Properties/Rules/Laws of Logarithms:

For *M* and *N* positive numbers and *r* a real number,

Product Rule:

Expansion →

$$\log_b(MN) = \log_b M + \log_b N$$

← **Compression**

Why? $b^{\log_b(MN)} = MN$ **and** $b^{(\log_b M + \log_b N)} = b^{\log_b M} \cdot b^{\log_b N} = MN$

Expand and simplify:

$$\log_5(25x)$$

$$\log_5(25x) = \log_5 25 + \log_5 x = \boxed{2 + \log_5 x}$$

Compress(or Condense) and simplify:

$$\log_6 9 + \log_6 4$$

$$\log_6 9 + \log_6 4 = \log_6 36 = \boxed{2}$$

Quotient Rule:

Expansion →

$$log_b\left(\frac{M}{N}\right) = log_b M - log_b N$$

← **Compression**

Why? $b^{log_b\left(\frac{M}{N}\right)} = \frac{M}{N}$ **and** $b^{(log_b M - log_b N)} = \frac{b^{log_b M}}{b^{log_b N}} = \frac{M}{N}$

Expand and simplify:

$$log_3\left(\frac{x}{9}\right)$$

$$log_3\left(\frac{x}{9}\right) = log_3 x - log_3 9 = \boxed{log_3 x - 2}$$

Compress and simplify:

$$log_3 2 - log_3 6$$

$$log_3 2 - log_3 6 = log_3\left(\tfrac{1}{3}\right) = \boxed{-1}$$

<u>Power Rule:</u>

Expansion →

$$log_b\left(M^r\right) = r\,log_b M$$

← **Compression**

Why? $b^{log_b\left(M^r\right)} = M^r$ **and** $b^{r\,log_b M} = \left(b^{log_b M}\right)^r = M^r$

Expand and simplify:

$$log_7\left(7x^5\right)$$

$$log_7\left(7x^5\right) = log_7 7 + 5\,log_7 x = \boxed{1 + 5\,log_7 x}$$

Compress:

$$2\,log_3 x - 4\,log_3 y$$

$$2\,log_3 x - 4\,log_3 y = log_3 x^2 - log_3 y^4 = \boxed{log_3\left(\frac{x^2}{y^4}\right)}$$

Expand: $log_2\left[\frac{x^3(x+2)}{(x+3)^2}\right]$

$$log_2\left[\frac{x^3(x+2)}{(x+3)^2}\right]=log_2 x^3+log_2(x+2)-log_2\left[(x+3)^2\right]$$

$$=\boxed{3log_2 x+log_2(x+2)-2log_2(x+3)}$$

Compress: $3log_5(3x+1)-2log_5(2x-1)-log_5 x$

$$3log_5(3x+1)-2log_5(2x-1)-log_5 x=log_5\left[(3x+1)^3\right]-log_5\left[(2x-1)^2\right]-log_5 x$$

$$=\boxed{log_5\left(\frac{(3x+1)^3}{x(2x-1)^2}\right)}$$

Change of Base Formula:

Suppose that $y = log_b x$**. Then** $b^y = x$ **and therefore** $log_a\left(b^y\right) = log_a x$**. From the Power Rule, you get** $y\,log_a b = log_a x$**, and solving for *y* yields** $y = \frac{log_a x}{log_a b}$**. So**

$$log_b x = \frac{log_a x}{log_a b}.$$

Calculators have a logarithm key for base 10, log, called the common logarithm. They also have a logarithm key for base *e*, ln, called the natural logarithm. $e = 2.7182818...$

$$log_b x = \frac{log\,x}{log\,b}$$

Or

$$log_b x = \frac{ln\,x}{ln\,b}$$

Example:

Calculate $log_3 5$ **to 3 decimal places.**

$$log_3 5 = \frac{log\, 5}{log\, 3}$$

Or

$$log_3 5 = \frac{ln\, 5}{ln\, 3}$$

1.465