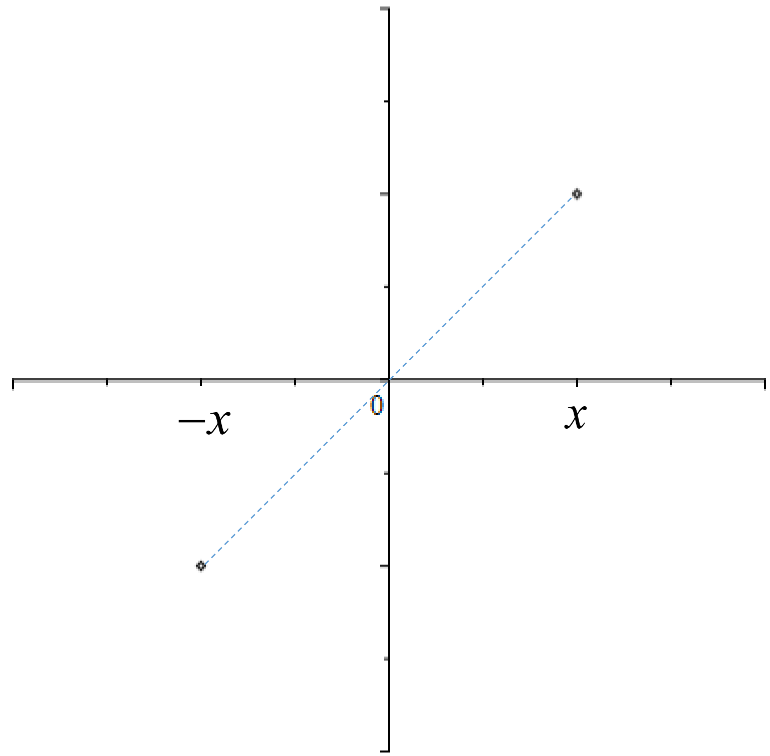
Even and Odd Functions:

A function *f* is even if $f(-x) = f(x)$ **for all *x* in the domain of *f*.**



The graph has *y*-axis symmetry.

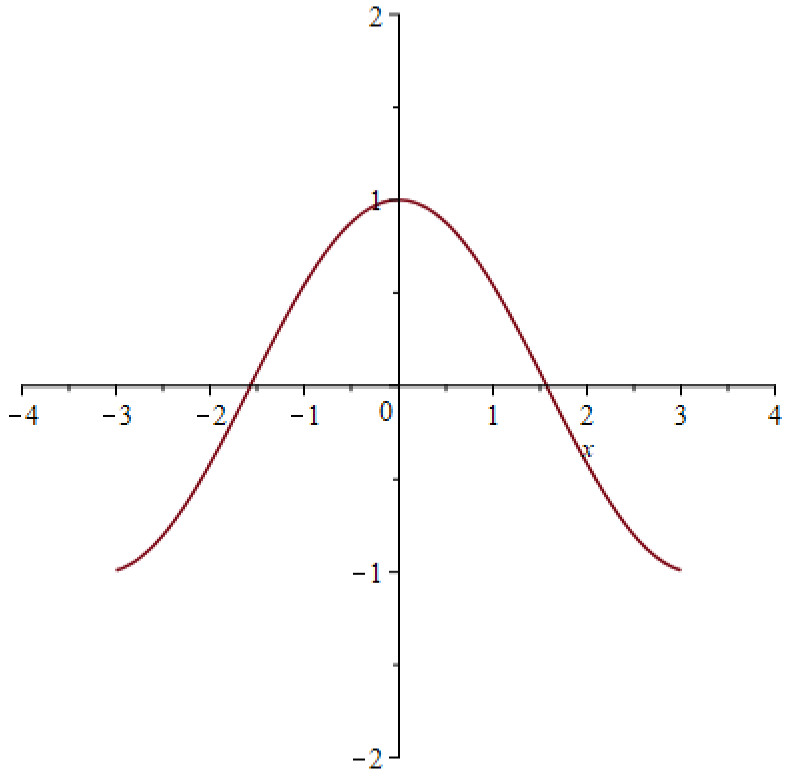
A function *f* is odd if $f(-x) = -f(x)$ for all *x* in the domain of *f*.



The graph has origin symmetry.

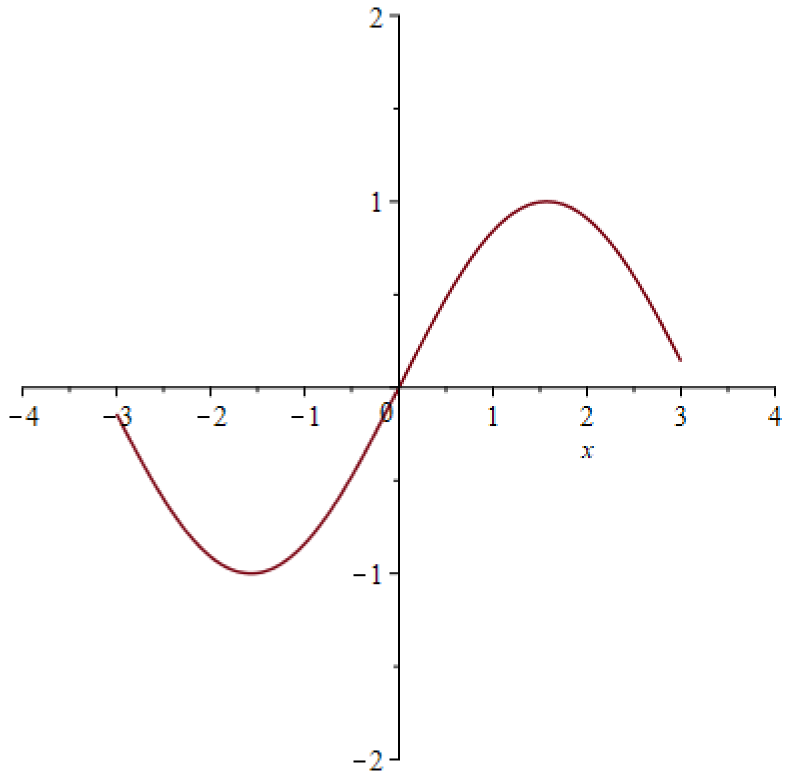
Determine if the following functions are odd, even, neither, or both.

1.



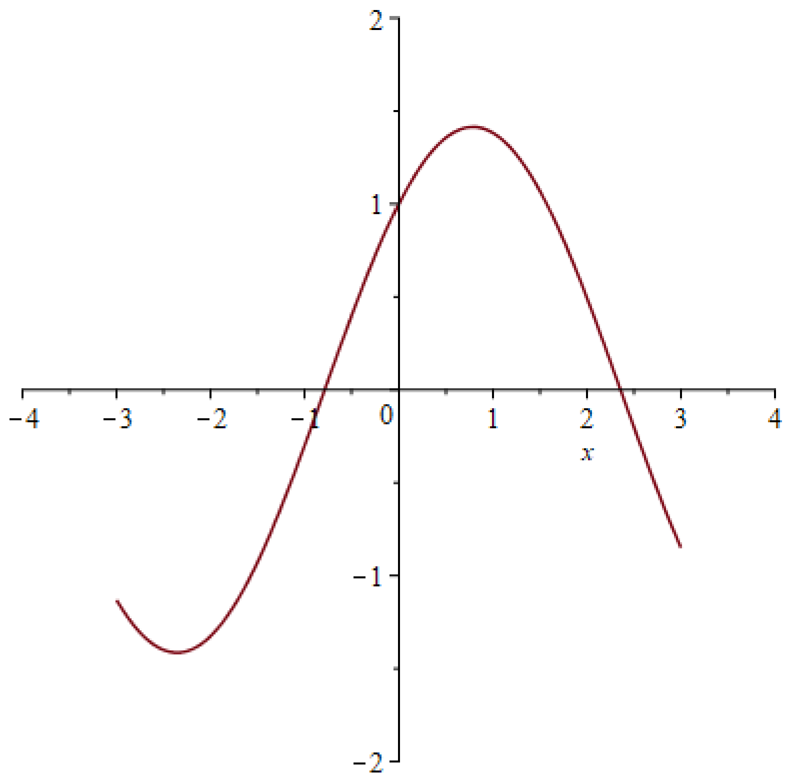
The graph has y-axis symmetry, so it's even.

2.



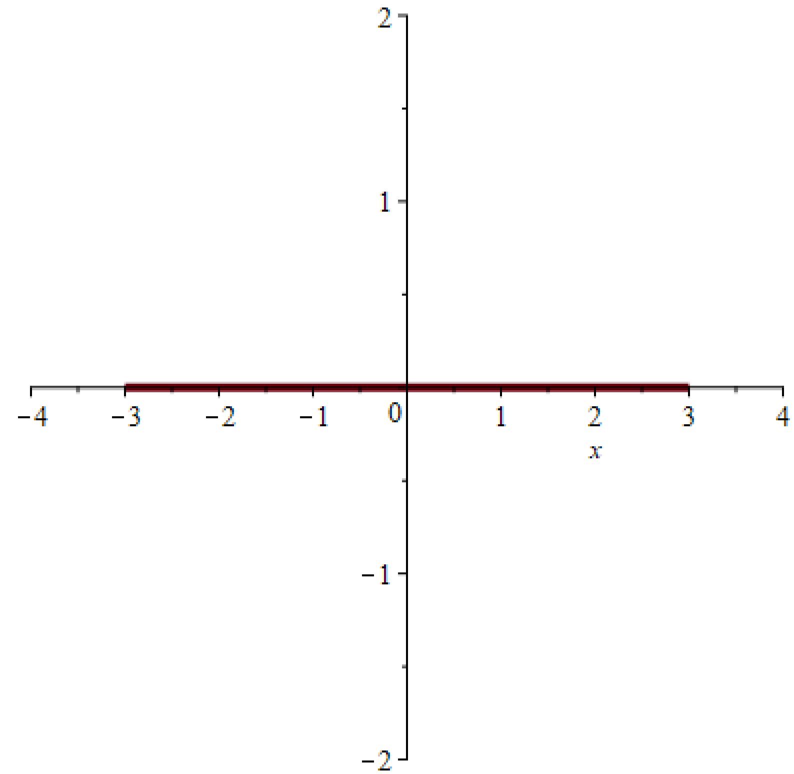
The graph has origin symmetry, so it's odd.

3.



The graph has neither y-axis nor origin symmetry, so it’s neither.

4.



The graph has both *y*-axis and origin symmetry, so it’s both.

5.

$$f(x) = 2x^4 - x^2$$

$$f(-x) = 2(-x)^4 - (-x)^2$$

$$= 2x^4 - x^2 = f(x)$$

$$\neq -\left(2x^4 - x^2\right) = -f(x)$$

$$\boxed{Even}$$

6.

$$f(x) = x^3 + x$$

$$f(-x) = (-x)^3 + (-x)$$

$$= -x^3 - x = -\left(x^3 + x\right) = -f(x)$$

$$\neq x^3 + x = f(x)$$

$$\boxed{Odd}$$

7. $f(x) = x + x^2$

$$f(-x) = (-x) + (-x)^2$$

$$= -x + x^2 \neq f(x) \neq -f(x)$$

$\boxed{Neither}$

8. $f(x) = (x+1)^2 - (x-1)^2 - 4x = x^2 + 2x + 1 - x^2 + 2x - 1 - 4x = 0$

$$\Rightarrow f(-x) = 0 = f(x)$$

$$\Rightarrow f(-x) = 0 = -0 = -f(x)$$

$\boxed{Both}$

9.

$$f(x)=\begin{cases} x^3 & ;x\geq 0 \\ -x^3 & ;x\leq 0 \end{cases}$$

$$f(-x)=\begin{cases} (-x)^3 & ;-x\geq 0 \\ -(-x)^3 & ;-x\leq 0 \end{cases}=\begin{cases} -x^3 & ;x\leq 0 \\ x^3 & ;x\geq 0 \end{cases}=\begin{cases} x^3 & ;x\geq 0 \\ -x^3 & ;x\leq 0 \end{cases}=f(x)$$

$$\neq\begin{cases} -x^3 & ;x\geq 0 \\ x^3 & ;x\leq 0 \end{cases}=-f(x)$$

$\boxed{Even}$