

Properties/Rules/Laws of Logarithms:

For M and N positive numbers and r a real number,

Product Rule:

Expansion



$$\log_b(MN) = \log_b M + \log_b N$$



Compression

Why? $b^{\log_b(MN)} = MN$

and $b^{(\log_b M + \log_b N)} = b^{\log_b M} \cdot b^{\log_b N} = MN$

Expand and simplify:

$$\log_5(25x)$$


$$\log_5(25x) = \log_5 25 + \log_5 x = \boxed{2 + \log_5 x}$$


Compress(or Condense) and simplify:

$$\log_6 9 + \log_6 4$$

$$\log_6 9 + \log_6 4 = \log_6 36 = \boxed{2}$$

Quotient Rule:

Expansion

$$\log_b \left(\frac{M}{N} \right) = \log_b M - \log_b N$$


Compression

Why? $b^{\log_b \left(\frac{M}{N} \right)} = \frac{M}{N}$ and $b^{(\log_b M - \log_b N)} = \frac{b^{\log_b M}}{b^{\log_b N}} = \frac{M}{N}$

Expand and simplify:

$$\log_3 \left(\frac{x}{9} \right)$$

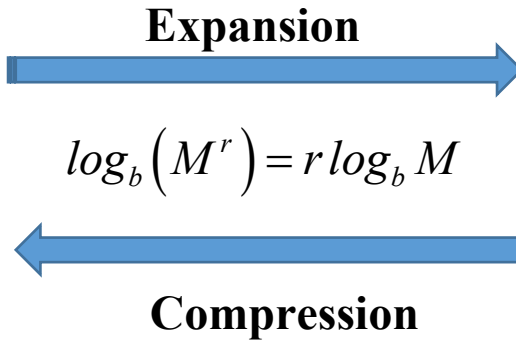
$$\log_3 \left(\frac{x}{9} \right) = \log_3 x - \log_3 9 = \boxed{\log_3 x - 2}$$

Compress and simplify:

$$\log_3 2 - \log_3 6$$

$$\log_3 2 - \log_3 6 = \log_3 \left(\frac{1}{3} \right) = \boxed{-1}$$

Power Rule:



Why? $b^{\log_b(M^r)} = M^r$

and $b^{r \log_b M} = (b^{\log_b M})^r = M^r$

Expand and simplify:

$$\log_7(7x^5)$$

$$\log_7(7x^5) = \log_7 7 + 5\log_7 x = \boxed{1 + 5\log_7 x}$$

Compress:

$$2\log_3 x - 4\log_3 y$$

$$2\log_3 x - 4\log_3 y = \log_3 x^2 - \log_3 y^4 = \boxed{\log_3 \left(\frac{x^2}{y^4} \right)}$$

Expand: $\log_2 \left[\frac{x^3(x+2)}{(x+3)^2} \right]$

$$\begin{aligned} \log_2 \left[\frac{x^3(x+2)}{(x+3)^2} \right] &= \log_2 x^3 + \log_2(x+2) - \log_2 \left[(x+3)^2 \right] \\ &= \boxed{3\log_2 x + \log_2(x+2) - 2\log_2(x+3)} \end{aligned}$$

Compress: $3\log_5(3x+1) - 2\log_5(2x-1) - \log_5 x$

$$\begin{aligned} 3\log_5(3x+1) - 2\log_5(2x-1) - \log_5 x &= \log_5 \left[(3x+1)^3 \right] - \log_5 \left[(2x-1)^2 \right] - \log_5 x \\ &= \boxed{\log_5 \left(\frac{(3x+1)^3}{x(2x-1)^2} \right)} \end{aligned}$$

Change of Base Formula:

Suppose that $y = \log_b x$. Then $b^y = x$ and therefore $\log_a (b^y) = \log_a x$. From the Power

Rule, you get $y \log_a b = \log_a x$, and solving for y yields $y = \frac{\log_a x}{\log_a b}$. So

$$\log_b x = \frac{\log_a x}{\log_a b}.$$

Calculators have a logarithm key for base 10, \log , called the common logarithm. They also have a logarithm key for base e , \ln , called the natural logarithm. $e = 2.7182818...$

$$\log_b x = \frac{\log x}{\log b}$$

Or

$$\log_b x = \frac{\ln x}{\ln b}$$

Example:

Calculate $\log_3 5$ to 3 decimal places.

$$\log_3 5 = \frac{\log 5}{\log 3}$$

Or

$$\log_3 5 = \frac{\ln 5}{\ln 3}$$

1.465