

Preference Ballot:

It's a ballot in which the voter lists his/her choices in order of preference.



Example:

In an election with candidates of A, B, C, and D, a preference ballot might be

For Mayor	1st Choice	2nd Choice	3rd Choice	4th Choice
JOHN ADAMS	☐	☒	☐	☐
THOMAS JEFFERSON	☐	☐	☐	☐
ABRAHAM LINCOLN	☒	☐	☐	☐
GEORGE WASHINGTON	☐	☐	☐	☐

Ballot
1 st : B
2 nd : C
3 rd : A
4 th : D

Preference Ballots

- In order to get more information from voters, a better way to hold an election is to allow voters to rank order the candidates rather than just pick their favorite. This shows not only who you want to win, but also who you want to lose (and everyone in-between).

Preference Table:

It's a table that summarizes all the preference ballots cast in an election.

Ballot 1st C 2nd E 3rd D 4th A 5th B	Ballot 1st A 2nd D 3rd B 4th C 5th E	Ballot 1st B 2nd D 3rd A 4th C 5th D	Ballot 1st A 2nd E 3rd C 4th D 5th E	Ballot 1st C 2nd B 3rd D 4th A 5th B	Ballot 1st D 2nd E 3rd B 4th A 5th C	Ballot 1st A 2nd C 3rd D 4th E 5th B
Ballot 1st B 2nd E 3rd A 4th C 5th D	Ballot 1st A 2nd D 3rd C 4th B 5th E	Ballot 1st D 2nd C 3rd B 4th A 5th E	Ballot 1st A 2nd C 3rd D 4th E 5th B	Ballot 1st C 2nd B 3rd D 4th A 5th E	Ballot 1st D 2nd E 3rd B 4th A 5th C	Ballot 1st A 2nd C 3rd D 4th E 5th B
Ballot 1st B 2nd E 3rd A 4th C 5th D	Ballot 1st C 2nd D 3rd B 4th A 5th E	Ballot 1st A 2nd E 3rd C 4th D 5th B	Ballot 1st C 2nd D 3rd B 4th A 5th E	Ballot 1st A 2nd D 3rd B 4th C 5th E	Ballot 1st D 2nd C 3rd E 4th A 5th B	Ballot 1st D 2nd C 3rd B 4th E 5th A

Example:

Number of votes	120	100	80	75
1st	A	B	A	D
2nd	B	C	D	C
3rd	C	D	C	B
4th	D	A	B	A

1) How many people voted in the election?

2) How many voted for the order A, D, C, B?

3) How many chose A as their first choice?

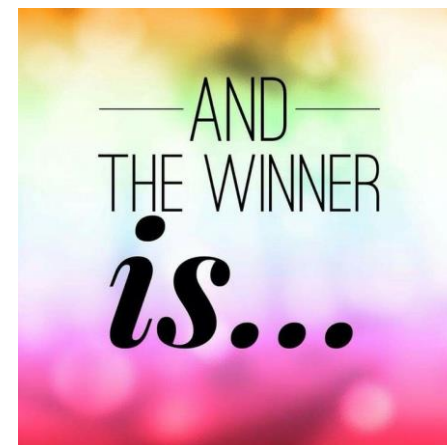
4) How many chose A as their second choice?

5) How many chose A as their fourth choice?

Methods of Determining a Winner in an Election:

The Plurality Method:

The candidate with the most first-place votes is the winner.



Example: In the following election, who is the plurality winner?

Number of votes	120	100	80	75
1 st	A	B	B	D
2 nd	B	C	A	C
3 rd	C	D	C	B
4 th	D	A	B	A

Plurality Method

- ▶ This is the method that many people are most comfortable with. In this method, the person with the most 1st place votes wins (so only look at the 1st row to determine the winner).

The Borda Count Method:

Each last-place vote is given 1 point. Each next-to-last-place vote is given 2 points, and so on. The candidate with the most total points is the winner.

Example: In the following election, who is the Borda count winner?

Number of votes	15	10	8	6
1 st	A	B	C	B
2 nd	C	C	A	A
3 rd	B	A	B	C



Candidate	3 rd –place points	2 nd –place points	1 st –place points	Total
A	$10 \cdot 1 = 10$	$14 \cdot 2 = 28$	$15 \cdot 3 = 45$	83
B	$23 \cdot 1 = 23$	$0 \cdot 2 = 0$	$16 \cdot 3 = 48$	71
C	$6 \cdot 1 = 6$	$25 \cdot 2 = 50$	$8 \cdot 3 = 24$	80

Who is the plurality winner?

The Plurality-with-Elimination Method:

The candidate with a majority of first-place votes is the winner. If no candidate received a majority of first-place votes, then the candidate or candidates with the fewest first-place votes is(are) eliminated. If a candidate now has a majority of first-place votes, then that candidate is the winner. If not, continue the elimination process, until a candidate receives a majority of first-place votes.

Example: In the following election, who is the plurality-with-elimination winner?



Number of votes	12	10	9	5
1 st	A	B	C	A
2 nd	B	C	B	C
3 rd	C	A	A	B

What's a majority?



Number of votes	12	10	9	5
1 st	A	B	B	A
2 nd	B	A	A	B

Who is the plurality winner?

Who is the Borda count winner?

	3rd	2nd	1st	Total
A	19	0	51	
B	5	42	30	
C	12	30	27	

The Pairwise Comparison Method:

The candidates are compared in pairs. If a candidate is preferred over another, then the preferred candidate receives 1 point. If it's a tie, then both candidates receive $\frac{1}{2}$ point. The candidate with the most total points is the winner.

Example: In the following election, who is the pairwise comparison winner?



Number of votes	17	10	9
1 st	A	B	C
2 nd	C	C	B
3 rd	B	A	A



Pair	Comparison	Points
A vs. B	A is preferred to B by 17. B is preferred to A by 19.	B gets 1 point, A gets 0 points
A vs. C	A is preferred to C by 17. C is preferred to A by 19.	C gets 1 point, A gets 0 points
B vs. C	B is preferred to C by 10. C is preferred to B by 26.	C gets 1 point, B gets 0 points

	A	B	C
Total	0	1	2

Who is the plurality winner?

Number of votes	17	10	9
1st	A	B	C
2nd	C	C	B
3rd	B	A	A

Who is the Borda count winner?

	3rd	2nd	1st	Total
A	19	0	51	
B	17	18	30	
C	0	54	27	

Who is the plurality-with-elimination winner?

All of the methods of determining the winner of an election can fail. Here are some simple examples:

1. If two or more candidates all receive the largest number of first-place votes, then the Plurality Method fails.
2. If two or more candidates all receive the largest total number of Borda points, then the Borda Count Method fails.
3. If two or more candidates all receive the largest total number of points in the pairwise comparisons, then the Pairwise Comparison Method fails.
4. If the election has just two candidates and both receive the same number of first-place votes, then the Plurality with Elimination Method fails.

